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SECOND EDITION
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APPLIED MATHEMATICS FOR TODAY: AN INTRODUCTION

Second Edition

D. DOTTORI
G. KNILL
R. McVEAN
J. SEYMOUR

ISBN 0-07-077842-6

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of the National Symbol for Metric Conversion"

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Government Specifications Board."

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Second Edition

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McGRAW-HILL RYERSON LIMITED

Toronto Montreal New York St. Louis San Francisco
Auckland Bogota´ Guatemala Hamburg
Lisbon London Madrid Mexico New Delhi Panama Paris
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APPLIED MATHEMATICS FOR TODAY: AN INTRODUCTION, SECOND EDITION

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ISBN 0-07-077842-6

8 9 10 AP 9 8 7

Illustrations by Samuel Daniel

Printed and bound in Canada

Canadian Cataloguing in Publication Data

Main entry under title:

A M T, APPLIED MATHEMATICS FOR TODAY: AN INTRODUCTION

THE RYERSON MATHEMATICS PROGRAM

INCLUDES INDEX

ISBN 0-07-077842-6

1. MATHEMATICS — 1961- I. DOTTORI, DINO, 1937-
II. TITLE: APPLIED MATHEMATICS FOR TODAY: AN INTRODUCTION

QA39.2.D678 1980 510 C80-094138-1

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The metric usage in this text has been reviewed by the Metric Screening Office of the Canadian Government Specifications Board.

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REVIEW AND PREVIEW TO CHAPTER 1

DECIMAL FRACTIONS

EXERCISE 1 *Addition and Subtraction*

1. Add

(a) $\begin{array}{r}31.3\\122.5\\\hline\end{array}$ (b) $\begin{array}{r}12.920\\113.756\\38.209\\\hline\end{array}$ (c) $\begin{array}{r}0.725\\1.983\\21.007\\\hline\end{array}$ (d) $\begin{array}{r}8.0062\\0.0981\\17.9273\\\hline\end{array}$

(e) $\begin{array}{r}101.76\\9.03\\8.88\\\hline\end{array}$ (f) $\begin{array}{r}2.01\\56.83\\12.95\\127.88\\\hline\end{array}$ (g) $\begin{array}{r}9.98\\0.23\\7.01\\12.06\\\hline\end{array}$ (h) $\begin{array}{r}157.2\\3.8\\1037.2\\8.6\\\hline\end{array}$

(i) $\begin{array}{r}126.102\\75.398\\0.013\\16.237\\\hline\end{array}$ (j) $\begin{array}{r}918.1\\10\ 376.2\\4.9\\106.7\\\hline\end{array}$

2. Subtract

(a) $\begin{array}{r}41.9\\36.2\\\hline\end{array}$ (b) $\begin{array}{r}107.8\\89.9\\\hline\end{array}$ (c) $\begin{array}{r}1.006\\0.908\\\hline\end{array}$ (d) $\begin{array}{r}5073.29\\0.86\\\hline\end{array}$

(e) $\begin{array}{r}18.357\\9.270\\\hline\end{array}$ (f) $\begin{array}{r}56.923\\31.886\\\hline\end{array}$ (g) $\begin{array}{r}482.360\\7.092\\\hline\end{array}$ (h) $\begin{array}{r}2.0096\\0.8500\\\hline\end{array}$

(i) $\begin{array}{r}100.73\\10.89\\\hline\end{array}$ (j) $\begin{array}{r}12.008\\0.569\\\hline\end{array}$

3. Simplify

(a) 10.036 − 5.092 (b) 4.029 + 7.63 + 88.95
(c) (5.07 + 32.5) − 17.3 (d) 156.88 − (42.93 + 71.26)
(e) 5.661 + 7.29 + 0.931 + 100.035
(f) 86.52 − 18.91 − 7.03
(g) Subtract 0.7356 from 0.9583
(h) Subtract 0.5736 from 1
(i) Subtract 63.091 from the sum of 56.013 and 11.987

4. Simplify

(a) 16.375 − 8.125 (b) 6.375 − 8.125
(c) 6.25 − (3.5 + 1.7) (d) (7.63 + 2.15) − 5.63
(e) 1 − 0.6715 (f) 9.276 − 1
(g) (3.275 + 4.281) − (1.175 + 4.108)
(h) 68.57 + 21.65 − 75.74 + 8.19
(i) Subtract the sum of 577.8 and 121.9 from the sum of 847.1 and 257.6.

EXERCISE 2 *Multiplication and Division*

1. Complete the table in your notebook.

n	$100n$	$10n$	$0.1n$	$0.001n$
12	1200		1.2	
35.1				
	586			
				0.0643
		83.95		
			0.37	

2. Multiply

(a) $\begin{array}{r} 16.8 \\ 3 \\ \hline \end{array}$ (b) $\begin{array}{r} 156.2 \\ 12 \\ \hline \end{array}$ (c) $\begin{array}{r} 0.0392 \\ 25 \\ \hline \end{array}$ (d) $\begin{array}{r} 893.2 \\ 5.7 \\ \hline \end{array}$

(e) $\begin{array}{r} 0.0911 \\ 15.3 \\ \hline \end{array}$ (f) $\begin{array}{r} 163.89 \\ 3.24 \\ \hline \end{array}$ (g) $\begin{array}{r} 0.0038 \\ 0.95 \\ \hline \end{array}$ (h) $\begin{array}{r} 0.9172 \\ 56.3 \\ \hline \end{array}$

(i) $\begin{array}{r} 4.073 \\ 5.91 \\ \hline \end{array}$ (j) $\begin{array}{r} 0.5692 \\ .918 \\ \hline \end{array}$

3. Divide

(a) $\frac{148}{8}$ (b) $\frac{71.12}{5.6}$ (c) $\frac{0.5389}{17}$ (d) $\frac{55.3}{1.75}$

(e) $\frac{0.1967}{35}$ (f) $\frac{351.45}{0.45}$ (g) $\frac{0.004\ 56}{0.57}$ (h) $\frac{0.8778}{16.5}$

(i) $\frac{611.1}{0.0105}$ (j) $\frac{513.52}{0.655}$

4. Simplify

(a) 43.56×5.002 (b) 0.932×5.17

(c) $(2.640 \div 1.5) \times 8.96$ (d) $(83.06 + 1.532) \times 0.073$

(e) $\frac{2.4 \times 0.077}{0.0005}$ (f) $\frac{205.53}{0.85 \times 62}$

(g) $0.756 \times (1.513 \div 0.89)$ (h) $(17.3 \times 0.0728) \div 0.052$

EXERCISE 3 *Problems Using Decimal Fractions*

English	62
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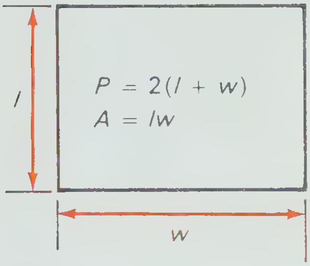
1. Calculate the average of the marks listed at left (to the nearest tenth).

2. To assemble 5 electrical units requires 13.5 h. How much time is required to assemble one unit?

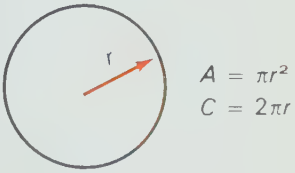
3. Complete the table in your notebook for the given rectangles.

Length cm	Width cm	Perimeter cm	Area cm²
4.3	7.1		
1.75	21.3		
0.9			1.17
	7.3		109.5
	10.3	42.8	

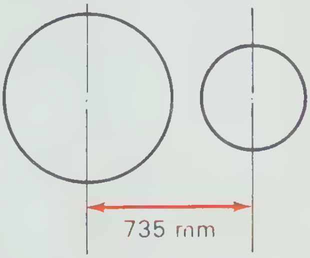


4. A stack of 1728 sheets of steel is 2100 mm high. How thick is each sheet?

5. Calculate the area and the circumference of a circle with radius 0.9 cm. (Use $\pi = 3.14$)



6. The distance shown between the centres of the pulleys at right may vary ±12.5 mm. Determine the maximum and minimum distances between the centres.



7. A water bed has a volume of 0.462 m³. If the density of water is 985 kg/m³ at room temperature, calculate the mass of the bed to the nearest kilogram.

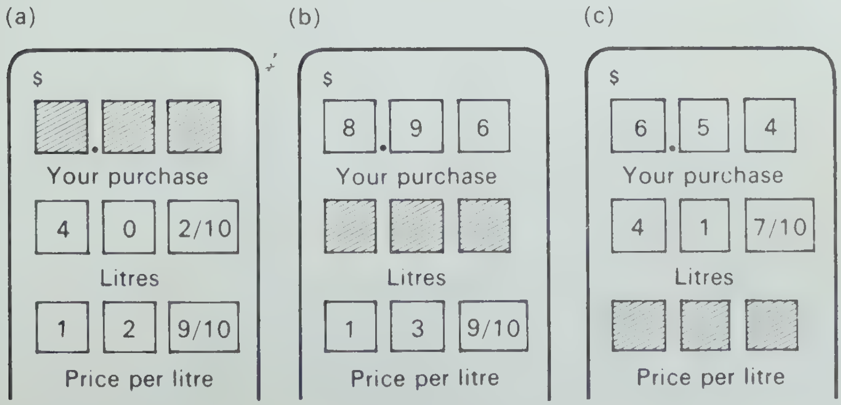
8. If a motorist travels 298.7 km using 65.8 L of gasoline, how many litres of gasoline does he use travelling 100 km?

9. A man earns $7.23/h.

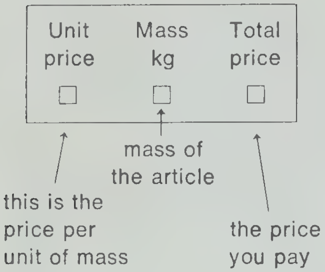
(a) Calculate his total wages for a 40 h week.

(b) If he is paid time and a half for overtime, calculate his wages for a 45 h week.

10. Calculate the missing values in each of the gasoline pump indicators shown below.



gasoline pumps



11. Many articles sold by mass are marked by a sticker similar to the one shown at left. Calculate the missing quantity on each of the stickers.

(a)

Unit price	Mass kg	Total price
$1.19	1.75	▨

(b)

Unit price	Mass kg	Total price
▨	4.65	$4.98

(c)

Unit price	Mass kg	Total price
$0.89	▨	$3.38

(d)

Unit Price	Mass kg	Total Price
▨	3.45	$18.94

EXERCISE 4 *Percent*

1. Compute the following percentages.

(a) 25% of $18.50
(b) 50% of $85.50
(c) 75% of $24.95
(d) 7% of $9.95
(e) 10% of $37.80
(f) 20% of $219.60
(g) 70% of $105.00
(h) 85% of $75.22
(i) 150% of $28.60
(j) 120% of $47.50
(k) 33$\frac{1}{3}$% of $69.95
(l) 66$\frac{2}{3}$% of $99.95

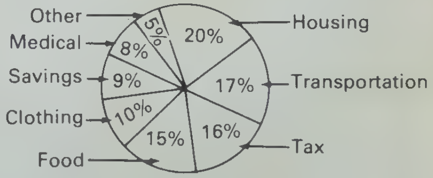
2. Some companies charge 1$\frac{1}{2}$% of the unpaid balance for extending credit. Find the credit charge for each of the following unpaid balances.

(a) $9.95
(b) $108.57
(c) $256.80
(d) $63.78
(e) $2175.69
(f) $3751.50
(g) $1046.80
(h) $58.14
(i) $857.29
(j) $65.13
(k) $63.67
(l) $64.72

3. Ace Stationery Supply has a delivery van that cost $7857.00 when new. Find the value of the van after 3 years if it depreciated at 30% per year.

4. The Jones family buys materials to build a cottage for $7659.00. The materials are subject to 11% Federal tax followed by 7% Provincial tax. How much did the Jones family have to pay for the materials?

5. Ron and Cecile Labelle budget their money as shown. How much do they spend on each item if their total annual income is $37 585?



6. Express the first number as a percent of the second.

(a) $21.80, $43.60
(b) $33.32, $99.95
(c) $6.50, $20.00
(d) $15.39, $69.95
(e) $849.15, $4995
(f) $937.65, $3125.50
(g) $150, $100
(h) $112.80, $95.60

CHAPTER 1

Number Applications

1.1 COMPUTATIONS

The hand held calculator sold at a reasonable cost has made accurate and rapid computation possible for everyone. However, while calculators have provided speed and accuracy, it is still the user who determines what buttons will be pressed. The following sequence of operations is suitable for most machines.

Problem: 26.8 + 35.7 − 42.6

Buttons Pressed	*Display*
C	0.00
2 6 • 8	26.8
+	
3 5 • 7	35.7
−	62.5
4 2 • 6	42.6
=	19.9

Problem: 38.6 ÷ 104

Buttons Pressed	*Display*
C	0.00
3 8 • 6	38.6
÷	
1 0 4	104
=	0.371 153 8

Problem: 6.25 × 0.217

Buttons Pressed	*Display*
C	0.00
6 • 2 5	6.25
X	
• 2 1 7	0.217
=	1.356 25

EXERCISE 1-1

1. Find the indicated sum or difference.

(a) $\begin{array}{r} 3125 \\ +\ 4862 \\ \hline \end{array}$ (b) $\begin{array}{r} 127.8 \\ -\ \ 63.5 \\ \hline \end{array}$ (c) $\begin{array}{r} 78.75 \\ +\ 18.92 \\ \hline \end{array}$ (d) $\begin{array}{r} 5.125 \\ +\ 17.63 \\ \hline \end{array}$ (e) $\begin{array}{r} 2118 \\ -\ 1315 \\ \hline \end{array}$

(f) $\begin{array}{r} 63.7 \\ +\ 21.54 \\ \hline \end{array}$ (g) $\begin{array}{r} 85.9 \\ -\ 28.75 \\ \hline \end{array}$ (h) $\begin{array}{r} 32.75 \\ -\ 18.6 \\ \hline \end{array}$ (i) $\begin{array}{r} 0.3711 \\ -\ 0.1004 \\ \hline \end{array}$ (j) $\begin{array}{r} 0.2578 \\ +\ 0.0173 \\ \hline \end{array}$

2. Find the indicated product or quotient.

(a) $\begin{array}{r} 5683 \\ \times\ 2114 \\ \hline \end{array}$ (b) $\dfrac{2168}{125}$ (c) $\dfrac{32.57}{8.75}$ (d) $\begin{array}{r} 63.75 \\ \times\ 1.375 \\ \hline \end{array}$ (e) $\begin{array}{r} 58.6 \\ \times\ 21.25 \\ \hline \end{array}$

(f) $\dfrac{63.25}{14.7}$ (g) $\begin{array}{r} 0.683 \\ \times\ 1.257 \\ \hline \end{array}$ (h) $\begin{array}{r} 0.7981 \\ \times\ 0.5613 \\ \hline \end{array}$ (i) $\begin{array}{r} 0.7534 \\ 0.0485 \\ \hline \end{array}$ (j) $\begin{array}{r} 6.175 \\ \times\ 4.358 \\ \hline \end{array}$

3. Express the following times as decimals.
Examples: (i) $3\frac{1}{4}$ h = 3.25 h
(ii) 5 h 45 min = $5\frac{45}{60}$ h = 5.75 h

(a) $3\frac{1}{2}$ h (b) $7\frac{3}{4}$ h (c) $5\frac{3}{5}$ h (d) $6\frac{1}{4}$ h
(e) 30 min (f) 48 min (g) 15 min (h) 24 min
(i) 3 h 36 min (j) 8 h 12 min (k) 5 h 48 min (l) 1 h 24 min

4. To find a person's wages we multiply the hourly rate by the number of hours worked. Find the following wages.
(a) \$4.00/h for 40 h (b) \$3.80/h for 25 h
(c) \$9.60/h for 36 h (d) \$8.55/h for 34 h
(e) \$8.40/h for $20\frac{1}{2}$ h (f) \$6.00/h for 18 h 30 min
(g) \$7.20/h for $7\frac{1}{4}$ h (h) \$5.50/h for 25 h 30 min

5. Find the hourly rate for the following jobs.
(a) \$280 wages earned for 40 h work.
(b) \$221.00 wages earned for 34 h work.
(c) \$270.00 wages earned for 25 h work.
(d) \$281.52 wages earned for 36 h work.
(e) \$200.70 wages earned for 18 h work.
(f) \$177.63 wages earned for 24.5 h work.

6. Find the yearly wages for the following jobs at 40 h per week for 52 weeks including holidays with pay.
(a) \$210 per week (b) \$7.50/h
(c) \$4.25/h (d) \$6.80/h
(e) \$9.25/h (f) \$10.80/h
(g) \$980 per month (h) \$325 every two weeks
(i) \$485 twice a month (j) \$12.55/h

7. Which of the following is the best rate of pay for 40 h per week for 52 weeks including holidays?
(a) \$16 000 annual salary
(b) \$1335 per month
(c) \$665 twice a month
(d) \$630 every two weeks
(e) \$310 per week
(f) \$7.75/h

1.2 ORDER OF OPERATIONS

When several mathematical operations are to be performed, the result depends on the order in which the operations are performed. We can illustrate this by interpreting

$3 + 2 \times 4$ in two ways:

$$\begin{aligned} & 3 + 2 \times 4 \\ &= 3 + 8 \\ &= 11 \end{aligned} \qquad \text{or} \qquad \begin{aligned} & 3 + 2 \times 4 \\ &= 5 \times 4 \\ &= 20 \end{aligned}$$

Which answer is correct?

To eliminate confusion, mathematicians have agreed to perform the operations in the following order:

B	Perform operations in brackets first.
E	Evaluate all powers — exponents.
D M	Perform division and/or multiplication in the order in which they appear from left to right.
A S	Perform addition and/or subtraction in the order in which they appear from left to right.

EXAMPLE 1. *Simplify* (a) $(11 + 4) \times 2 \div 3 - (12 - 2) \div 5$
(b) $12 \times 6 + 10 \div 5 - 3\,[4 - \frac{1}{2}(10 - 4)]$

Solution When there are two levels of brackets, work on the inner brackets first.

(a)
$$\begin{aligned} & \underbrace{(11 + 4)} \times 2 \div 3 - \underbrace{(12 - 2)} \div 5 \\ =\ & \underbrace{15 \times 2} \div 3 - 10 \div 5 \\ =\ & \underbrace{30 \div 3} - 10 \div 5 \\ =\ & 10 - \underbrace{10 \div 5} \\ =\ & \underbrace{10 - 2} \\ =\ & 8 \end{aligned}$$

$$
\begin{aligned}
(b)\quad & 12 \times 6 + 10 \div 5 - 3\left[4 - \tfrac{1}{2}\underbrace{(10 - 4)}\right] \\
&= 12 \times 6 + 10 \div 5 - 3\left[4 - \underbrace{\tfrac{1}{2} \times 6}\right] \\
&= 12 \times 6 + 10 \div 5 - 3\left[\underbrace{4 - 3}\right] \\
&= 12 \times 6 + 10 \div 5 - \underbrace{3\left[1\right]} \\
&= \underbrace{12 \times 6} + 10 \div 5 - 3 \\
&= 72 + \underbrace{10 \div 5} - 3 \\
&= \underbrace{72 + 2} - 3 \\
&= \underbrace{74 - 3} \\
&= 71
\end{aligned}
$$

EXERCISE 1-2

3^2
$= 3 \times 3$
$= 9$

A

1. Simplify.

(a) $3 \times 2 - 5$
(b) $7 - 3 \times 2$
(c) $5 - 2 + 3$
(d) $5 - (2 + 3)$
(e) $6 \times 3 - 5$
(f) $(6 - 5) \times 3$
(g) $3^2 - 5$
(h) $4^2 - (5 - 3)$
(i) $(2 + 3)^2$
(j) $2^2 + 3^2$
(k) $18 - 2^3 + 2 \times 3$
(l) $125 \div 25 \times 2^2$

B

2. Simplify.

(a) $3 + [2 + (7 - 4)]$
(b) $8 - [6 - (10 - 7)]$
(c) $25 \div 5 + 3 \times 7$
(d) $63 \div 9 - 5 \times 2 + 3$
(e) $65 \div (5 + 24 \div 3)$
(f) $(27 - 2)(12 \div 4) \div (11 + 4)(2 + 7)$
(g) $(23 - 3)(15 - 5) \div (6 + 4)$
(h) $(25 + 3) \div (9 - 2)(8 \div 4)$
(i) $\dfrac{5 + 3(2 - 7 + 9)}{17}$
(j) $\dfrac{3(5 + 2) - 3(7 - 5)}{5}$
(k) $7 + \dfrac{1 - 2(6 - 7)}{3}$
(l) $19 - \dfrac{(15 \div 5 + 3)}{(8 - 5)}$

3. Simplify.

(a) $[(28 + 5) \div 3 - (16 - 4) \div 4] + 2$
(b) $\frac{3}{4}(5^2 + 11) - \frac{2}{3}(3^2 + 3)$
(c) $3(5^2 - 4^2) + 2(3^2 - 2^2)$
(d) $(6 \div 3 - 2)(5 + 4 - 7)$
(e) $(8 \times 5 \div 4)^2 - (8 \div 4 + 1)^3$
(f) $144 \div 12 \times (5 - 2)$

4. Simplify using a calculator.

(a) $38.75 - (63.71 - 41.65)$
(b) $(915.2 + 697.3) \div 31.25$
(c) $\dfrac{(64.2 + 20.45) \times 5.375}{21.62}$

(d) $3.751(25 + 63) \div 110 - 0.008$
(e) $83.25 + 5[78.6 - (21.83 + 37.6)]$
(f) $\left(13.75 + \frac{275}{4}\right) \times \left(\frac{8.25 + 13.25}{2}\right)$

C 5. Evaluate the following for the given values of the variables.
(a) $P = 2(l + w)$; $l = 35.6$, $w = 21.7$
(b) $S = \frac{1}{2}(a + 1)n$; $a = 2$, $l = 100$, $n = 50$
(c) $S = \frac{1}{2}n(n + 1)$; $n = 100$
(d) $d = vt + 4.9t^2$; $v = 6.3$, $t = 5.5$
(e) $D = \sqrt{l^2 + w^2 + h^2}$; $l = 12$, $w = 3$, $h = 4$
(f) $S = \frac{r^n - 1}{r - 1}$; $r = 2$, $n = 10$

6. Typing speed is calculated using the formula

$$S = \frac{w - 10e}{m}$$

where w is the number of words
e is the number of errors
m is the time in minutes.

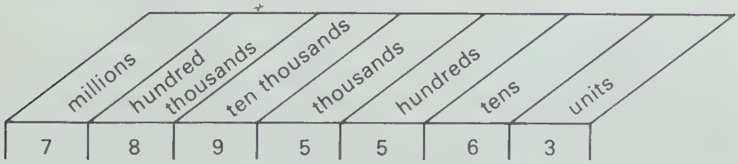
Find the typing speed of the following.

Words w	Number of errors e	Time in Minutes m	Typing Speed s
300	2	5	
400	5	10	
450	10	10	
500	8	12	

1.3 ROUNDING

Numbers are used to describe quantities in two ways:
(i) as an exact number which results from counting;
(ii) as an approximate number which results from measurement or estimation.

These numbers can be rounded to a measure of accuracy determined by the use of the number. When we say the population of New York is 8 000 000 we are using a number rounded to the nearest million.



We can round the number	7 895 563
to the nearest million	8 000 000
to the nearest hundred thousand	7 900 000
to the nearest ten thousand	7 900 000
to the nearest thousand	7 896 000
to the nearest hundred	7 895 600
to the nearest ten	7 895 560

The general rules for rounding are:
1. If the digits to be dropped begin with 5 or more, increase the last significant digit by 1.
2. If the digits to be dropped begin with 4 or less, the digits are dropped with no change.

EXAMPLE 1. *Round off the value 6.3946*
(a) to the nearest hundredth
(b) to the nearest tenth

Solution

(a) 6.39 | 46 ≐ 6.39
less than 5,
drop digits

(b) 6.3 | 946 ≐ 6.4
5 or more,
add 1

EXERCISE 1-3

A 1. Round off to the nearest cent.
(a) $1.2543 (b) $84.6557 (c) $12.8148 (d) $6.2851
(e) $4.775 (f) $5.317 (g) $61.208 (h) $8.753
(i) $9.718 (j) $8.5151 (k) $8.256 (l) $9.5555

2. Round off to one decimal place (tenths).
(a) 6.375 (b) 0.2163 (c) 5.218 47 (d) 62.315 206
(e) 27.205 (f) 32.251 (g) 69.597 (h) 89.975
(i) 99.999 (j) 8.0527 (k) 3.7124 (l) 91.508 21

3. Round off to the nearest ten.
(a) 2751 (b) 63.98 (c) 557.35 (d) 219.97
(e) 6227 (f) 27.354 (g) 10.751 (h) 439.428
(i) 5019 (j) 228.051 (k) 192.604 (l) 50.721

4. Round off to the nearest unit.
(a) 2132 (b) 54.785 (c) 7.815 (d) 61.025
(e) 50.975 (f) 43.521 (g) 0.9875 (h) 69.5102
(i) 3.4991 (j) 21.0634 (k) 0.1025 (l) 117.251

5. Round off to the nearest hundred.
(a) 25 432 (b) 165 756 (c) 248.725 (d) 3182.99
(e) 495.95 (f) 749.50 (g) 1595.20 (h) 1949.50
(i) 2106.9 (j) 351.25 (k) 289.95 (l) 425 757

6. Round off to the nearest hundredth.
(a) 10.725 54 (b) 5.025 75 (c) 16.371 (d) 12.405
(e) 0.125 75 (f) 0.071 25 (g) 0.045 65 (h) 3.9765
(i) 12.4894 (j) 128.462 (k) 465.564 (l) 158.815

7. The circumference of the earth at the equator is approximately 39 960 km. Express this value correct to
(a) the nearest 10 km
(b) the nearest 100 km
(c) the nearest 1000 km

1.4 ESTIMATING

A quick estimate is helpful to locate a decimal, to check on calculations, or to provide an approximate cost of a job or business transaction.

To estimate, we round the numbers to the highest place value, and perform the operations mentally.

EXAMPLE 1. *Estimate the answers.*

(a) 569×0.315 (b) $\dfrac{63.71}{0.0283}$ (c) $\dfrac{41.2 \times 27}{6110}$

Solution We round to the highest place value.

(a) $569 \times 0.315 \doteq 600 \times 0.3$
$\doteq 180$

actual

179.29

(b) $\dfrac{63.71}{0.0283} \doteq \dfrac{60}{0.03}$
$\doteq 2000$

actual

2251.2367

(c) $\dfrac{41.2 \times 27}{6110} \doteq \dfrac{40 \times 30}{6000}$

$\doteq \dfrac{4 \times 3}{60}$

$\doteq \dfrac{12}{60}$

$\doteq 0.2$

actual

0.1820622

EXERCISE 1-4

A **1.** Estimate the answers.

(a) 3.2×6.9 (b) $142 \div 7.3$ (c) 18.9×5.01
(d) $(4.8)^2$ (e) $5.8 \div 2.97$ (f) $14.04 \div 6.852$
(g) $(3.05)^3$ (h) 147.8×1.92 (i) $17.76 \div 5.81$
(j) $28 \times 42 \div 11$ (k) $296 \div 11.5$ (l) 0.895×31.25

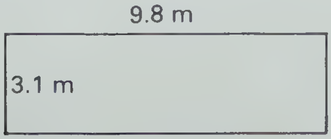
2. Estimate the answers and check with a calculator.

(a) $\dfrac{512 \times 296}{311}$ (b) $\dfrac{18.75 \times 41.6}{83.2}$
(c) $\dfrac{21.6 + 39.7 + 11.4}{68.7}$ (d) $\dfrac{53.2 \times 68.4}{81.1 + 18.5}$
(e) $\dfrac{63.5 + 21.2}{11.5 \times 0.965}$ (f) $\dfrac{(3.165 + 18.2) \div 4.85}{124.6 \div 5.12}$

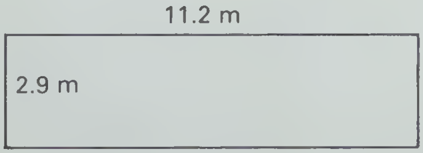
3. It costs \$28/m² to pave a driveway. Give estimates of the cost for paving the following driveways.

(a) 30 m² (b) 48 m² (c) 28 m² (d) 21 m²

(e)



(f)



4. At Fashion Textiles, velour costs \$9.95/m. Estimate the total cost of the following bolts of material.
(a) 14.5 m (b) 17.8 m (c) 22.1 m (d) 28.6 m (e) 19.1 m

5. Estimate the cost of copper pipe at \$1.82/m for the following lengths
(a) 120 pieces at 5 m each
(b) 93 pieces at 3 m each

6. Estimate the cost of
(a) 195 stacking chairs at \$29.95 each
(b) 3125 ceiling tiles at $51\frac{1}{2}$¢ each

7. Estimate the total number of people who attended from the following average attendances:
(a) 7 home games of NFL football at 56 218 per game.
(b) 81 home games of AL baseball at 20 733 per game.
(c) 38 home games of NHL hockey at 11 842 per game.
(d) 41 home games of NBA basketball at 10 361 per game.

8. Canadians save an average of 11.2% of their take home pay. Estimate the amount each of the following saves, given the take home pay shown:
(a) Jones \$9802
(b) Smith \$12 437
(c) Kostyk \$10 851
(d) Kelly \$9127

1.5 EXPONENTS

The product $2 \times 2 \times 2$ can be written using exponents.

$$\underbrace{2 \times 2 \times 2}_{\text{3 factors}} = 2^3 \qquad \left\{\begin{array}{ll} 2^3 & \text{power} \\ 2 & \text{base} \\ 3 & \text{exponent} \end{array}\right.$$

3^5 means $3 \times 3 \times 3 \times 3 \times 3 = 243$
5^3 means $5 \times 5 \times 5 = 125$

We can write 72 by listing the prime factors and using exponents.

$$\begin{aligned} 72 &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^3 \times 3^2 \end{aligned}$$

Beginning with $10^5 = 10 \times 10 \times 10 \times 10 \times 10$ we reduce the *exponent* by one, by dividing the right side by 10. Thus:

$$\begin{aligned} 10^4 &= 10 \times 10 \times 10 \times 10 \\ 10^3 &= 10 \times 10 \times 10 \\ 10^2 &= 10 \times 10 \end{aligned}$$

Continuing the process —

$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = \frac{1}{10}$$
$$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$$
$$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$$

This leads to two special definitions

$10^0 = 1$	and	$10^{-n} = \frac{1}{10^n}$
$a^0 = 1$		$a^{-n} = \frac{1}{a^n}$

EXERCISE 1-5

A **1.** Express each of the following using exponents.

(a) $3 \times 3 \times 3 \times 3$ (b) $4 \times 4 \times 4$ (c) $5 \times 5 \times 5 \times 5$

(d) $(-3)(-3)(-3)(-3)(-3)$ (e) $7 \times 7 \times 7 \times 7 \times 7$

(f) $9 \times 9 \times 9$ (g) 5 (h) $2 \times 2 \times 2 \times 2 \times 2 \times 2$

2. Evaluate:

(a) 3^3 (b) 5^2 (c) 7^2 (d) 7^3

(e) 5^3 (f) 2^6 (g) 2^4 (h) 4^2

(i) 8^2 (j) 10^4 (k) 1^5 (l) 0^7

(m) 7^0 (n) 2^{-1} (o) 10^{-3} (p) 3^{-2}

3. List the prime factors of each of the following using exponents.

(a) 12 (b) 36 (c) 800 (d) 400

(e) 90 (f) 144 (g) 63 (h) 225

(i) 108 (j) 2500 (k) 96 (l) 720

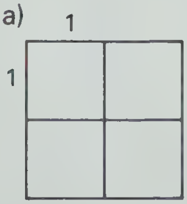
4. Simplify.

(a) $3^2 + 4^2$ (b) $5^2 + 12^2$ (c) $7^2 + 24^2$ (d) $8^2 + 15^2$

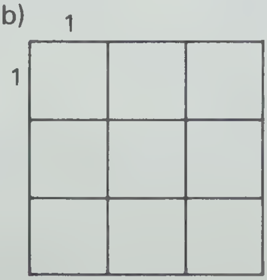
(e) $3^3 + 4^3$ (f) $5^3 + 12^3$ (g) $7^3 + 24^3$ (h) $8^3 + 15^3$

B **5.** Write the area of each of the following using exponents.

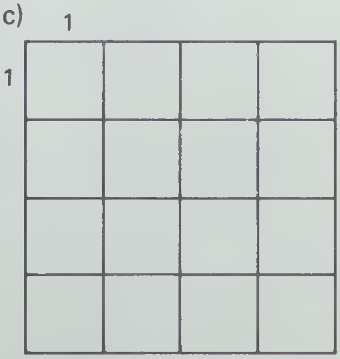
(a)



(b)



(c)



C 6. Use your calculator to compute the following to seven significant figures.

(a) 1.05^{12} (b) 1.08^{15} (c) 1.04^{10}

(d) 1.015^{8} (e) 1.012^{10} (f) 1.014^{12}

(g) $\frac{1}{1.07^{10}}$ (h) $\frac{1}{1.015^{10}}$ (i) $\frac{1}{1.045^{12}}$

1.6 SIGNIFICANT DIGITS AND STANDARD NOTATION

The accuracy of any number is often indicated by stating the number of significant digits in the number.

42.6 is correct to 3 significant digits } means { the value is closer to 42.6 than to 42.5 or 42.7

In certain values, the number of significant digits is indicated simply by writing the number.

Value	Number of Significant Digits
75	2
302.65	5
0.009 35	3
5	1

The use of zero in a number can cause some confusion in determining the number of significant digits:

1570 can mean either

(a) closer to 1570 than to 1569 or 1571
(b) closer to 1570 than to 1560 or 1580

$10^3 = 1000$
$10^2 = 100$
$10^1 = 10$
$10^0 = 1$

$10^{-1} = \frac{1}{10}$
$10^{-2} = \frac{1}{100}$
$10^{-3} = \frac{1}{1000}$
$10^{-4} = \frac{1}{10\,000}$

To indicate clearly the number of significant digits, the number may be expressed in standard notation.

Standard Notation

$1570 = \boxed{1.570} \times \boxed{10^3}$ indicates 4 significant digits
$1570 = \boxed{1.57} \times \boxed{10^3}$ indicates 3 significant digits

a value between 1 and 10

a power of 10 to locate the decimal

Standard notation is also used to express awkward numbers in a simpler form.

EXAMPLE 1. *Express the following numbers in standard notation.*

(a) 15 750 000 000 (to 4 significant digits)
(b) 0.000 000 72 (to 2 significant digits)

Solution (a)
$$\begin{aligned} 15\,750\,000\,000 &= 1.575 \times 10\,000\,000\,000 \\ &= 1.575 \times 10^{10} \end{aligned}$$

(b)
$$\begin{aligned} 0.000\,000\,72 &= 7.2 \times 0.000\,000\,01 \\ &= 7.2 \times \frac{1}{10\,000\,000} \\ &= 7.2 \times \frac{1}{10^7} \\ &= 7.2 = 10^{-7} \end{aligned}$$

EXAMPLE 2. *Express the value 500 in standard notation to indicate:*

(a) 3 significant digits
(b) 2 significant digits
(c) 1 significant digit

Solution

(a) $500 = 5.00 \times 10^2$
(b) $500 = 5.0 \times 10^2$
(c) $500 = 5 \times 10^2$

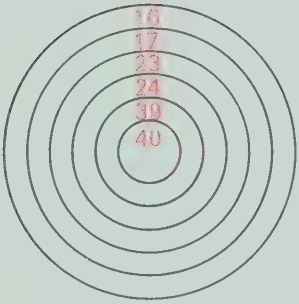
EXERCISE 1-6

A **1.** Classify as an exact or approximate number.

(a) The planet Pluto is 5 910 000 000 km from earth.
(b) The speed of light is 299 800 km/s.
(c) The number of seats in the stadium is 31 316.
(d) The thickness of a sheet of paper is 0.0006 mm.
(e) The total offence of the football team was 382 m.
(f) This book contains 13 chapters.

2. State the number of significant digits indicated for each of the following numbers.

(a) 36.2 (b) 10.56 (c) 2 (d) 0.0637
(e) 51 625 (f) 3.2×10^5 (g) 5.11×10^{-7}
(h) 7.0×10^3 (i) 45 012 (j) 3.00×10^{-2}



B 3. Complete the following table.

	Value	Number of Significant Digits	Standard Notation
(a)	5 600 000	2	
(b)	86.1		
(c)		3	1.25×10^3
(d)	0.0036		
(e)			4.316×10^{-1}
(f)	5.1	2	
(g)	0.015 60	4	
(h)			3.20×10^{-4}

4. Express the following numbers in standard notation.

(a)	893	(b)	16.32	(c)	0.0256
(d)	3 656 000 000	(e)	0.000 32	(f)	0.991
(g)	0.05	(h)	410 000	(i)	0.0630
(j)	201.1	(k)	1600	(l)	0.000 000 357

5. Express each of the following in decimal form.

(a)	3.2×10^2	(b)	5.0×10^{-1}	(c)	8.9×10^{-5}
(d)	3.261×10^3	(e)	4.0×10^{-2}	(f)	5.7×10^0
(g)	2.10×10^{-1}	(h)	1.03×10^{-9}		

REVIEW EXERCISE

1. Perform the following computations.

(a) $\begin{array}{r} 0.265 \\ +\ 46.75 \\ \hline \end{array}$ (b) $\begin{array}{r} 125.7 \\ -\ 32.8 \\ \hline \end{array}$ (c) $\begin{array}{r} 63.275 \\ -\ 14.88 \\ \hline \end{array}$ (d) $\begin{array}{r} 411.75 \\ +\ 6.389 \\ \hline \end{array}$

(e) $\begin{array}{r} 31.65 \\ \times\ 8.275 \\ \hline \end{array}$ (f) $\frac{5.9375}{1.25}$ (g) $\frac{58.175}{8.95}$ (h) $\begin{array}{r} 6.285 \\ \times\ 12.51 \\ \hline \end{array}$

2. Express the following as decimals.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$

(e) $\frac{5}{8}$ (f) $\frac{3}{4}$ (g) $\frac{7}{8}$ (h) $\frac{1}{5}$

(i) $\frac{2}{5}$ (j) $\frac{3}{5}$ (k) $\frac{4}{5}$ (l) $\frac{3}{2}$

3. Express the following as decimals.

(a) $3\frac{3}{4}$ h (b) 5 h 15 min (c) 7 h 24 min (d) 126 min

4. Find the hourly rate:
(a) \$225.60 for 40 h work.
(b) \$380.80 for 34 h work.
(c) \$359.00 for 50 h work.

5. Which pay system pays more money?
(a) \$8.85/h for a 40 h week.
(b) \$765.00 paid twice a month.

6. Simplify.
(a) $5 + 3 \times 2 - 4 \div 2$ (b) $6 \div 3 \times 2 - 3$
(c) $7 + 2(5 - 3) + 4$ (d) $6 - 4(5 - 4) + 3$
(e) $6.25 + 3.7 \times 2.5 - 4.75$ (f) $6.25 \div 0.25 \times 3.1 - 7.8$
(g) $85.3 + 2.5(4.68 + 2.7) - 5.2$ (h) $(5.8 + 2.7)(6.3 - 4.8)$
(i) $5.3^2 - 2.7^2 + 8.4 \times 2.1$ (j) $(6.3 - 2.7)^2 \times (4.8 - 3.1)^2$

7. Round off to the indicated unit.
(a) to 1 decimal place: 3.25, 6.587, 14.058, 91.615
(b) to the nearest ten: 58 525, 6351, 15 854.25, 21.857
(c) to the nearest hundredth: 6.125, 3.141 59, 58.271 43, 0.025 75

8. Round to the indicated accuracy.
(a) three significant digits:
3.141 59, 585 207, 6158.6, 0.125 65, 0.003 725
(b) two significant digits:
25 420 000, 3 185 208, 15.751, 0.381 52, 0.005 555
(c) one significant digit:
6.751, 0.035 275, 48 125 000, 0.55, 1.925

9. Estimate the following:
(a) 5.8×2.1 (b) $151 \div 28$ (c) 18.4×6.2
(d) $\dfrac{2.8 \times 91.5}{32.6}$ (e) $\dfrac{58 \div 4.6}{7.2 - 4}$ (f) $\dfrac{526}{8.5 \times 2.8}$
(g) the cost of 51 m of costume material at \$3.95/m
(h) 88 bags of fertilizer at \$7.95 per bag
(i) 2 season tickets per game for 11 home games at \$8.50 per ticket
(j) the total number of bolts if there are 86 boxes and there were 53 in one box when checked

10. State the number of significant figures indicated by each of these numbers.
(a) 532 (b) 63.1 (c) 16.32 (d) 9.8
(e) 1.03 (f) 0.5 (g) 0.032 (h) 70.02
(i) 3.18×10^2 (j) 21.32×10^8
(k) 7.653×10^{-1} (l) 4.1×10^{-1}

11. Complete the following tables in your books.

(a)

Decimal Value	Standard Notation	
	Three Significant Digits	Two Significant Digits
563.7	5.64×10^2	5.6×10^2
48.63		
0.7325		
193.98		
0.004 982		
7.87		
0.5849		
0.000 910 3		

Test your skill: Divide 300 by $\frac{1}{3}$ and add 3.

(b)

Decimal Value	Standard Notation
36.2	
	4.3×10^5
13 760	
	8.2×10^{-1}
0.0064	
3.9	
	4.75×10^{-5}
	1.86×10^0
0.916	

$10^x \cdot 10^y = 10^{x+y}$
$10^x \div 10^y = 10^{x-y}$

(c)

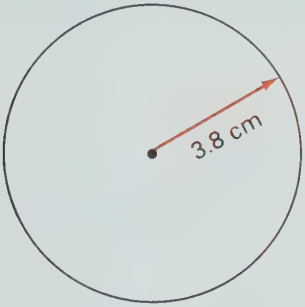
Problem (Decimal Form)	Problem (Standard Notation)	Answer	Answer (Decimal Form)
3100 × 565	$3.1 \times 10^3 \times 5.65 \times 10^2$	1.75×10^6	1 750 000
450 × 56 000			
0.39 × 175 000			
76 × 0.0035			
0.018 × 0.0063			
71.5 ÷ 43 000	$\frac{7.15 \times 10^1}{4.3 \times 10^4}$	1.66×10^{-3}	
56 000 ÷ 1.65			
0.095 ÷ 680			
6.6 ÷ 0.0035			
0.003 92 ÷ 0.000 062			

12. The formula for finding the circumference of a circle is

$$\underset{\text{circumference}}{C} = 2\pi \underset{\text{radius}}{r}$$

Calculate in centimetres the circumference of the circles below.

(a)



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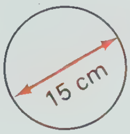
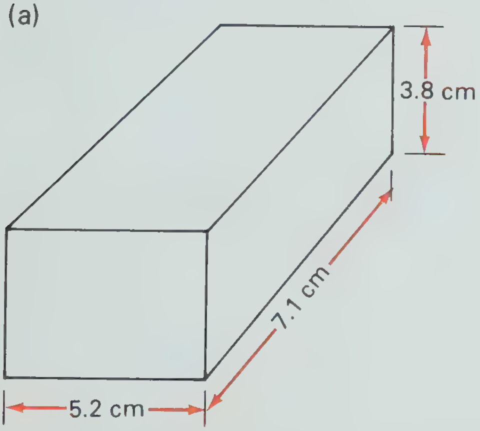


13. Find the volume of each using V = lwh

(a)



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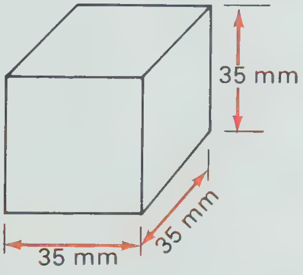
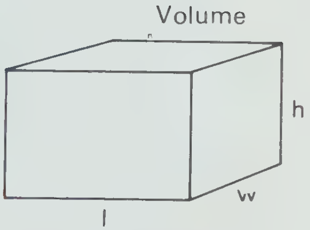
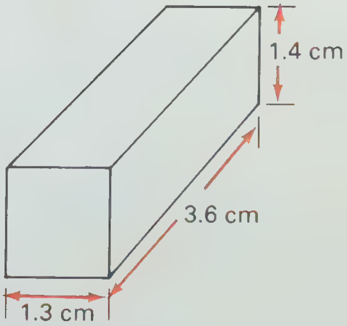




$V = l \times w \times h$
$= lwh$

14. (a) Calculate the area of a rectangle with length 5.5 cm and width 1.8 cm.
(b) Calculate the volume of the box at right.
(c) What is the cost of 4.5 kg of roast beef at $8.49/kg?
(d) If the sum of the 6 marks on John's report card is 372, what is his average mark?
(e) If the radius of a circle is 7.3 cm, what is the area?
(f) A car uses 77.5 L of gasoline on a trip of 504 km. Determine the average number of litres per 100 km consumed.



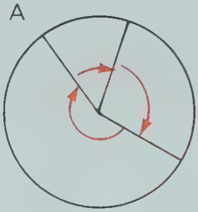
REVIEW AND PREVIEW TO CHAPTER 2

EXERCISE 1

Substitute and evaluate in each of the following (use $\pi = 3.14$)

1. $P = 2l + 2w$;	(a) $l = 5.6$ cm, $w = 3.4$ cm (b) $l = 12.0$ cm, $w = 9.3$ cm
2. $A = \dfrac{h(a+b)}{2}$	(a) $a = 4.2$ cm, $b = 6.8$ cm, $h = 5.0$ cm (b) $a = 14$ cm, $b = 7.2$ cm, $h = 9.0$ cm
3. $C = 2\pi r$	(a) $r = 14.3$ cm (b) $r = 428$ cm
4. $A = \pi(R^2 - r^2)$	(a) $r = 6.3$ cm, $R = 9.4$ cm (b) $r = 12.0$ cm, $R = 13.5$ cm
5. $V = \pi r^2 h$	(a) $r = 3.6$ cm, $h = 9.2$ cm (b) $r = 1.75$ m, $h = 10.2$ m
6. $V = \frac{1}{3}a^2 h$	(a) $a = 755$ m, $h = 482$ m (b) $a = 4.6$ cm, $h = 4.6$ cm
7. $V = \frac{1}{2}abh$	(a) $a = 9.2$ m, $b = 7.4$ m, $h = 0.2$ m (b) $a = 6.0$ cm, $b = 4.1$ cm, $h = 1.2$ cm
8. $\dfrac{A}{\pi r^2} = \dfrac{\theta}{360^\circ}$	(a) $\theta = 110^\circ$, $r = 10$ cm (b) $\theta = 205^\circ$, $r = 10$ cm

Match these figures with the formulae on the left.

A


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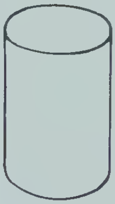

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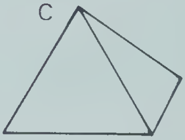

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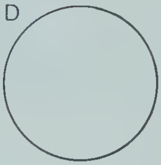

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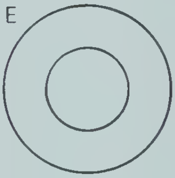

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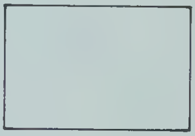

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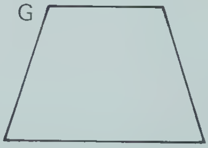

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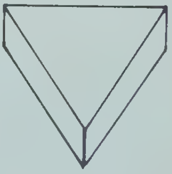


EXERCISE 2

Calculate

1. $$\begin{array}{r} 333.7 \\ 46.9 \\ 298.8 \\ +\ 561.6 \\ \hline \end{array}$$

2. $$\begin{array}{r} 200.3 \\ -\ 149.4 \end{array}$$

3. $$\begin{array}{r} 39.6 \\ \times\ 46.7 \\ \hline \end{array}$$

4. $$\frac{291.81}{21.3}$$

5. $481.7 + 387.9 + 2163.7$

6. $5064.6 - 2938.7$

7. 146.3×11.8

8. $579.88 \div 10.9$

Statistics

Statistics is the branch of mathematics that deals with the collection, organization and interpretation of facts or numbers called data.

2.1 READING AND CONSTRUCTING GRAPHS

Graphs are one way in which statistical information is organized and displayed. We shall study graphing before dealing with the collection of data.

Graphs are used to display data in an eye appealing and informative way. Different types of graphs are used to display different types of data.

A good graph contains:

(i) a title.

(ii) labelled and scaled axes.

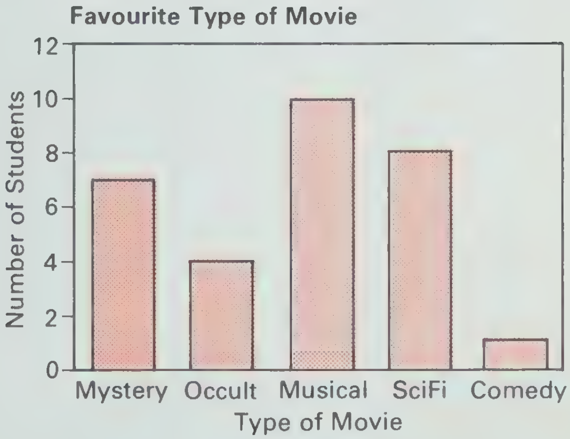
I The Bar Graph

A bar graph is used to *compare similar things.* The bar graph at the right shows the favourite type of movie as selected by members of a class at North High School.

(a) How many students prefer
  (i) mysteries?
  (ii) comedies?
  (iii) musicals?
  (iv) occult?
  (v) science fiction?

(b) Rank the movies in popularity from highest to lowest.

(c) What was the total number of students surveyed?



II The Line Graph

A line graph is used *to show how one thing changes.* The line graph at the right shows the attendance at the Granada Theatre for a one week period.

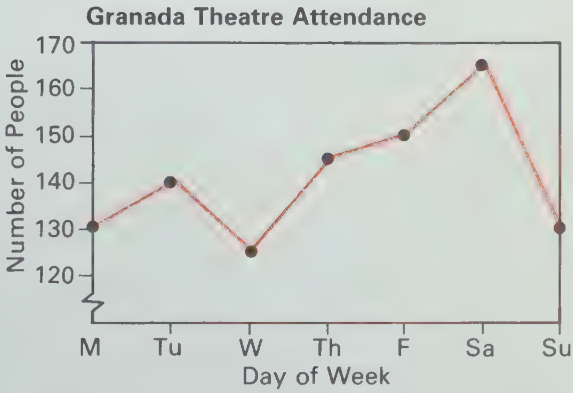
(a) What was the attendance on Tuesday?

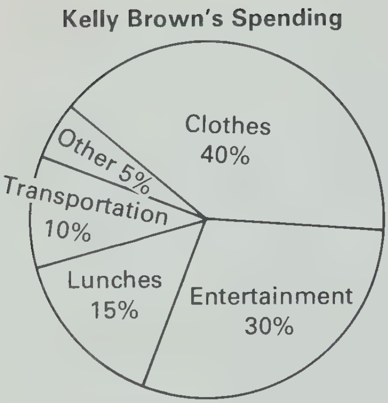
(b) What was the attendance on Friday?

(c) What day had the lowest attendance?

(d) What day had the highest attendance?

(e) What was the total attendance for the week?

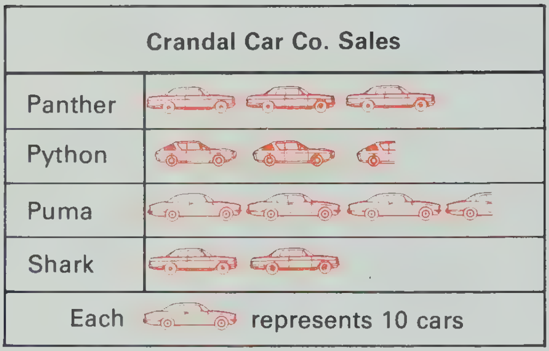




III The Circle Graph

A circle graph is used to show how one thing is divided. The circle graph at left shows how Kelly Brown spends what she earns from her part time job.

(a) What is the greatest expense?
(b) What is the least expense?
(c) What expenses could be included in "other"?
(d) If Kelly earns $200 a month, how much does she spend on each item?



IV The Pictograph

Pictographs use pictures or drawings to represent numbers. The pictograph at left shows the number of custom cars sold during one year by the Crandal Car Company.

(a) How many Panthers were sold?
(b) How many Sharks?
(c) How many Pythons?
(d) What were the total sales for the year?

EXAMPLE 1. The manager of the Blue Line Hockey Club spent the team's gate receipts in the following way:

Ice Rental	$ 720
Uniforms	$ 600
Travel	$ 480
Equipment	$ 360
Miscellaneous	$ 240
Total	$2400

Construct a circle graph to show this information.

Solution

First express each expense as a percent of the total:

Ice Rental: $\frac{720}{2400} \times 100 = 30\%$

Uniforms: $\frac{600}{2400} \times 100 = 25\%$

Travel: $\frac{480}{2400} \times 100 = 20\%$

Equipment: $\frac{360}{2400} \times 100 = 15\%$

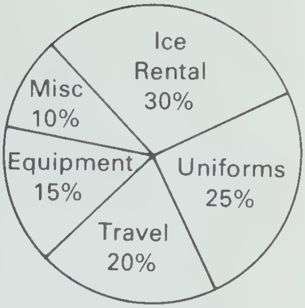
Miscellaneous: $\frac{240}{2400} \times 100 = 10\%$

Check 100%

Then write each percent as a decimal and multiply by 360° (the number of degrees in a circle).

Ice Rental:	0.3 × 360° = 108°
Uniforms:	0.25 × 360° = 90°
Travel:	0.2 × 360° = 72°
Equipment:	0.15 × 360° = 54°
Miscellaneous:	0.1 × 360° = 36°

Draw a circle. Construct each angle *from the centre*. Label each section and write a title for the graph.



Blue Line Hockey Club Expenses

EXERCISE 2-1

1. What type of graph would you use to illustrate each of the following?

(a) The temperature change during a day.

(b) The favourite type of TV program for the members of your class.

(c) How the Brown family spends its weekly income.

(d) The world's population growth since 1900.

(e) The lengths of the world's five longest rivers.

(f) The way a city spends your tax dollar.

2. Construct graphs to illustrate the following data.

(a) The number of points scored during one game by the starting five members of a basketball team are:

Player	Points
Russell	16
Smith	8
Barry	11
Cowens	7
Lanier	13

(b) The price of a share of Tricorp over a 5 day period is:

Day	Selling Price
Monday	$1.30
Tuesday	$1.40
Wednesday	$1.20
Thursday	$1.10
Friday	$1.50

(c) The students council of Beacon High School spent the $4000 they raised as follows:

Concerts:	$1200
Dances:	$1600
Graduation	$ 800
Miscellaneous	$ 400

(d) Six of the planets that orbit the sun have moons. The following table gives the number of moons for these planets.

Planet	Number of Moons
Earth	1
Mars	2
Jupiter	15
Saturn	10
Uranus	5
Neptune	2

Which planets have no moons?

(e) Tom has a mass of 75 kg. His mass is made up as follows.

Muscle:	37.5 kg
Bone:	13.5 kg
Fat:	15 kg
Other:	9 kg

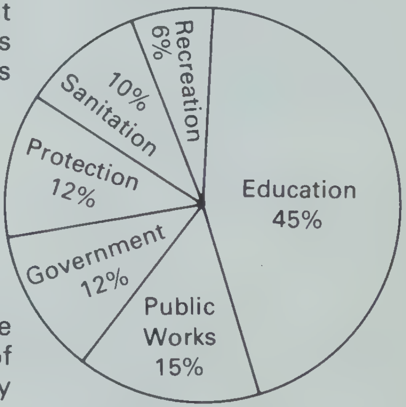
(f) The pollution index in Bay City on Friday was recorded as follows:

Time	00:00	01:00	02:00	03:00	04:00	05:00
Index	16	20	23	24	21	18

Time	06:00	07:00	08:00	09:00	10:00	11:00
Index	16	9	4	0	5	8

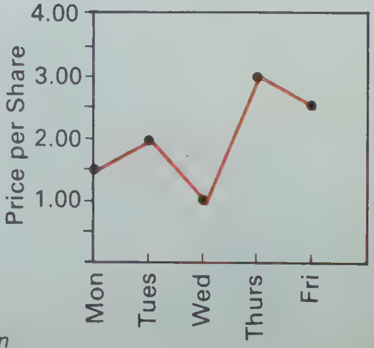
3. The circle graph shows how Carson City taxes were spent last year. If the total expenditure was $20 000 000, how much was spent for
(a) education?
(b) public works?
(c) sanitation?
(d) protection?
(e) recreation?
(f) government?

Carson City Taxes



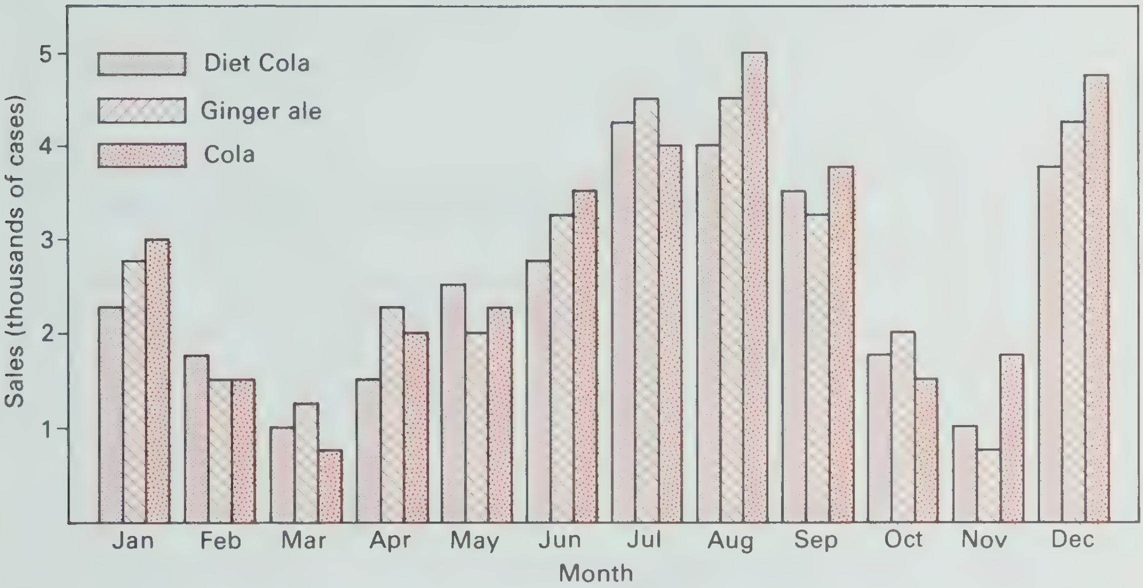
4. The line graph shows the changes in the price of a share of Abco stock over a five day period.
(a) What would you pay for 200 shares on Tuesday?
(b) If you sold 300 shares on Friday, how much would you receive?
(c) If you bought 250 shares on Monday and sold them on Thursday, what would your profit be, excluding expenses?

Abco Stock



5. In order to keep enough stock in the warehouse, the manager of the Lucky Seven Supermarket needs to predict the monthly sales for each type of soft drink. To help him make his predictions, he uses a bar graph showing last year's sales. The following graph shows the Lucky Seven sales for last year.

Lucky Seven Soft Drink Sales



(a) What month had the greatest cola sales?
(b) What month had the greatest diet cola sales?
(c) Which type is most popular in July?
(d) Which type is most popular in February?
(e) What were the cola sales for April?
(f) What were the ginger ale sales for January?
(g) What were the diet cola sales for September?
(h) How many more cases of cola were sold in April than November?
(i) March had more sales than November in one type. What was it?
(j) Find the total sales for each month.
(k) Which month had the greatest sales? Give possible reasons.
(l) Which month had the least sales? Give possible reasons.

2.2 MEAN, MEDIAN AND MODE

A graph does not always provide enough information about data. Some times we want to know where the *middle* is. Something that tells us about the middle of the data is called a measure of central tendency (measure of the middle).

Three measures of central tendency are:

(i) the mean.
(ii) the median.
(iii) the mode.

I The Mean

The mean is commonly called the average. The mean of a set of numbers is found by adding them and dividing the result by the number of values added.

EXAMPLE 1. *Terry bowled six games and had the following scores: 152, 165, 179, 157, 183 and 172. Find his mean score.*

Solution

$$\text{mean} = \frac{152 + 165 + 179 + 157 + 183 + 172}{6} = \frac{1008}{6} = 168$$

II The Median

If a set of numbers is arranged in order from smallest to largest (or largest to smallest), then the number in the middle is called the median.

For an odd number of values, there will be one number in the middle. For an even number of values, there will be two numbers in the middle. In the second case, the median is the mean of the two middle numbers.

EXAMPLE 2. *Find the median of the following test marks: 65, 83, 71, 54, 60, 91, 76.*

Solution *We first arrange the values in order:*

54, 60, 65, 71, 76, 83, 91

The median is the middle value 71.

EXAMPLE 3. *Find the median of the following test marks: 62, 78, 67, 55, 70, 71*

Solution *Arrange the values in order.*

55, 62, 67, 70, 71, 78

The two numbers in the middle are 67 and 70. The median is

$$\frac{67 + 70}{2} = \frac{137}{2} = 68.5$$

III The Mode

The mode of a set of numbers is the value that occurs most often. If every number occurs just once, there is no mode. There may be more than one mode.

EXAMPLE 4. *Determine the mode(s) of the following sets of numbers.*
(a) 6, 7, 9, 6, 7, 6, 8, 9, 4
(b) 7, 9, 12, 9, 6, 9, 7, 8, 7, 8, 7, 9
(c) 9, 7, 6, 5, 8, 10, 4, 3

Solution (a) 4, 6, 6, 6, 7, 7, 8, 9, 9
The mode is 6.
(b) 6, 7, 7, 7, 7, 8, 8, 9, 9, 9, 9, 12
The modes are 7 and 9.
(c) 3, 4, 5, 6, 7, 8, 9, 10
There is no mode.

EXERCISE 2-2

A **1.** Determine the modes of the following sets of data.
(a) 6, 9, 13, 4, 6, 8, 9, 6, 9, 9
(b) 9, 11, 10, 9, 13, 10, 8, 16
(c) 6, 12, 11, 4, 9, 3, 8, 7
(d) 21, 24, 21, 30, 29, 24, 21, 26, 20

B **2.** Determine the mean, median and mode of the following sets of data.
(a) 10, 9, 13, 14, 9
(b) 36, 43, 55, 41, 31, 40
(c) 31, 34, 36, 35, 41, 31, 30
(d) 33, 9, 40, 68, 59, 9, 68, 38
(e) 91, 90, 84, 86, 91
(f) 10.1, 9.4, 9.8, 8.2, 9.6, 10.5
(g) 116, 110, 144, 135, 124, 110, 157

2.3 MEAN, MEDIAN OR MODE?

The mean, median and mode are all used to determine the "middle" of data.

The mean is used for data such as:
- (a) bowling averages.
- (b) the average yearly rainfall for an area.
- (c) the fuel consumption for a car.

The median is used for:
- (a) test marks.
- (b) days absent by employees.
- (c) salaries.

The mode is useful for data like:
- (a) shoe sizes.
- (b) shirt sizes.
- (c) the number of cylinders in an engine.

EXAMPLE 1. *A company pays its employees the following annual salaries:*

1 President at \$300 000
1 Vice President at \$80 000
1 Treasurer at \$70 000
1 Accountant at \$40 000
3 Salesmen at \$30 000 each
4 Stenographers at \$20 000 each
Find the mean, median and mode of the salaries.

Solution (a) Mean:

The sum of the salaries is:

$$
\begin{aligned}
1 \times \$300\,000 &= \$300\,000\\
1 \times \$80\,000 &= \$80\,000\\
1 \times \$70\,000 &= \$70\,000\\
1 \times \$40\,000 &= \$40\,000\\
3 \times \$30\,000 &= \$90\,000\\
4 \times \$20\,000 &= \underline{\$80\,000}\\
\text{Total} \qquad & \quad \$660\,000
\end{aligned}
$$

There are 11 employees

$\therefore$ *the mean is* $\frac{660\,000}{11} = \$60\,000$

(b) Median: \$30 000
(c) Mode: \$20 000

As you can see, the *mean* is influenced by extreme values. In addition, for this company, no one earns the mean salary.

Which measure of central tendency best indicates the level of salary paid by this company?

EXERCISE 2-3

1. The word "average" is often used in place of either the mean, median or mode. What "average" would best describe the following data?

(a) Your bowling average.
(b) The most popular hat size.
(c) The amount of snow your area gets in a year.
(d) The average mark on a math test.
(e) The average number of paperclips per box.
(f) The points scored by a basketball player per game.
(g) The most popular dress size.

2. In 65 games Jim had 143 goals scored against him. What was the average number of goals scored against him per game?

3. On one weekend, Larry's Men's Wear sold eight sports jackets of the following sizes: 38, 40, 42, 38, 40, 44, 40, 44. What was the average size of jacket sold?

4. The table at right gives Sarah's bowling scores for a four week period. Determine her average after:
(a) Week 1
(b) Week 2
(c) Week 3
(d) Week 4

Week	Game 1	Game 2	Game 3
1	173	182	167
2	159	167	148
3	184	191	186
4	160	150	173

5. When Shawn filled the tank of her car with gas, the odometer read 11 384.0 km. When she filled it the next time, the odometer read 11 673.6 km. She added 36.2 L of gasoline. How many kilometres did she drive for every litre of gasoline?

6. The Alpine Land Development Company pays the following annual salaries:

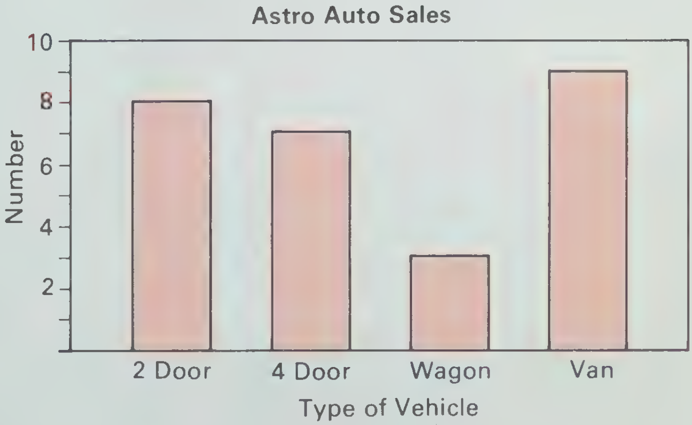
1 President at $250 000
1 Vice President at $150 000
1 Executive Director at $100 000
1 Treasurer at $95 000
2 Accountants at $40 000 each
4 Salesmen at $35 000 each
3 Stenographers at $20 000 each
2 Receptionists at $20 000 each

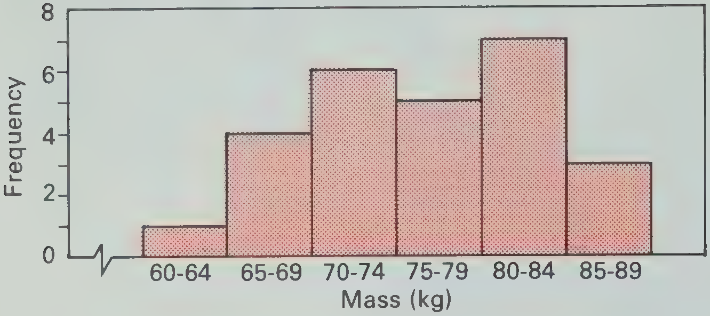
(a) Find the mean, median and mode of the salaries paid by Alpine.
(b) Which average best indicates the salary paid by the company?

2.4 THE HISTOGRAM

Histograms and bar graphs look alike but are quite different.

A bar graph is used to compare similar things. The bar graph at right compares the sales of four different types of vehicle sold by Astro Auto Sales during one week.
(a) How many vans were sold?
(b) How many station wagons were sold?




Trojan Football Team Mass of Players

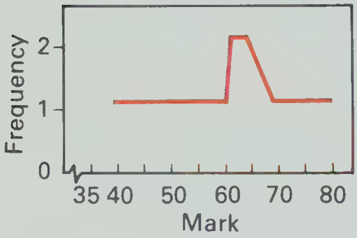
A histogram looks like a bar graph and behaves like a line graph. A histogram shows how often one thing occurs. The one at the left shows how the players masses are distributed on the Trojan Football team.

(a) How many players have a mass between 70 and 74 kg?
(b) Between 80 and 84 kg?
(c) Between 60 and 64 kg?

Data is sometimes difficult to graph and interpret. For example, the following are the marks, out of 100, earned by thirty students on a history test.

39	42	48	41	52	55
59	62	62	63	63	68
69	69	70	73	74	75
76	77	78	79	81	83
84	85	86	87	92	95

A graph of these results would be meaningless since the marks are so varied.


History Test Marks

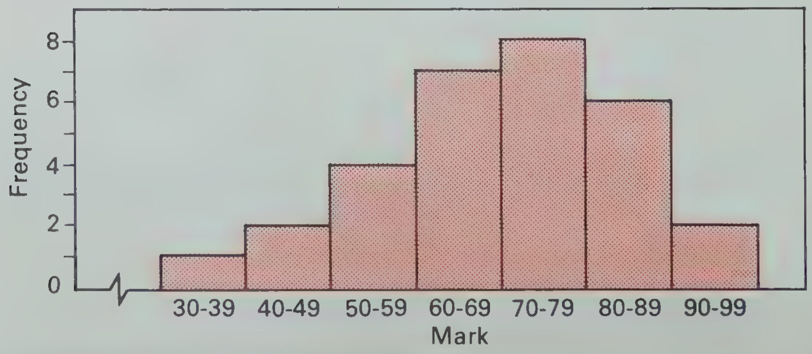
"Frequency" indicates how often something happens.

However, if we group the data into classes of the same width, a meaningful graph can be drawn.

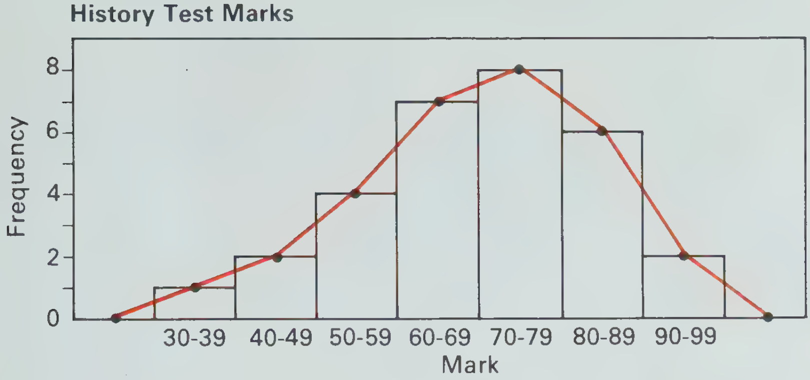
The table shows the History test marks grouped in 10 mark intervals.

Interval	Tally	Frequency
30-39	|	1
40-49	||	2
50-59	||||	4
60-69	𝍸 ||	7
70-79	𝍸 |||	8
80-89	𝍸 |	6
90-99	||	2

The following is a histogram representing this data. The bars are constructed with the frequency of each class represented by the height of a bar.



A histogram is sometimes replaced by a frequency polygon. A frequency polygon is formed by joining the mid-points at the top of each class.



Both the histogram and frequency polygon provide a method of viewing the shape of data and a pictorial look at the frequency of occurrence in each class.

EXERCISE 2-4

A **1.** The histogram shows the masses of students in a grade 10 math class.

(a) How many students were in the class?

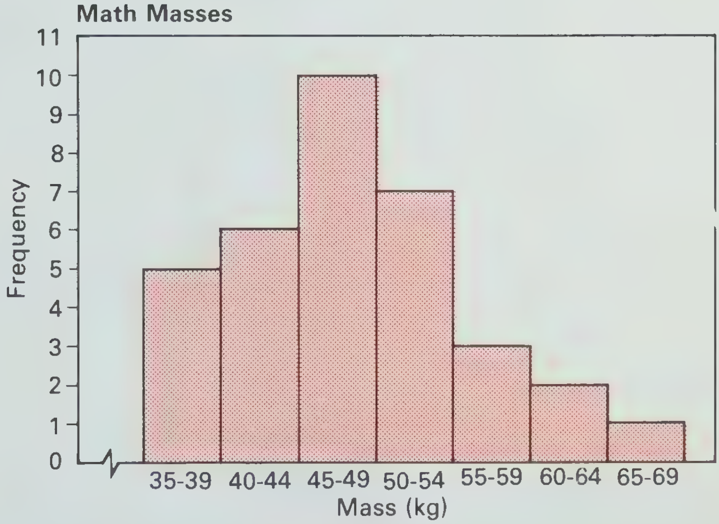
(b) How many students have masses between

(i) 35 and 39 kg?
(ii) 50 and 54 kg?
(iii) 55 and 59 kg?
(iv) 45 and 49 kg?
(v) 40 and 49 kg?

(c) How many students have masses

(i) less than 45 kg?
(ii) greater than 49 kg?
(iii) less than 55 kg?

(d) What figure best represents the average mass of a student in the class?



B 2. Complete the table and draw a histogram to illustrate the following data.

Height of Math Students in Centimetres		
Interval	*Tally*	*Frequency*
150-154	I	
155-159	𝍸	
160-164	𝍸 𝍸	
165-169	𝍸 𝍸 II	
170-174	𝍸 𝍸 𝍸 III	
175-179	𝍸 𝍸 III	
180-184	𝍸 I	
185-189	III	

3. The following are the marks earned by students on an English test out of 100.

73	81	42	55	64	91	63
71	87	54	65	79	80	89
35	50	66	78	67	52	54
86	87	78	77	51	64	70
74	75	88	81	91	95	65

(a) Construct a frequency distribution table using 10 mark intervals.
(b) Draw a histogram to illustrate the data.

4. In order to determine whether or not to install a stop sign at an intersection, the speeds of 50 vehicles passing through the intersection were recorded. The following are the speeds in kilometres per hour.

51	52	55	52	43	49	49	56	51	51
48	53	49	55	52	54	43	55	53	52
45	42	50	58	53	55	45	58	56	55
46	39	47	54	50	58	52	48	55	49
54	59	51	49	48	51	53	49	59	51

(a) Construct a frequency distribution table using intervals of 3 km/h.
(b) Draw a histogram to illustrate the data.
(c) If the speed limit is 50 km/h, should a stop sign be installed?

5. The number of tickets issued by two policemen, Corporal Brown and Corporal Towers, in the last 30 working days is as follows.

Corporal Brown

10	19	17	16	18	2
8	17	15	9	13	6
6	8	6	10	10	12
9	4	6	11	7	9
15	2	5	12	2	5

Corporal Towers

2	15	15	18	6	15
7	19	12	17	14	19
16	3	4	11	18	7
18	7	6	5	2	14
6	6	16	3	5	18

(a) Construct a frequency distribution table for each policeman.
(b) Construct a histogram for each policeman.
(c) Which policeman is more consistent in issuing tickets?

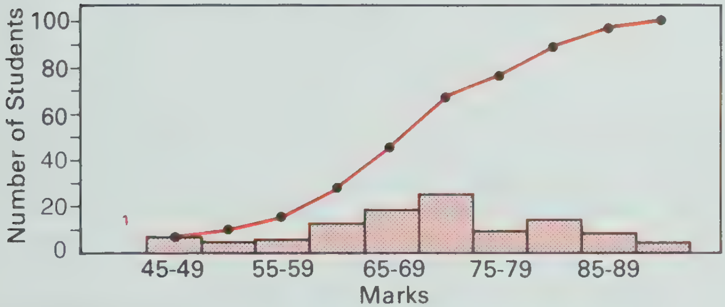
2.5 CUMULATIVE FREQUENCY

The following frequency table shows the marks of 100 students for an English test. The third row gives the number of students with a mark in the designated range or lower.

Interval	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94
Frequency	6	4	5	12	17	23	9	13	8	3
Cumulative Frequency	6	10	15	27	44	67	76	89	97	100

(a) How many students had a mark lower than 65?
(b) How many students had a mark lower than 80?
(c) How many students had a mark between 70 and 74 or lower?

The following is a cumulative frequency graph of this data.



History Marks Cumulative Frequency Graph

EXERCISE 2-5

B 1. The frequency table gives the speed of 50 vehicles passing through an intersection in km/h. The speeds have been grouped into intervals.
(a) Complete the cumulative frequency column.
(b) Draw the cumulative frequency graph.
(c) How many cars had speeds less than 50 km/h?
(d) How many cars had speeds less than 60 km/h?
(e) How many cars had speeds between 50 and 54 km/h or less?

Interval	Frequency	Cumulative Frequency
35-39	6	
40-44	10	
45-49	11	
50-54	13	
55-59	5	
60-64	3	
65-69	2	

2. The frequency table gives the masses of the Blue Jay players grouped into intervals.
(a) Complete the cumulative frequency column.
(b) Draw the cumulative frequency graph.
(c) How many players have masses less than 80 kg?
(d) How many players have masses between 85 and 89 kg or less?

Interval in Kilograms	Frequency	Cumulative Frequency
70-74	2	
75-79	5	
80-84	7	
85-89	6	
90-94	4	
95-99	1	

2.6 PERCENTILES

The following table gives the height of the Expos in centimetres grouped into intervals. The Expos have 25 players. The third column gives the cumulative frequency. The fourth column gives the cumulative frequency as a percent of the total number of players.

Interval	Frequency	Cumulative Frequency	Cumulative Frequency %
170-172	2	2	8%
173-175	1	3	12%
176-178	3	6	24%
179-181	6	12	48%
182-184	8	20	80%
185-187	4	24	96%
188-190	1	25	100%

24% of the Expos have heights equal to or less than 178 cm
80% of the Expos have heights equal to or less than 184 cm

Ranking things as a certain percent of the total is called percentile ranking.

Percentile ranking is often used to determine rankings on tests.

If your mark is in the 76th percentile of a test, that means that 76% of the students had marks equal to or lower than yours. It also means that 24% (100-76) of the students had marks higher than yours.

EXAMPLE 1. *Sarah got 77 on an English test. The marks of the other students in her class were as follows.*

86 76 43 75 55 80
67 82 92 77 57 75
52 65 71 80 63 61
70 67 69 50 70 67

Determine her percentile rank.

Solution *A total of 25 students wrote the test.*
2 students (including Sarah) had marks of 77.
18 students had marks lower than 77.
∴20 students had marks equal to or less than 77.
20 students is $\frac{20}{25} \times 100 = 80\%$ *of the class.*
Therefore Sarah's mark is in the 80th percentile.

EXERCISE 2-6

A **1.** (a) What does a mark in the 60th percentile indicate?
(b) What does a mark in the 92nd percentile indicate?

B **2.** Fifty students wrote a Geography test. Tom's mark of 75 put him in the 80th percentile.
(a) How many students had marks higher than 75?
(b) How many students had marks equal to or less than 75?

3. Three hundred students wrote a college entrance exam. Sharon's mark of 85 put her in the 92nd percentile.
(a) How many students had marks less than or equal to 85?
(b) How many students had marks higher than 85?

4. Peter had a mark of 64 on a driving test. A total of 25 students took the test. The other 24 marks were as follows.

51 73 64 52 55 90
40 64 80 67 61 42
62 60 85 68 64 73
65 72 81 50 67 75

Determine Peter's percentile rank.

5. Frank and Bob are in different math classes. They both got 72 on a math test. The marks of the other students in their classes were as follows.
Frank's class:

65 55 80 92 73 85
84 61 54 47 70 70
72 54 57 68 64 69
81 60 81 69 60 70

Bob's class:

74 71 67 55 50 64 98
69 67 81 56 92 50
40 85 53 54 61 66
71 71 73 70 79 60
77 69 51 45 66 68

(a) Find Frank's percentile rank in his class.
(b) Find Bob's percentile rank in his class.
(c) Assuming that the competition is the same in both classes, whose mark of 72 is the best?
(d) Why doesn't a mark of 72 mean the same thing for all tests?

REVIEW EXERCISE

1. Construct graphs to illustrate the following data.

(a) The maximum speeds of several animals are given in the table.

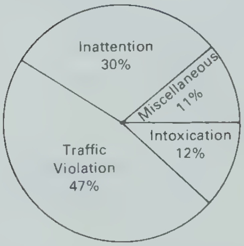
Animal	Speed (kilometres per hour)
Cheetah	110
Rabbit	55
Greyhound	65
Human	40
Lion	80
Giraffe	50

(b) The temperature in Winnipeg, Manitoba on a day in September was recorded as follows.

Time	Temp °C	Time	Temp °C
06:00	5	13:00	17
07:00	7	14:00	16
08:00	8	15:00	15
09:00	12	16:00	13
10:00	13	17:00	12
11:00	16	18:00	11
12:00	17		

(c) It cost Sarah $5000 to attend college last year. Her expenses were as follows:

Tuition	$1600	Food	$1000
Travel	$ 400	Books	$ 200
Room	$1200	Personal	$ 600



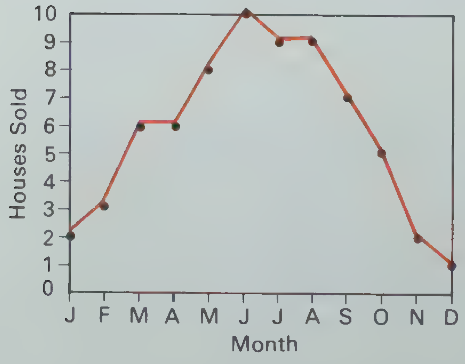
2. The circle graph shows the causes of fatal automobile accidents.

(a) Out of 500 fatal accidents, how many are caused by:

(i) inattention?

(ii) a traffic violation?

(iii) intoxication?



3. The graph shows the number of houses sold during one year by the Gibraltar Realty Company.

(a) During what month were sales the highest?

(b) During what months were the sales the same?

(c) By now much did the sales in September exceed those in February?

(d) What month had the fewest sales?
(e) What were the total sales for the year?

4. Determine the mean, median, and mode of the following sets of data.

(a) 15, 19, 15, 12, 20, 25, 20, 17, 16, 13, 15.
(b) 149, 127, 113, 137, 134, 131, 105.
(c) 36, 50, 46, 44, 42, 50, 41, 51.

5. The table gives Mark's bowling scores for a five week period. Determine his average after

(a) Week 1
(b) Week 2
(c) Week 3
(d) Week 4
(e) Week 5

Week	Game 1	Game 2	Game 3
1	200	201	205
2	195	215	220
3	219	194	187
4	204	212	196
5	191	201	235

6. The following are the number of hits per game for the Blue Jays over a three week period.

7 6 6 8 4 8
12 9 1 10 5 11
3 7 12 7 9
5 9 14 3 6

(a) Construct a frequency distribution using three hit intervals.
(b) Draw a histogram to illustrate the data.

7. The frequency table gives the masses of the Steeler football players grouped into intervals.

(a) Complete the cumulative frequency column.
(b) Draw the cumulative frequency graph.
(c) How many players have masses less than 105 kg?
(d) How many players have masses between 105 and 109 kg or less?

Interval in kilograms	Frequency	Cumulative Frequency
80-84	1	
85-89	4	
90-94	5	
95-99	8	
100-104	11	
105-109	6	
110-114	3	
115-119	2	

8. Terry got 66 on a History test. The marks of the other students in the class were as follows.

80 50 64 56 53
52 60 66 63 83
55 53 70 59 59
58 85 72 56

Determine his percentile rank.

REVIEW AND PREVIEW TO CHAPTER 3

COMMON FRACTIONS AND DECIMALS

EXERCISE 1 *Addition and Subtraction*

1. Determine the value for ▨ to form equivalent fractions.

(a) $\frac{1}{4} = \frac{\text{▨}}{8}$ (b) $\frac{1}{2} = \frac{\text{▨}}{10}$ (c) $\frac{2}{3} = \frac{\text{▨}}{12}$
(d) $\frac{15}{18} = \frac{\text{▨}}{6}$ (e) $\frac{6}{\text{▨}} = 3$ (f) $\frac{7}{\text{▨}} = \frac{14}{18}$
(g) $2\frac{1}{4} = \frac{\text{▨}}{4}$ (h) $7\frac{3}{5} = \frac{\text{▨}}{5}$ (i) $12\frac{1}{8} = \frac{\text{▨}}{8}$

In the problems below, express all answers in lowest terms.

2. Simplify

(a) $\frac{5}{8} + \frac{1}{8}$ (b) $\frac{4}{5} - \frac{1}{5}$ (c) $\frac{3}{5} + \frac{2}{10}$
(d) $\frac{5}{6} + \frac{1}{6}$ (e) $1\frac{3}{4} + \frac{2}{3}$ (f) $2\frac{1}{2} - 1\frac{1}{3}$
(g) $5\frac{1}{4} + \frac{3}{5}$ (h) $\frac{7}{8} - \frac{5}{6}$ (i) $3\frac{1}{4} - \frac{7}{8}$
(j) $1\frac{1}{6} + \frac{5}{9}$ (k) $4\frac{3}{7} - 2\frac{1}{4}$ (l) $1\frac{1}{8} - \frac{5}{7}$

3. Simplify

(a) $1\frac{1}{3} + \frac{1}{4} - \frac{5}{6}$ (b) $2\frac{1}{2} - \frac{3}{5} + \frac{5}{6}$ (c) $\frac{2}{3} + \frac{5}{6} - \frac{1}{2}$
(d) $2\frac{3}{4} + \frac{1}{3} + \frac{7}{8}$ (e) $2\frac{1}{3} + 1\frac{1}{5} - 3\frac{8}{15}$ (f) $5\frac{1}{3} + \frac{3}{10} - 1\frac{1}{6}$

How many whole numbers from 0 to 10 can you construct with 5 5's and the operations +, −, ×, ÷?

e.g. $\frac{5+5}{5} - \frac{5}{5} = 1$

EXERCISE 2 *Multiplication and Division*

1. Simplify

(a) $\frac{2}{3} \times \frac{5}{7}$ (b) $\frac{1}{4} \times \frac{2}{5}$ (c) $\frac{8}{15} \times \frac{3}{10}$
(d) $1\frac{1}{2} \times \frac{5}{9}$ (e) $3\frac{1}{5} \times \frac{15}{16}$ (f) $2 \times \frac{1}{3}$
(g) $5 \times \frac{5}{8}$ (h) $\frac{3}{4} \times 1\frac{1}{3}$ (i) $3\frac{2}{3} \times 1\frac{1}{8}$

2. Simplify

(a) $\frac{3}{4} \div \frac{2}{5}$ (b) $\frac{5}{8} \div \frac{2}{3}$ (c) $1\frac{1}{4} \div \frac{3}{5}$
(d) $\frac{5}{12} \div \frac{3}{4}$ (e) $3\frac{1}{5} \div 2\frac{1}{2}$ (f) $2 \div \frac{1}{2}$
(g) $\frac{7}{8} \div 2$ (h) $2\frac{5}{6} \div 1\frac{1}{3}$ (i) $\frac{2}{3} \div 1\frac{1}{2}$

3. Simplify

(a) $1\frac{1}{2} \times \frac{3}{4} \times 3\frac{1}{2}$ (b) $5\frac{1}{2} \times \frac{5}{22} \times \frac{3}{5}$ (c) $\frac{3}{5}(\frac{2}{3} + \frac{1}{4})$
(d) $(2\frac{1}{7} - \frac{5}{6}) \times \frac{7}{8}$ (e) $3 \times \frac{5}{6} \div 1\frac{1}{3}$ (f) $(\frac{2}{5} + \frac{1}{10}) \times (1\frac{1}{2} - \frac{5}{12})$
(g) $\frac{1}{5} + \frac{2}{3} \times 1\frac{3}{4}$ (h) $(\frac{3}{4})^2 - \frac{1}{4}$ (i) $(\frac{2}{3} + \frac{1}{5})^2$

EXERCISE 3 *Operations with Decimals*

1. (a) $\begin{array}{r} 4.32 \\ 5.94 \\ +\ 3.05 \\ \hline \end{array}$ (b) $\begin{array}{r} 56.9 \\ 38.2 \\ +\ 97.8 \\ \hline \end{array}$ (c) 3.74 + 2.81 + 3.74
(d) 14.7 + 3.9 + 8.6
(e) 0.750 + 0.086 + 1.920

2. (a) $\begin{array}{r} 86.32 \\ -\ 31.84 \\ \hline \end{array}$ (b) $\begin{array}{r} 104.7 \\ -\ \ 85.2 \\ \hline \end{array}$ (c) 35.9 − 26.8
(d) 426.8 − 59.7

3. (a) $\begin{array}{r} 4.73 \\ \times\ \ 9.5 \\ \hline \end{array}$ (b) $\begin{array}{r} 97.5 \\ \times\ 0.38 \\ \hline \end{array}$ (c) 56.4 × 8.21
(d) 0.157 × 2.54

4. (a) $4.7\overline{)26.461}$ (b) $0.53\overline{)1.325}$
(c) 436.54 ÷ 7.3 (d) 60.918 ÷ 85.2

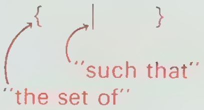
Rational and Irrational Numbers

CHAPTER 3

3.1 RATIONAL NUMBERS AS DECIMALS

Throughout history, we have invented and developed tools and appliances as a need arose. In a similar way, we have also invented new sets of numbers to solve new problems as they occurred. The following sets are some of those which you have already encountered.

Natural numbers	$N = \{1,2,3,\ldots\}$
Whole numbers	$N_0 = \{0,1,2,3,\ldots\}$
Integers	$I = \{\ldots,-2,-1,0,1,2,3,\ldots\}$
Rational numbers	$Q = \left\{\frac{a}{b}\,\middle\vert\, a,b \in I,\ b \neq 0\right\}$



Notice that it is not possible to list the elements in the set Q, since no fraction has a "next larger" fraction.

EXAMPLE 1. *By division, express the following rational numbers in decimal form:*

(a) $\frac{-3}{16}$ (b) $\frac{2}{11}$

Solution (a)

$$\begin{array}{r} 0.1875 \\ 16\overline{)3.0000} \\ \underline{16} \\ 140 \\ \underline{128} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

$$\frac{-3}{16} = -0.1875$$

(b)

$$\begin{array}{r} 0.1818 \\ 11\overline{)2.0000} \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \end{array}$$

$$\frac{2}{11} = 0.\dot{1}\dot{8}$$

In the same way, **any rational number may be expressed as a terminating or repeating decimal.** It is also possible to express such a decimal in the form $\frac{a}{b}$, $b \neq 0$.

EXAMPLE 2. *Express the following decimals in the form* $\frac{a}{b}$*,* $b \neq 0$.

(a) 0.45 (b) 3.2 (c) $0.\dot{3}\dot{6}$

$0.2\dot{9}\dot{3} = 0.293\ 93\ldots$

The **period** is 93, and the **length of the period** is 2.

Solution (a) $0.45 = \frac{45}{100} = \frac{9}{20}$ (b) $3.2 = 3\frac{2}{10} = \frac{32}{10} = \frac{16}{5}$

(c) For a repeating decimal, the procedure is to eliminate the repeating portion of the decimal by subtraction.
If we let $x = 0.363\ 636\ldots$, then $100x = 36.3636\ldots$

$$x = 0.\underbrace{363\ 636\ldots}, \quad 100x = 36.\underbrace{3636\ldots}$$

the decimal portion is the same

If $x = 0.3636\ldots$
$\therefore\ 10x = 3.63636\ldots$
$\therefore\ 100x = 36.3636\ldots$
etc.

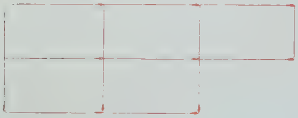
$$\begin{aligned} \therefore\quad 100x &= 36.3636\ldots \\ x &= 0.3636\ldots \\ \text{Subtracting: } 99x &= 36 \\ \therefore\quad x &= \frac{36}{99} \\ &= \frac{4}{11} \end{aligned}$$

EXERCISE 3-1

A 1. Complete the following table:

	Fraction form	Decimal form		Fraction form	Decimal form
(a)	$\frac{1}{4}$		(g)	$\frac{5}{4}$	
(b)		0.3	(h)		4.3
(c)	$-\frac{1}{10}$		(i)		0.625
(d)	$2\frac{1}{2}$		(j)	$\frac{73}{1000}$	
(e)		-0.75	(k)		-0.001
(f)	$\frac{3}{100}$		(l)		10.2

Remove 2 matches so that only 3 squares are left.



2. State the period and the length of the period for each of the following repeating decimals:
(a) $0.\dot{2}$ (b) $0.\dot{5}\dot{6}$ (c) $-0.3\dot{9}\dot{8}$ (d) $0.\dot{1}56\dot{9}$
(e) $3.\dot{5}6\dot{7}$ (f) $0.93\dot{5}$ (g) $0.25\dot{0}$ (h) $-1.98\dot{0}$
Express the decimals in parts (g) and (h) as non-repeating decimals.

3. Complete the table

	x	$10x$	$100x$	$1000x$
(a)	$0.333\ldots$	$3.333\ldots$		
(b)	$0.2929\ldots$	$2.929\ 29\ldots$		
(c)	$0.\dot{5}\dot{6}$	$5.656\ 56\ldots$		
(d)	$3.2\dot{5}$			

	x	$10x$	$100x$	$1000x$
(e)	$1.5\dot{2}\dot{6}$			
(f)		4.343 43 . .		
(g)			17.555 . . .	
(h)	$-0.60\dot{3}$			
(i)				293.232 32 . .

B **4.** Express each of the following rational numbers as a **terminating** or **repeating** decimal. State the period and the length of the period.
(a) $\frac{4}{5}$ (b) $\frac{4}{9}$ (c) $\frac{15}{9}$ (d) $\frac{2}{3}$ (e) $\frac{17}{20}$
(f) $-\frac{3}{11}$ (g) $\frac{24}{11}$ (h) $\frac{3}{7}$ (i) $\frac{11}{30}$ (j) $\frac{7}{55}$

5. Express each terminating decimal as a rational number in the form $\frac{a}{b}$, $b \neq 0$.

$Q = \left\{\frac{a}{b} \middle| a, b \in I,\ b \neq 0\right\}$

(a) 0.7 (b) 1.25 (c) −0.35 (d) 4.5
(e) 0.375 (f) −2.36 (g) 0.0025 (h) −1.01

6. Express each repeating decimal in the form $\frac{a}{b}$, $b \neq 0$.

(a) 0.555 . . . (b) $0.\dot{7}$ (c) $0.\dot{2}\dot{7}$ (d) $-0.\dot{1}\dot{3}$
(e) $1.\dot{6}$ (f) $0.\dot{3}6\dot{9}$ (g) −0.543 543 . . . (h) $0.2\dot{4}$
(i) $0.5\dot{9}$ (j) $0.2\dot{3}\dot{5}$ (k) $1.4\dot{3}$ (l) $0.20\dot{8}$

7. Complete the following calculations by first changing all non-decimal rational numbers to decimal numbers.

(a) $2.7 + 3\frac{1}{2}$ (b) $5.23 + 9\frac{3}{4}$ (c) $\frac{29}{5} + 0.8$
(d) $3\frac{7}{10} - 2.8$ (e) $35.6 - 21\frac{16}{100}$ (f) $\frac{1}{4} - 0.06$
(g) $6\frac{1}{2} \times 2.4$ (h) $5\frac{1}{8} \times 0.1$ (i) $7.52 \times \frac{3}{100}$
(j) $76.25 \div \frac{1}{2}$ (k) $8\frac{3}{8} \div 2.5$ (l) $5.25 \div 2\frac{5}{8}$

8. What decimal fraction of an hour is (i) 6 min, (ii) 12 min, (iii) 15 min, (iv) 30 min, (v) 36 min, (vi) 45 min, (vii) 54 min?
(b) If a ferry boat makes a round trip each 36 min, how many round trips does it make in a day if the first trip starts at 0730 h and the last trip ends at 2230 h?

9. If a jogger runs $3\frac{1}{4}$ times around a 500 m track, how many kilometres does he run?

3.2 IRRATIONAL NUMBERS

In section 3.1 the relationship was illustrated between rational numbers in fraction form and in decimal form.

$\pi = 3.141\ 592\ 653\ 589$
$793\ 238\ 462\ 642\ldots$

$\sqrt{2} = 1.41\ldots$

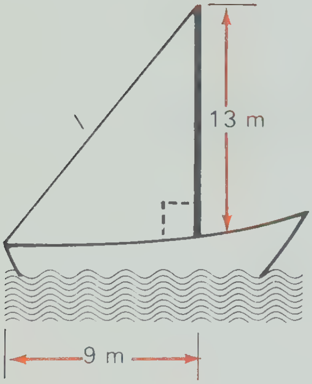
$$\frac{3}{16} = \boxed{0.1875}$$
$$\frac{2}{11} = \boxed{0.1\dot{8}}$$

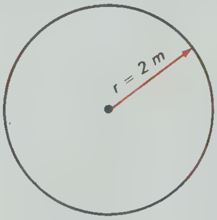
a terminating
or
repeating decimal

A **non-terminating, non-repeating** decimal is called an **irrational number,** and cannot be represented in the form $\frac{a}{b}$, $b \neq 0$. The set of irrational numbers is denoted by the symbol $\bar{Q}$.

$$\left.\begin{matrix}0.121\ 121\ 112\ldots\\ 18.921\ 992\ 199\ 921\end{matrix}\right\}\text{ are irrational numbers constructed by forming a changing pattern.}$$

Irrational numbers also occur in other forms in many types of problems.





$$\begin{aligned}A &= \pi r^2\ (\text{m}^2)\\ &= \pi \times 2^2\ (\text{m}^2)\\ &= 4\pi\ \text{m}^2\end{aligned}$$

$$\begin{aligned}l^2 &= 9^2 + 13^2\ (\text{m}^2)\\ &= 81 + 169\ (\text{m}^2)\\ &= 250\ (\text{m}^2)\\ l &= \sqrt{250}\ \text{m}\end{aligned}$$

these values are both irrational numbers

In both of the preceding problems, an **approximate value** may be used for each of the irrational numbers to complete the calculation.

$$\pi \doteq 3.14 \qquad \sqrt{250} \doteq 15.81$$

$(+5)^2 = 25$
$(-5)^2 = 25$
but $\sqrt{25} = 5$

The expression $\sqrt{250}$ **is called the square root of 250** and represents the number which may be squared to give 250.

The symbol $\sqrt{\ }$ is known as the **radical sign** and is used to represent the **positive** square root of any number.

The square roots of some numbers are rational.

EXAMPLE 1. *Evaluate (a) $\sqrt{49}$ (b) $\sqrt{36}$*

Divide by prime factors

2	36
2	18
3	9
3	3
	1

Solution

(a) $\sqrt{49} = \sqrt{7 \times 7}$
$= 7$
check $7 \times 7 = 49$

(b) $\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3}$
$= \sqrt{(2 \times 3) \times (2 \times 3)}$
$= 6$
check $6 \times 6 = 36$

If the factors of a number can be grouped into two identical sets, the number has a rational square root.

$$\begin{aligned}\sqrt{144} &= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}\\ &= \sqrt{(2 \times 2 \times 3) \times (2 \times 2 \times 3)}\\ &= 2 \times 2 \times 3\\ &= 12\end{aligned}$$

EXAMPLE 2. *Show that the following expressions do not represent rational numbers.*

Solution

(a) $\sqrt{12}$ (b) $\sqrt{500}$

(a) $\sqrt{12} = \sqrt{2 \times 2 \times 3}$
$= \sqrt{(2) \times (2) \times 3}$

(b) $\sqrt{500} = \sqrt{2 \times 2 \times 5 \times 5 \times 5}$
$= \sqrt{(2 \times 5) \times (2 \times 5) \times 5}$

2	500
2	250
5	125
5	25
5	5
	1

An expression such as $\sqrt{3}$, which does not have a rational value, is called a radical.

EXAMPLE 3. *Show that* $\sqrt{3} \doteq 1.732$

Solution $1.732 \times 1.732 = 2.999\,824$
$\doteq 3.000$ *to 4 significant digits*
$\sqrt{3} \doteq 1.732$

The set of real numbers denoted by the letter R, is the union of the sets of rational and irrational numbers.

$$R = Q \cup \bar{Q}$$

EXERCISE 3-2

A **1.** Classify the following as rational or irrational numbers.
7, $\frac{3}{4}$, 1, $\sqrt{16}$, $5.313\,313\,331\ \ldots$, $0.2\dot{9}$, $-\frac{5}{6}$, $0.292\,929\ \ldots$, $\sqrt{10 \times 10}$, 103, $\sqrt{3 \times 3 \times 5 \times 5}$, $\sqrt{2 \times 2 \times 7 \times 7 \times 11}$

2. State two elements of $\bar{Q}$ in each of the following forms:
(a) radical form.
(b) decimal form.

3. Complete the following:
(a) $\sqrt{64} = ▨$ (b) $\sqrt{36} = ▨$ (c) $\sqrt{▨} = 7$
(d) $\sqrt{1} = ▨$ (e) $\sqrt{▨} = 0$ (f) $\sqrt{▨} = 11$
(g) $\sqrt{\frac{9}{4}} = ▨$ (h) $\sqrt{▨} = \frac{1}{6}$ (i) $\sqrt{10 \times 10} = ▨$

B **4.** Find the value of the following expressions by grouping the factors:
(a) $\sqrt{144}$ (b) $\sqrt{225}$ (c) $\sqrt{324}$
(d) $\sqrt{64}$ (e) $\sqrt{256}$ (f) $\sqrt{784}$

5. Group factors of the following expressions to show which represent rational numbers and which represent irrational numbers. Evaluate the rational expressions.

(a) $\sqrt{72}$ (b) $\sqrt{75}$ (c) $\sqrt{400}$ (d) $\sqrt{196}$
(e) $\sqrt{147}$ (f) $\sqrt{784}$ (g) $\sqrt{441}$ (h) $\sqrt{1875}$

6. Verify each of the following statements.
(a) $\sqrt{2} \doteq 1.414$ (b) $\sqrt{21} \doteq 4.583$
(c) $\sqrt{10} \doteq 3.162$ (d) $\sqrt{83} \doteq 9.110$

7. The following value of π is correct to 9 significant digits

$$\pi = 3.141\,592\,65$$

By division, calculate the number of correct digits in each of the following approximations of π.
(a) $\frac{22}{7}$ (b) $\frac{32}{9}$ (c) $\frac{355}{113}$

8. A square rug has an area of 16 m^2.
(a) What is the cost of the rug at \$9.50/m^2?
(b) What is the length of one side of the rug?
(c) How much edge binding is required to go around the rug?
(d) What is the cost of the binding at \$0.65/m?

1 ha = 10 000m^2

9. A square field has an area of 1 ha. At \$4.25/m, what will the cost of fencing the field be?

10. Is it possible to give, as a decimal, the measure of the side of a square that covers exactly 2 m^2? Explain your answer.

3.3 RADICALS *Part 1*

The simplification of expressions containing radicals is often possible without replacing the radical with an approximate value.

The following examples illustrate a method of multiplying radicals.

EXAMPLE 1. *Simplify*

(a) $(\sqrt{3})^2$ (b) $\sqrt{9} \times \sqrt{16}$

Solution

(a)
$$\begin{aligned}(\sqrt{3})^2 &= \sqrt{3} \times \sqrt{3}\\ &= 3 \text{ (definition)}\end{aligned}$$
or
$$\begin{aligned}(\sqrt{3})^2 &= \sqrt{3} \times \sqrt{3}\\ &= \sqrt{3 \times 3}\\ &= \sqrt{9}\\ &= 3\end{aligned}$$

(b)
$$\begin{aligned}\sqrt{9} \times \sqrt{16} &= 3 \times 4\\ &= 12\end{aligned}$$
or
$$\begin{aligned}\sqrt{9} \times \sqrt{16} &= \sqrt{9 \times 16}\\ &= \sqrt{144}\\ &= 12\end{aligned}$$

In general, if $a, b \geqq 0$, $\sqrt{a}\times\sqrt{b} = \sqrt{ab}$

This idea may be used to simplify certain radicals. **In simplest form, a radical should have the lowest value under the radical sign.**

EXAMPLE 2. *Simplify*

(a) $\sqrt{20}$ (b) $3\sqrt{18}$

Solution (a)
$$\begin{aligned}\sqrt{20} &= \sqrt{4} \times \sqrt{5}\\ &= 2 \times \sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$
$$\therefore \quad \sqrt{20} = 2\sqrt{5}$$

This is called an **entire radical** This is called a **mixed radical**

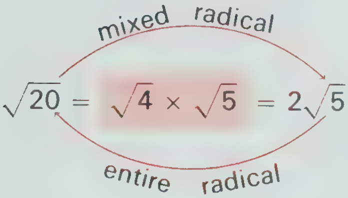
(b)
$$\begin{aligned}3\sqrt{18} &= 3 \times \sqrt{9} \times \sqrt{2}\\ &= 3 \times 3 \times \sqrt{2}\\ &= 9\sqrt{2}\end{aligned}$$

Insert the numbers from 1 to 9 so that the sum of the rows, columns and diagonals is 15

EXAMPLE 3. *(a) Express $\sqrt{45}$ as a mixed radical.*
(b) Express $3\sqrt{5}$ as an entire radical.

Solution (a) $\sqrt{45} = \sqrt{9} \times \sqrt{5}$
$= 3 \times \sqrt{5}$
$3\sqrt{5}$

(b) $3\sqrt{5} = \sqrt{9} \times \sqrt{5}$
$= \sqrt{45}$



$$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

EXAMPLE 4. *Simplify (a) $3\sqrt{2} \cdot 5\sqrt{3}$ (b) $2\sqrt{6} \times 3\sqrt{3}$*

Solution (a) $3\sqrt{2} \cdot 5\sqrt{3} = 3 \times 5 \times \sqrt{2} \times \sqrt{3}$
$= 15\sqrt{6}$

(b) $2\sqrt{6} \times 3\sqrt{3} = 2 \times 3 \times \sqrt{6} \times \sqrt{3}$
$= 6\sqrt{18}$
$= 6 \times \sqrt{9} \times \sqrt{2}$
$= 6 \times 3 \times \sqrt{2}$
$= 18\sqrt{2}$

EXERCISE 3-3

A

1. Simplify

(a) $(\sqrt{2})^2$ (b) $\sqrt{5} \times \sqrt{3}$ (c) $\sqrt{7} \times \sqrt{7}$
(d) $\sqrt{5} \times \sqrt{2}$ (e) $3\sqrt{3} \times 2\sqrt{5}$ (f) $\sqrt{11} \times \sqrt{5}$
(g) $2\sqrt{7} \times \sqrt{3}$ (h) $(\sqrt{6})^2$ (i) $\sqrt{10} \times \sqrt{10}$
(j) $3\sqrt{6} \times \sqrt{6}$ (k) $\sqrt{7} \times \sqrt{2}$ (l) $(2\sqrt{3})^2$

B

2. Express as mixed radicals in simplest form

(a) $\sqrt{12}$ (b) $\sqrt{27}$ (c) $\sqrt{54}$ (d) $\sqrt{32}$
(e) $\sqrt{72}$ (f) $2\sqrt{18}$ (g) $3\sqrt{8}$ (h) $5\sqrt{24}$
(i) $7\sqrt{28}$ (j) $\sqrt{98}$ (k) $3\sqrt{63}$ (l) $\sqrt{128}$

$\sqrt{\sqrt{16}} = ?$

3. Express as entire radicals

(a) $3\sqrt{5}$ (b) $4\sqrt{11}$ (c) $10\sqrt{3}$ (d) $5\sqrt{3}$
(e) $2\sqrt{15}$ (f) $9\sqrt{2}$ (g) $4\sqrt{10}$ (h) $8\sqrt{3}$

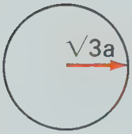
4. Simplify

(a) $\sqrt{5} \times \sqrt{12}$ (b) $\sqrt{3} \times \sqrt{6}$ (c) $3\sqrt{2} \times 5\sqrt{8}$
(d) $\sqrt{6} \times 2\sqrt{10}$ (e) $\sqrt{10} \times \sqrt{2}$ (f) $5\sqrt{6} \times 3\sqrt{15}$
(g) $5\sqrt{2} \times \sqrt{6}$ (h) $\sqrt{5} \times \sqrt{50}$ (i) $3\sqrt{5} \times 2\sqrt{5}$
(j) $\sqrt{6} \times \sqrt{10} \times \sqrt{2}$ (k) $\sqrt{18} \times \sqrt{27}$ (l) $4\sqrt{20} \times 5\sqrt{8}$

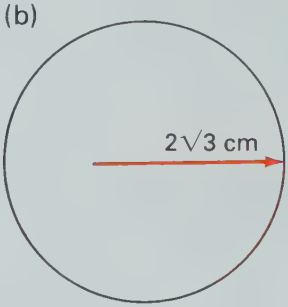
$A = \pi r^2$
Use $\pi = 3.14$

5. Find the areas of the following circular figures.

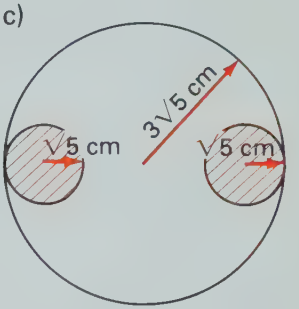
(a)



(b)



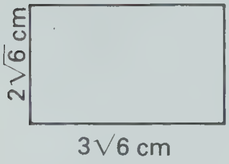
(c)



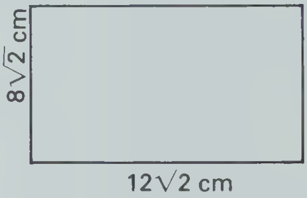
$A = lw$

6. Find the areas of the following rectangular figures.

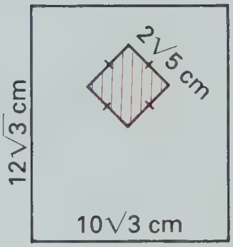
(a)



(b)



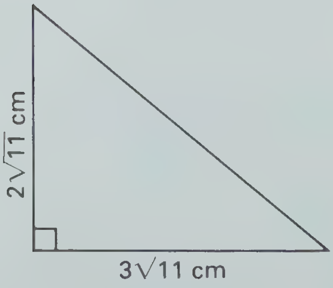
(c)



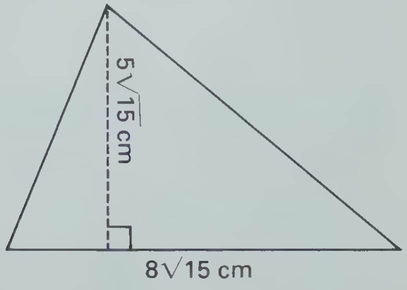
$A = - b h$

7. Find the area of the following triangular figures.

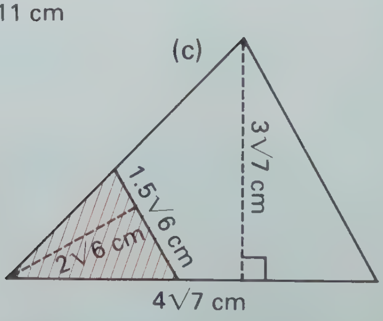
(a)



(b)



(c)



3.4 RADICALS *(Part II)*

In certain cases, it is possible to simplify radical expressions by addition and subtraction. Since the radical $\sqrt{2}$ represents a number, the distributive property may be applied as it is in the simplification of algebraic expressions.

EXAMPLE 1. *Simplify* (a) $5a + 4a$ (b) $3\sqrt{2} + 5\sqrt{2}$

Solution (a) $5a + 4a$ (b) $3\sqrt{2} + 5\sqrt{2}$

$$= (5 + 4)a \qquad = (3 + 5)\sqrt{2}$$

$$= 9a \qquad = 8\sqrt{2}$$

This idea may be used only with **like radicals** such as $3\sqrt{2}$ and $5\sqrt{2}$. However, it is sometimes necessary to express radicals in simplest form before the addition or subtraction is possible.

EXAMPLE 2. *Simplify* $\sqrt{27} - 2\sqrt{3} + 3\sqrt{12}$

Solution $\sqrt{27} - 2\sqrt{3} + 3\sqrt{12} = \sqrt{9} \cdot \sqrt{3} - 2\sqrt{3} + 3\sqrt{4} \cdot \sqrt{3}$

$$= 3\sqrt{3} - 2\sqrt{3} + 6\sqrt{3}$$

$$= 7\sqrt{3}$$

EXAMPLE 3. *Simplify* $\sqrt{20} + 8\sqrt{7} + 3\sqrt{45} - 2\sqrt{28}$

Solution $\sqrt{20} + 8\sqrt{7} + 3\sqrt{45} - 2\sqrt{28}$

$$= \sqrt{4} \times \sqrt{5} + 8\sqrt{7} + 3\sqrt{9} \times \sqrt{5} - 2\sqrt{4} \times \sqrt{7}$$

$$= 2\sqrt{5} + 8\sqrt{7} + 9\sqrt{5} - 4\sqrt{7}$$

$$= 11\sqrt{5} + 4\sqrt{7}$$

EXERCISE 3-4

A

1. Simplify

(a) $2\sqrt{2} + 8\sqrt{2}$ (b) $5\sqrt{3} + 2\sqrt{3}$ (c) $8\sqrt{5} - 2\sqrt{5}$

(d) $4\sqrt{7} + \sqrt{7}$ (e) $2\sqrt{10} + 2\sqrt{10}$ (f) $9\sqrt{3} - 5\sqrt{3}$

(g) $12\sqrt{11} - \sqrt{11}$ (h) $\sqrt{6} + \sqrt{6}$ (i) $10\sqrt{13} - \sqrt{13}$

2. Simplify

(a) $2\sqrt{3} + 6\sqrt{3} + \sqrt{3}$ (b) $15\sqrt{2} - 11\sqrt{2} + 3\sqrt{2}$

(c) $8\sqrt{7} + 3\sqrt{7} - 10\sqrt{7}$ (d) $\sqrt{5} + \sqrt{5} + 3\sqrt{5}$

(e) $10\sqrt{10} + 3\sqrt{10} - 13\sqrt{10}$ (f) $12\sqrt{11} - 2\sqrt{11} - 5\sqrt{11}$

n	n^2	$\sqrt{n}$
1	1	1.000 000
2	4	1.414 214
3	9	1.732 051
4	16	2.000 000
5	25	2.236 068
6	36	2.449 490
7	49	2.645 751
8	64	2.828 427
9	81	3.000 000
10	100	3.162 278
11	121	3.316 625
12	144	3.464 102
13	169	3.605 551
14	196	3.741 657
15	225	3.872 983
16	256	4.000 000
17	289	4.123 106
18	324	4.242 641
19	361	4.358 899
20	400	4.472 136
21	441	4.582 576
22	484	4.690 416
23	529	4.795 832
24	576	4.898 979
25	625	5.000 000
26	676	5.099 020
27	729	5.196 152
28	784	5.291 503
29	841	5.385 165
30	900	5.477 226
31	961	5.567 764
32	1 024	5.656 854
33	1 089	5.744 563
34	1 156	5.830 952
35	1 225	5.916 080
36	1 296	6.000 000
37	1 369	6.082 763
38	1 444	6.164 414
39	1 521	6.244 998
40	1 600	6.324 555
41	1 681	6.403 124
42	1 764	6.480 741
43	1 849	6.557 439
44	1 936	6.633 250
45	2 025	6.708 204
46	2 116	6.782 330
47	2 209	6.855 655
48	2 304	6.928 203
49	2 401	7.000 000

Express all answers in simplest form.

3. Simplify

(a) $3\sqrt{8} + 6\sqrt{8}$
(b) $2\sqrt{18} + 3\sqrt{18}$
(c) $2\sqrt{27} - 3\sqrt{3}$
(d) $7\sqrt{6} - \sqrt{24}$
(e) $\sqrt{40} + \sqrt{90}$
(f) $\sqrt{18} + 3\sqrt{8}$
(g) $4\sqrt{12} - \sqrt{27}$
(h) $2\sqrt{45} - \sqrt{20}$
(i) $3\sqrt{27} + 2\sqrt{12}$

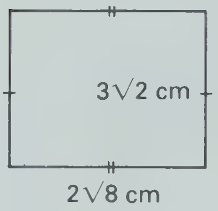
4. Simplify

(a) $5\sqrt{2} + \sqrt{18} - \sqrt{72}$
(b) $6\sqrt{5} + 3\sqrt{5} + 8\sqrt{2} - 3\sqrt{2}$
(c) $3\sqrt{5} + 8 + 2\sqrt{5} - 2$
(d) $\sqrt{32} - 5\sqrt{2} + 2\sqrt{18}$
(e) $2\sqrt{48} + 5\sqrt{27}$
(f) $5\sqrt{12} + 3\sqrt{12} - 2\sqrt{27}$
(g) $\sqrt{63} + 2\sqrt{28} - 5\sqrt{7}$
(h) $\sqrt{72} + \sqrt{75} - 3\sqrt{3} - 7\sqrt{18}$

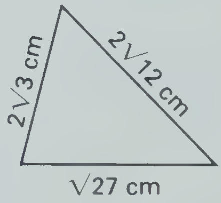
5. Find the perimeter of the following figures as a radical expression.

Perimeter means the distance around a figure.

(a)



(b)



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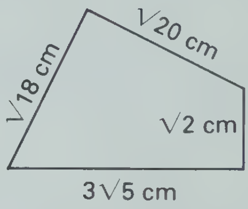
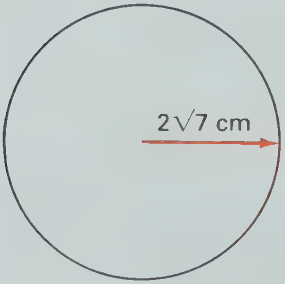


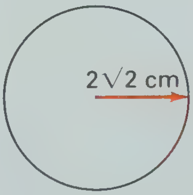
6. Find the circumference of the given circles as a radical expression.

$C = \pi d$
Use $\pi = 3.14$

(a)



(b)



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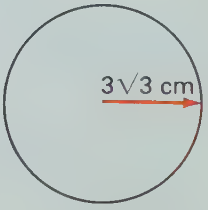


3.5 SQUARE ROOTS *Part 1*

Often it is not practical to leave a quantity expressed as a radical. We may require a decimal approximation for our calculations.

Estimating square roots

An estimate is an informed guess.

For numbers between 1 and 100:

EXAMPLE 1. *Estimate* (a) $\sqrt{7}$ (b) $\sqrt{30}$ (c) $\sqrt{78}$

Solution *From the squares of the whole numbers we know:*

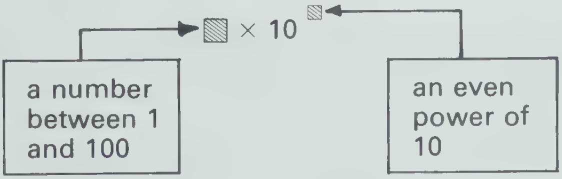
$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

(a) $\sqrt{4} < \sqrt{7} < \sqrt{9}$
$\therefore 2 < \sqrt{7} < 3$
Since 7 is closer to 9 than 4, estimate
$\sqrt{7} \doteq 2.7$

(b) $\sqrt{25} < \sqrt{30} < \sqrt{36}$
$\therefore 5 < \sqrt{30} < 6$
Since 30 is about midway between 25 and 36, estimate
$\sqrt{30} \doteq 5.5$

(c) $\sqrt{64} < \sqrt{78} < \sqrt{81}$
$\therefore 8 < \sqrt{78} < 9$
Since 78 is closer to 81 than 64, estimate
$\sqrt{78} \doteq 8.8$

For positive numbers less than 1, or greater than 100: First, express the number in the form of a factor times an even power of ten.



EXAMPLE 2. *Estimate* (a) $\sqrt{956}$ (b) $\sqrt{5284}$ (c) $\sqrt{0.029}$

Solution

(a) $956 = 9.56 \times 10^2$
$\sqrt{956} = \sqrt{9.56} \times \sqrt{10^2}$
$\sqrt{9} < \sqrt{9.56} < \sqrt{16}$
$\therefore 3 < \sqrt{9.56} < 4$
estimate 3.1
$\therefore \sqrt{956} \doteq 3.1 \times 10$
$= 31$
Check by squaring
$31^2 = 961$
$\doteq 956$

(b) $5284 = 52.84 \times 10^2$
$\sqrt{5284} = \sqrt{52.84} \times \sqrt{10^2}$
$\sqrt{49} < \sqrt{52.84} < \sqrt{64}$
$7 < \sqrt{52.84} < 8$
estimate 7.3
$\therefore \sqrt{5284} \doteq 7.3 \times 10$
$= 73$
Check by squaring
$73^2 = 5328$
$\doteq 5285$

(c) $0.029 = 2.9 \times 10^{-2}$
$\sqrt{0.029} = \sqrt{2.9} \times \sqrt{10^{-2}}$
$\sqrt{1} < \sqrt{2.9} < \sqrt{4}$
$1 < \sqrt{2.9} < 2$
estimate 1.7
$\sqrt{0.029} \doteq 1.7 \times 10^{-1}$
$= 0.17$
Check by squaring
$0.17^2 = 0.0289$
$\doteq 0.029$

EXERCISE 3-5

1. Which of the following is the best estimate of the square root? Check by using your calculator.

(a) $\sqrt{247} \doteq (13.6, 15.7, 17.2)$ (b) $\sqrt{9.53} \doteq (3.1, 3.4, 3.6)$
(c) $\sqrt{0.384} \doteq (0.58, 0.60, 0.62)$ (d) $\sqrt{74.2} \doteq (8.6, 8.8, 9.0)$
(e) $\sqrt{5790} \doteq (76.1, 78.4, 80.5)$ (f) $\sqrt{0.0784} \doteq (0.24, 0.26, 0.28)$
(g) $\sqrt{372} \doteq (17.8, 19.3, 20.9)$ (h) $\sqrt{3.6} \doteq (1.7, 1.8, 1.9)$

2. Evaluate the following expressions.

(a) $3\sqrt{4}$ (b) $2\sqrt{9}$ (c) $\sqrt{4} + \sqrt{36}$
(d) $2\sqrt{36} - \sqrt{25}$ (e) $4\sqrt{100} - 4\sqrt{49}$ (f) $3\sqrt{16} + 3\sqrt{144}$
(g) $2\sqrt{64} - 2\sqrt{49}$ (h) $\sqrt{36} + \sqrt{121}$

3. Evaluate

(a) $\sqrt{1.44}$ (b) $\sqrt{1.69}$ (c) $\sqrt{\frac{25}{36}}$ (d) $\sqrt{\frac{81}{100}}$
(e) $\sqrt{0.25}$ (f) $\sqrt{0.49}$ (g) $\sqrt{2.25}$ (h) $\sqrt{\frac{144}{49}}$

4. Express in the form

$$\underbrace{▨}_{\text{a number between 1 and 100}} \times 10^{▨ \leftarrow \text{an even power of 10}}$$

then find the positive square root of each of the following numbers.

(a) $900 = 9 \times 10^2$ (b) $2500 = 25 \times 10^2$ (c) $0.0049 = 49 \times 10^{-4}$
$\sqrt{900} = ▨$ $\sqrt{2500} = ▨$ $\sqrt{0.0049} = ▨$
(d) 6400 (e) 40 000 (f) 0.16 (g) 14 400
(h) 0.0016 (i) 36 000 000

5. Use estimation to match the following numbers with their square roots.

Numbers		Square Roots	
(a) 53	(f) 8.2	(i) 4.12	(vi) 0.14
(b) 187	(g) 473	(ii) 44.5	(vii) 13.7
(c) 0.93	(h) 1984	(iii) 7.28	(viii) 2.28
(d) 17	(i) 0.15	(iv) 0.39	(ix) 2.86
(e) 0.02	(j) 5.2	(v) 21.7	(x) 0.96

6. Estimate the square roots of the following numbers to two significant digits.

(a) 32	(b) 60	(c) 21	(d) 38
(e) 593	(f) 750	(g) 1480	(h) 5920
(i) 0.42	(j) 0.085	(k) 0.124	(l) 0.0072

7. Estimate the square roots of the following numbers.

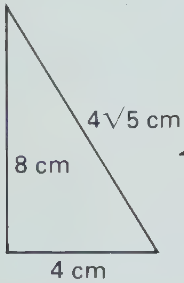
(a) 87.4	(b) 12.7	(c) 58.4	(d) 6.25
(e) 574	(f) 1382	(g) 0.168	(h) 0.0384

8. Simplify the following expressions and estimate the decimal equivalent.

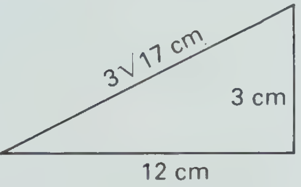
(a) $5\sqrt{6} + 2\sqrt{6}$	(b) $2\sqrt{12} + 5\sqrt{3}$	(c) $5\sqrt{2} - \sqrt{32}$
(d) $2\sqrt{9} + 4\sqrt{19}$	(e) $5\sqrt{8} - 2\sqrt{2}$	(f) $4\sqrt{27} + 5\sqrt{12}$

9. Find the perimeter of the triangles to the nearest centimetre.

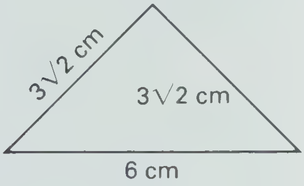
(a)



(b)



(c)



3.6 SQUARE ROOTS *Part II*

Two common ways of handling a large number of square root problems are to use either a calculator or a set of square root tables.

A table of square roots is usually accompanied by a table of squares.

Finding Square Roots From Tables

1. For Whole Numbers From 1 to 100

Take the answer directly from the tables.

2. For Fractions Greater Than 1 and Numbers Greater Than 100

Bracket the number in the n^2 column and estimate the next digit. The estimate may be improved by dividing and averaging.

EXAMPLE 1. *Find* (a) $\sqrt{43}$ (b) $\sqrt{59.6}$ (c) $\sqrt{7705}$

Solution

(a) *From the tables* $\sqrt{43} \doteq 6.557$

(b) *From tables* $\sqrt{59} \doteq 7.681$

$\sqrt{60} \doteq 7.745$

Estimate $\sqrt{59.6} \doteq 7.72$

Divide $59.6 \div 7.72 = \sqrt{7.72}$

$\therefore \sqrt{59.6} \doteq 7.72$

(*c*) *From table of squares* $7569 = 87^2$

$7744 = 88^2$

Estimate $7705 \doteq 87.8^2$

$\sqrt{7705} \doteq 87.8$

Divide $7705 \div 87.8 = 87.76$

$$\text{Average } \frac{87.8 + 87.76}{2} = 87.78$$

$\therefore \sqrt{7705} \doteq 87.78$

3. *For Numbers Between 0 and 1 or Numbers Greater Than 9801*

Use the form ▨×10$^{▨}$

EXAMPLE 2. *Find* (*a*) $\sqrt{0.53}$ (*b*) $\sqrt{86\,490}$

Solution

(*a*) $0.53 = 53 \times 10^{-2}$

$\therefore \sqrt{0.53} = \sqrt{53 \times 10^{-1}}$

$\doteq 7.280 \times 10^{-1}$

$\doteq 0.728$

(*b*) $86\,490 = 8.6490 \times 10^4$

$\therefore \sqrt{86\,490} = \sqrt{8.6490} \times 10^2$

$\sqrt{8} = 2.828$

$\sqrt{9} = 3.000$

Estimate $\doteq 2.95 \times 10^2$

$= 295$

Divide $86\,490 \div 295 = 293$

$$\text{Average } \frac{295 + 293}{2} = 294$$

$\sqrt{86\,490} \doteq 294$

or

$86\,490 = 864.9 \times 10^2$

From the tables $841 = 29^2$

$900 = 30^2$

Estimate $864.9 \doteq 29.4^2$

$\therefore \sqrt{86\,490} \doteq 29.4 \times 10$

$= 294$

EXERCISE 3-6

A 1. Using the table of squares and square roots, determine the following values.

(a) $\sqrt{23}$ (b) $\sqrt{74}$ (c) 46^2 (d) $\sqrt{95}$ (e) 39^2

(f) $\sqrt{50}$ (g) $\sqrt{61}$ (h) $\sqrt{17}$ (i) 83^2 (j) $\sqrt{8}$

2. Use the table of square roots and the fact that $\sqrt{100a} = 10\sqrt{a}$ to determine the following values

(a) $\sqrt{3600}$ (b) $\sqrt{500}$ (c) $\sqrt{1900}$ (d) $\sqrt{9500}$ (e) $\sqrt{4500}$

(f) $\sqrt{800}$ (g) $\sqrt{2700}$ (h) $\sqrt{1500}$ (i) $\sqrt{6300}$ (j) $\sqrt{7500}$

B 3. Use the table of squares to find the following values.

(a) $\sqrt{4489}$ (b) $\sqrt{6241}$ (c) $\sqrt{361}$ (d) $\sqrt{1156}$ (e) $\sqrt{2401}$

(f) $\sqrt{2916}$ (g) $\sqrt{841}$ (h) $\sqrt{9604}$ (i) $\sqrt{1936}$ (j) $\sqrt{7921}$

4. Use the table of square roots to evaluate the following expressions.

(a) $2\sqrt{3}$ (b) $5\sqrt{10}$ (c) $\dfrac{\sqrt{35}}{2}$ (d) $\sqrt{20} + \sqrt{3}$

(e) $\sqrt{4} + \sqrt{13}$ (f) $\dfrac{\sqrt{18}}{7}$ (g) $\dfrac{5\sqrt{2}}{2}$ (h) $\dfrac{\sqrt{7} + \sqrt{31}}{10}$

5. Use the table of squares and square roots to estimate the following square roots. Divide and average to improve your answer.

(a) $\sqrt{13.25}$ (b) $\sqrt{563.1}$ (c) $\sqrt{3.569}$ (d) $\sqrt{0.593}$

(e) $\sqrt{0.6932}$ (f) $\sqrt{0.0025}$ (g) $\sqrt{942.3}$ (h) $\sqrt{0.005\,971}$

6. Evaluate each expression using the method indicated in (a).

(a) $\sqrt{490} = 7\sqrt{10}$
$= 7 \times$ ▨
$=$ ▨
(b) $\sqrt{160}$ (c) $\sqrt{175}$

(d) $\sqrt{360}$ (e) $\sqrt{240}$ (f) $\sqrt{480}$

3.7 APPLICATIONS OF SQUARE ROOT

In some problems the approximate value of a square root is required. The method used in the calculation depends on the accuracy required and on the "tools" that are available.

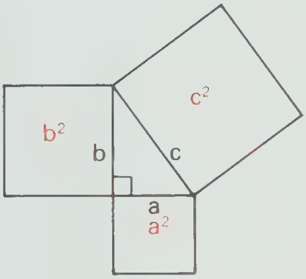
In any right-angled triangle, the relationship of the lengths of the sides is given by the formula.

$$c^2 = a^2 + b^2$$

From the formula we see that $c = \sqrt{a^2 + b^2}$

$a^2 = c^2 - b^2 \qquad a = \sqrt{c^2 - b^2}$

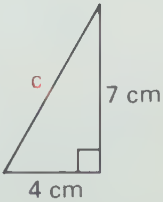
$b^2 = c^2 - a^2 \qquad b = \sqrt{c^2 - a^2}$



EXAMPLE 1. *(a) Find c if a = 7 cm and b = 4 cm*

Solution $c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$
$= \sqrt{7^2 + 4^2}$
$= \sqrt{49 + 16}$
$= \sqrt{65}$
$c = 8.062$ cm (from tables)
$\doteq 8$ cm



(b) Find b if c = 2.3 cm and a = 1.9 cm

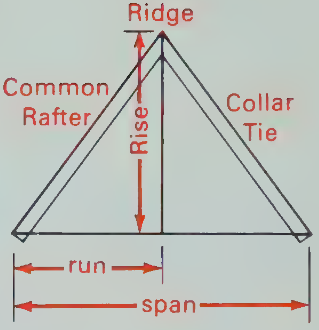
Solution

$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$= \sqrt{2.3^2 - 1.9^2}$
$= \sqrt{5.29 - 3.61}$
$= \sqrt{1.68}$
$b \doteq 1.3$ cm (estimate from tables)

EXERCISE 3-7

1. Find the lengths of the following common rafters.

Rafter Length	Rise	Run	Span
(a)	2.0 m	4.5 m	
(b)	2.4 m	7.2 m	
(c)	1.6 m	2.4 m	
(d)	2.0 m		12.0 m
(e)	1.2 m		9.6 m



2. When laying out a foundation it is important that the corners be square.

(a) Calculate the lengths of the marked diagonals for checking the layout.

(b) Make a model layout on corrugated cardboard using pins for stakes on a scale of 1 cm = 1 m (or use paper and compasses).

(i) Measure and layout *AB*.

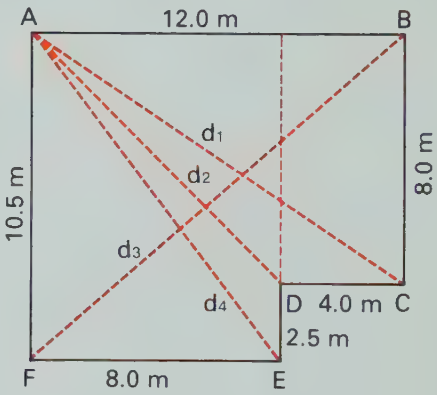
(ii) Calculate d_1 and place *C* the correct distance from *A* and *B*.

(iii) Calculate d_2 and place *D* the correct distance from *A* and *C*.

(iv) Calculate d_3 and place *F* the correct distance from *A* and *B*.

(v) Calculate d_4 and place *E* the correct distance from *A* and *F*.

Check your angles using a protractor.



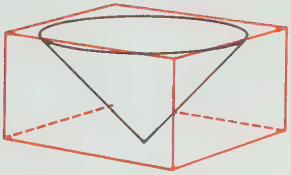
3. The voltage *V* required for a circuit is given by $V = \sqrt{PR}$ where *P* is the power in watts and *R* is the resistance in ohms.

(a) What voltage is required to light a 24 W bulb with a resistance of 6 ohms?

(b) What voltage is required to light a 100 W bulb with a resistance of 144 ohms?

(c) The current *I*, in amperes, is given by $I = \sqrt{\frac{P}{R}}$

Find the current for each of the above.

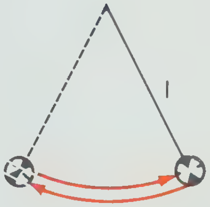


4. The radius of the base of a cone is given by the formula
$r = \sqrt{\frac{3V}{\pi h}}$ (r = radius, V = volume, h = height, π = 3.14)
A conical storage bin can have a maximum height of 3.0 m to fit in a shed. If the bin must hold 5 m^3, what must the radius be?

5. The time for an object to fall a distance h, is given by the formula
$t = \sqrt{\frac{h}{4.9}}$ (t = time in seconds, h = height in metres).
Calculate to the nearest tenth of a second, the time for an object to fall.
(a) 95 m (b) 115.3 m (c) 150 m.

6. A ship left port and sailed 28 km due east and then 17 km due north. To the nearest kilometre, how far was the ship from port? A neat sketch should be included in your solution.

7. The period of a simple pendulum is given by
$T = 2\pi\sqrt{\frac{l}{9.8}}$ where T is the period in seconds, l is the length in metres, and π = 3.14.



1 period is a complete swing

(a) A clockmaker is making a grandfather clock. If the pendulum has a length of 1.00 m, how long will it take for one complete swing?
(b) What is the period of a wall clock with pendulum 0.25 m long?

REVIEW EXERCISE

1. Complete the following table.

(a)

Fraction Form	$\frac{3}{4}$	$-2\frac{1}{8}$	$5\frac{3}{10}$	$-1\frac{7}{20}$	$\frac{49}{1000}$
Decimal Form					

(b)

Decimal Form	0.75	− 2.39	4.625	− 1.95	0.025
Fraction Form					

2. Simplify
(a) $\sqrt{3} \times \sqrt{3}$ (b) $(\sqrt{5})^2$ (c) $\sqrt{2} \times \sqrt{5}$
(d) $\sqrt{10} \times \sqrt{3}$ (e) $3 \times 2\sqrt{5}$ (f) $5\sqrt{7} \times 4$
(g) $3\sqrt{2} \times 2\sqrt{5}$ (h) $3\sqrt{5} \times 5\sqrt{6}$ (i) $3\sqrt{3} \times \sqrt{3}$
(j) $\frac{3}{4} \times 8\sqrt{2}$ (k) $15\sqrt{19} \times 2$ (l) $3\sqrt{7} \times 8\sqrt{2}$

3.
(a) $2\sqrt{3} + 5\sqrt{3}$ (b) $3\sqrt{7} + \sqrt{7}$ (c) $4\sqrt{2} + 7\sqrt{2}$
(d) $12\sqrt{11} + 12\sqrt{11}$ (e) $9\sqrt{6} + 5\sqrt{6}$ (f) $8\sqrt{15} - 3\sqrt{15}$
(g) $9\sqrt{17} - \sqrt{17}$ (h) $\sqrt{3} + 5\sqrt{3} - 2\sqrt{3}$ (i) $7\sqrt{5} - 6\sqrt{5}$
(j) $\sqrt{6} - \sqrt{6}$ (k) $12\sqrt{7} - 11\sqrt{7}$ (l) $11\sqrt{13} - 11\sqrt{13}$

4. Express the following as mixed radicals in simplest form

(a) $\sqrt{18}$ (b) $\sqrt{12}$ (c) $\sqrt{20}$ (d) $3\sqrt{8}$ (e) $2\sqrt{72}$
(f) $\sqrt{48}$ (g) $3\sqrt{45}$ (h) $5\sqrt{63}$ (i) $\sqrt{52}$ (j) $5\sqrt{50}$

5. Simplify.

(a) $\sqrt{3} \times \sqrt{6}$ (b) $\sqrt{10} \times \sqrt{2}$ (c) $5\sqrt{2} \times \sqrt{6}$
(d) $5\sqrt{6} \times 3\sqrt{2}$ (e) $8\sqrt{7} \times 3\sqrt{7}$ (f) $5\sqrt{6} \times 3\sqrt{15}$
(g) $\sqrt{5} \times \sqrt{50}$ (h) $\sqrt{18} \times \sqrt{27}$

6. Simplify

(a) $\sqrt{25} + 3\sqrt{9}$ (b) $3\sqrt{8} + 5\sqrt{8}$ (c) $\sqrt{40} + \sqrt{90}$
(d) $5\sqrt{8} - 3\sqrt{2}$ (e) $4\sqrt{12} - \sqrt{27}$ (f) $2\sqrt{48} + 5\sqrt{27}$
(g) $\sqrt{50} + 2\sqrt{18} - 10\sqrt{2}$ (h) $6\sqrt{5} - 3 + \sqrt{20} + 8$
(i) $2\sqrt{18} + \sqrt{8} - 3\sqrt{2}$ (j) $3\sqrt{8} - \sqrt{72}$
(k) $8\sqrt{2} - \sqrt{98}$ (l) $\sqrt{32} - 5\sqrt{2} + \sqrt{18}$

7. Use tables or calculator to find the following square roots.

(a) $\sqrt{75}$ (b) $\sqrt{15}$ (c) $\sqrt{0.95}$
(d) $\sqrt{4624}$ (e) $\sqrt{7569}$ (f) $\sqrt{729}$
(g) $\sqrt{29.16}$ (h) $\sqrt{43.56}$ (i) $\sqrt{0.0576}$

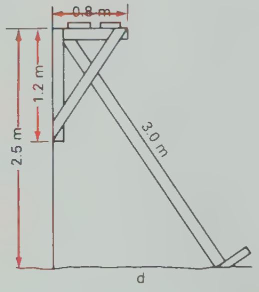
8. Estimate the following square roots to two significant figures.

(a) $\sqrt{379}$ (b) $\sqrt{41}$ (c) $\sqrt{0.63}$
(d) $\sqrt{8560}$ (e) $\sqrt{11.79}$ (f) $\sqrt{0.0837}$

9. A triangular scaffolding support has sides 1.2 m and 0.8 m.

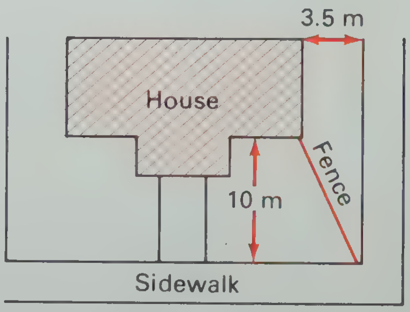
(a) How long must the bracing pieces be?

(b) The scaffold is 2.5 m above the ground and the scantling holding it against the wall is 3 m long. How far should the foot be from the wall?



10. A householder builds a fence from the corner of his house to the street corner to stop pedestrians from cutting across his lawn.

If the house is 3.5 m from the side street and 10 m from the front street, how long is the fence?



REVIEW AND PREVIEW TO CHAPTER 4

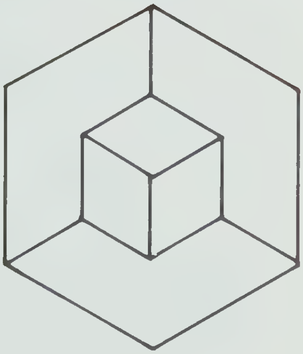
EXERCISE 1

Simplify the following

1. $5x + 10 + 6x + 3$
2. $3a + 2b + 5b + 6a$
3. $6y + 10y - 6 + 3y$
4. $5a - 7 - 2a + 8$
5. $7a - 12 - 9a - 6a$
6. $15 - 3x - 7y + 6x$
7. $16m - 3mn + 6m$
8. $14x - 3y - 7x - 8y$
9. $4a - 3b - 6a - 7 - 2b + 6$
10. $5x^2 + 6 - 5x - 6x + 3x^2$
11. $5xy + 3x - 7y + 6xy - 5x - 6y$
12. $6a^2 - 5a - 3a^2 + 6a - 4a + 2$
13. $6ab - 7b + 3a - 4ab - 6a$
14. $3a^2 - 5 + 6a - 7 - 6a^2 - 3$
15. $3a - 16x - 5 - 8x - 3 - 5a$

EXERCISE 2

Simplify the following using the distributive property.



1. $3(x + 7)$
2. $4(2a - 6)$
3. $5(3a^2 + 2a + 2)$
4. $2(a + 6)$
5. $3(x^2 - 2x - 7)$
6. $5(2x - 3y)$
7. $4(3a^2 + 2a - 4)$
8. $(3m - 5)$
9. $4(2a - 4) + 3(a + 2)$
10. $6(3x + 4) + 2(x + 7)$
11. $2(3b - 2) + 4(4b - 1)$
12. $3(2x^2 + 3x - 1) + (2x + 2)$
13. $7(3a^2 + 6a - 2) + (2a^2 - 3a + 1)$
14. $2(3b - 1) + (4b + 2) + 3(6 - b)$
15. $3(1 - 2x^2) + (3x - x^2 + 2) - 7$

EXERCISE 3

1. If $a = 2$, $b = 1$ and $c = 0$, evaluate the following:

(a) $2a + 3b$
(b) $6a - 3b$
(c) $5ab$
(d) $6ab - 2bc$
(e) $7abc$
(f) $a^2 + b^2$
(g) $2a^2 + 3b^2 - c$
(h) $2a^2b$
(i) $a^3 - 1$

2. If $I = Prt$, find the value of I when

(a) $P = 1000$, $r = 1$, $t = 3$
(b) $P = 500$, $r = 0.5$, $t = 2$
(c) $P = 200$, $r = 0.04$, $t = 6$
(d) $P = 2000$, $r = 0.06$, $t = 10$
(e) $P = 600$, $r = 0.12$, $t = 3$

CHAPTER 4

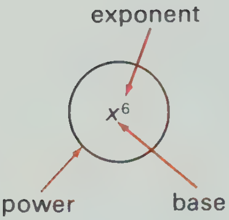
Polynomials

4.1 EXPONENTS IN MULTIPLICATION AND DIVISION

2^5 means $2 \times 2 \times 2 \times 2 \times 2$
$3 \times 3 \times 3 \times 3 = 3^4$
State the following using exponents
$y \times y \times y \times y = y^{\square}$
$5 \times 5 \times 5 \times 5 \times 5 = 5^{\square}$
We can write numbers as powers. For example:

$$125 = 5 \times 5 \times 5 = 5^3$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$
$$= 4 \times 4 \times 4 = 4^3$$
$$= 8 \times 8 = 8^2$$

We develop the rules for multiplying and dividing powers using positive exponents.



$a^1 = a$

$$a^2 \times a^3 = \underbrace{\overbrace{a \times a} \times \overbrace{a \times a \times a}}$$
$$= a^5$$
$$a^2 \times a^3 = a^{2+3} = a^5$$

$$a^7 \div a^3 = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times a \times a \times a \times a}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}}}$$
$$= a^4$$
$$a^7 \div a^3 = a^{7-3} = a^4$$

For powers with the same base
$$x^a \times x^b = x^{a+b} \qquad x^a \div x^b = x^{a-b}$$

EXAMPLE 1.

Simplify (a) $5x^3 \times 3x^4$

(b) $\dfrac{(8a^4b)(3a^3b^2)}{6a^2b}$

Solution

We arrange the factors so that we deal with the numerical factors first, then follow with the variables in alphabetical order.

(a) $5x^3 \times 3x^4 = 5 \times 3 \times x^3 \times x^4$
$= 15x^7$

(b) $\dfrac{(8a^4b)(3a^3b^2)}{6a^2b}$

$$= \frac{8 \times 3 \times a^4 \times a^3 \times b \times b^2}{6a^2b}$$
$$= \frac{24a^7b^3}{6a^2b}$$
$$= 4a^5b^2$$

EXERCISE 4-1

1. Evaluate

(a) 2^3 (b) 3^2 (c) 4^2 (d) 2^4

2. Simplify.

(a) $2^2 \times 5^2$ (b) $3^3 \times 2$

3. Simplify.

(a) $x^3 \times x^5$ (b) $a^2 \times a^{10}$ (c) $b \times b \times b$
(d) $m^2 \times m^3 \times m$ (e) $a^2 \times a^3 \times b \times b^4$ (f) $a^5 \times a^2 \times b^3 \times b$

4. Simplify

(a) $a^7 \div a^3$ (b) $b^3 \div b^2$ (c) $n^{12} \div n^3$
(d) $x^8 \div x^5$ (e) $a^5 \div a$ (f) $x^{10} \div x^9$

5. Simplify

(a) $(5a^3)(3a^6)$ (b) $(4x^2)(-2x^3)$ (c) $(3a^2b^3)(2ab^2)$
(d) $(2x)(5x^3)$ (e) $5mn \times 3m$ (f) $(-4x^3)(-3x^2)$
(g) $3y \times 5y \times 2y^2$ (h) $(a)(2a^2)(-3a^5)$
(i) $(-7x^3)(-2x^3)(-x^2)$

6. Simplify

(a) $12a^5 \div 3a^3$ (b) $21x^2y^5 \div 7xy$ (c) $8a^2 \div 8a$
(d) $48m^2n \div 6mn$ (e) $30m^2 \div (-6m^2)$
(f) $(-18ab^3) \div (-2ab)$ (g) $56pqr \div 8pr$
(h) $(-24a^2b^2) \div (8a^2b^2)$ (i) $(15 \times {}^{10}y^2) \div (-5xy)$

4.2 POWER LAWS

In the expression ab^3, the exponent 3 applies only to the factor b. If the 3 is to apply to both factors, the expression must be written:

$$\begin{aligned} &(ab)^3 \\ &= (ab)(ab)(ab) \\ &= a \times a \times a \times b \times b \times b \\ &= a^3b^3 \end{aligned}$$

The following table illustrates the power laws for exponents.

$(x^2)^3 = x^2 \times x^2 \times x^2$ $= x^6$ $(x^2)^3 = x^{2\times3} = x^6$	$(ab)^2 = (ab)(ab)$ $= a \times a \times b \times b$ $= a^2b^2$ $(ab)^2 = a^{1\times2} \times b^{1\times2}$ $= a^2b^2$	$\left(\frac{a}{b}\right)^3 = \frac{a\ a\ a}{b\ b\ b}$ $= \frac{a^3}{b^3}$ $\left(\frac{a}{b}\right)^3 = \frac{a^{1\times3}}{b^{1\times3}} = \frac{a^3}{b^3}$
$(a^2)^7 = a^{2\times7} = a^{14}$ $(b^5)^2 = b^{5\times2} = b^{10}$	$(x^4y)^2 = x^{4\times2}y^{1\times2} = x^8y^2$ $(2x^6)^3 = 2^{1\times3}x^{6\times3} = 8x^{18}$	$\left(\frac{2}{3}\right)^3 = \frac{2^{1\times3}}{3^{1\times3}} = \frac{8}{27}$ $\left(\frac{a}{5}\right)^2 = \frac{a^{1\times2}}{5^{1\times2}} = \frac{a^2}{25}$
$\boxed{(x^a)^b = x^{a\times b} = x^{ab}}$	$(xy)^a = x^ay^a$ $(x^ay^b)^c = x^{ac}y^{bc}$	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ $\left(\frac{x^a}{y^b}\right)^c = \frac{x^{ac}}{y^{bc}}$

Note that:
3×2^3
$= 3 \times 8$
$= 24$

But note that:
$(3 \times 2)^3$
$= 6^3$
$= 216$

EXAMPLE 1. *Simplify (a)* $(2x^4)^3$ *(b)* $(x^2y^3)^5$
(c) $\left(\dfrac{5x^3}{y}\right)^2$

Solution

(a) $(2x^4)^3 = 2^3 \times x^{4\times 3}$
$= 8x^{12}$

(b) $(x^2y^3)^5 = x^{2\times 5}\,y^{3\times 5}$
$= x^{10}y^{15}$

(c) $\left(\dfrac{5x^3}{y}\right)^2 = \dfrac{5^2\,x^{3\times 2}}{y^2}$
$= \dfrac{25x^6}{y^2}$

EXAMPLE 2. *Simplify (a)* $(3x^3)^2\,(2x^5)$
(b) $\dfrac{(4x^3)^2}{-8x}$

Solution

If the last digit of the number 3^3 is 7, what is the last digit of $(3^3)^3$?

(a) $(3x^3)^2 \times (2x^5) = 3^2\,(x^3) \times (2x^5)$
$= 9x^6 \times 2x^5$
$= 18x^{11}$

(b) $\dfrac{(4x^3)^2}{-\,8x} = \dfrac{4^2 \times (x^3)^2}{-\,8x}$
$= \dfrac{16x^6}{-\,8x}$
$= -\,2x^5$

EXERCISE 4-2

A 1. Simplify

(a) $(x^4)^2$
(b) $(a^3)^3$
(c) $(a^2b)^3$
(d) $(xy^3)^5$
(e) $(abc)^5$
(f) $(b^8)^3$
(g) $(2x^3)^3$
(h) $(a^5b^2)^3$
(i) $(3a^5)^2$
(j) $(3xy^2)^3$
(k) $(5a^8)^3$
(l) $(4x^2yz)^3$

2. Simplify

(a) $\left(\dfrac{x}{y}\right)^5$
(b) $\left(\dfrac{a^2}{3}\right)^2$
(c) $\left(\dfrac{x^2}{y}\right)^3$
(d) $\left(\dfrac{a}{b^5}\right)^4$
(e) $\left(\dfrac{3x}{y}\right)^2$
(f) $\left(\dfrac{2x^2}{w}\right)^3$
(g) $\left(\dfrac{5a^2}{2b^3}\right)^2$
(h) $\left(\dfrac{3a}{b^3}\right)^2$

B 3. Simplify

(a) $(-\,2a^3)^3$
(b) $(3x \times x^2)^2$
(c) $\left(\dfrac{12x^5}{4x^3}\right)^2$
(d) $(2a^3b \times 3a^4)$
(e) $\left(\dfrac{a^5b}{c^4}\right)^3$
(f) $\left(\dfrac{-\,32a^5b}{-\,16ab}\right)^2$
(g) $\left(\dfrac{48x^3y^7}{-\,12xy}\right)^3$
(h) $\left(\dfrac{(5a^2)\,(-\,12a^5)}{15a^2}\right)^3$
(i) $\left(\dfrac{-\,51a^4b^2c}{(17a^2b)\,(b^2c)}\right)^3$

4. Simplify

(a) $(2x^2)^3 \times (2x^5)$

(b) $\dfrac{(8ab^2)(3a^2b)}{12a^3}$

(c) $\dfrac{(3x^2y^5)^3}{9xy^2}$

(d) $\dfrac{(12m^2n^5)(-5mn^3)}{15m^3n^2}$

(e) $\dfrac{(-12x^2)(-4x^2y)}{(-6x^3)}$

(f) $\dfrac{-32m^{10}n^3}{(8m^5)(mn)}$

4.3 ZERO AND NEGATIVE EXPONENTS

In the division of powers, the subtraction of exponents presents two new problems.

Zero as an Exponent

$$\overset{\text{3 factors}}{x^3} = x \times x \times x$$

$$\frac{x^5}{x^5} = x^{5-5} = x^0 \qquad \text{and} \qquad \frac{x^5}{x^9} = x^{5-9} = x^{-4}$$

The problem is not in determining the exponents, but in deciding what these exponents mean. To evaluate 2^0, we simplify $2^3 \div 2^3$ in two ways.

(i) $\dfrac{2^3}{2^3} = 2^{3-3} = 2^0$

(ii) $\dfrac{2^3}{2^3} = \dfrac{8}{8} = 1$

$2^0 = 1$

$x^a \div x^b = x^{a-b}$

Negative Exponents

To evaluate a^{-2}, simplify $a^3 \div a^5$ in two ways.

(i) $a^3 \div a^5 = a^{3-5} = a^{-2}$

(ii)
$$a^3 \div a^5 = \frac{a^3}{a^5} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times a \times a} = \frac{1}{a^2}$$

$$a^{-2} = \frac{1}{a^2}$$

Use this idea to evaluate 3^{-2}, 2^{-4}, and 10^{-1}. These results suggest the following rules for exponents.

$$\boxed{x^0 = 1} \qquad \boxed{x^{-1} = \frac{1}{x}}$$

In both of these rules, $x \neq 0$.

EXAMPLE 1. *Evaluate* (*a*) $4^{-2} \times 3^0$
(*b*) $3^{-1} + 2^{-2}$

Solution

(*a*)
$$\begin{aligned} 4^{-2} \times 3^0 &= \frac{1}{4^2} \times 1 \\ &= \frac{1}{16} \end{aligned}$$

(*b*)
$$\begin{aligned} 3^{-1} + 2^{-2} &= \frac{1}{3} + \frac{1}{2^2} \\ &= \frac{1}{3} + \frac{1}{4} \\ &= \frac{4+3}{12} \\ &= \frac{7}{12} \end{aligned}$$

EXAMPLE 2. *Simplify* (*a*) $a^5 \div a^{-3}$ (*b*) $(x^{-3}y^2) \times (x^3y^{-4})$

Solution

(*a*)
$$\begin{aligned} a^5 \div a^{-3} &= a^{5-(-3)} \\ &= a^{5+3} \\ &= a^8 \end{aligned}$$

(*b*)
$$\begin{aligned} (x^{-3}y^2) \times (x^3y^{-4}) &= x^{-3} \times x^3 \times y^2 \times y^{-4} \\ &= x^0 \times y^{-2} \\ &= 1 \times y^{-2} \\ &= y^{-2} \end{aligned}$$

What time will it be
293 647 h after 20:00?

EXERCISE 4-3

A 1. Simplify
(a) $x^3 \times x^{-5}$ (b) $a^{-1} \times a^8$ (c) $y^0 \times y^4$
(d) $b^{-1} \times b^{-3}$ (e) $a^{10} \times a^3 \times a^{-5}$ (f) $x \times x^0$
(g) $b^{-5} \times b^5$ (h) $m \times m^{-1} \times m^0$

2. Simplify
(a) $x^{10} \div x^5$ (b) $b^9 \div b^{12}$ (c) $a^0 \div a^3$
(d) $x^{-3} \div x^2$ (e) $m^{-5} \div m^0$ (f) $n^4 \div n^{-3}$
(g) $0 \div y^5$ (h) $m^{-8} \div m^{-4}$

3. Evaluate
(a) 4^0 (b) 2^{-1} (c) $(-3)^0$
(d) 10^{-1} (e) 3^{-2} (f) 10^0
(g) 4^{-2} (h) 2^{-3} (i) 2^{-4}

B 4. Simplify
(a) $3x^5 \times 5x^{-2}$ (b) $(a^2b^5)(a^3b^{-8})$ (c) $(2y^5)(3y^{-5})$
(d) $x^{10} \times x^{-3} \times x^{-5}$ (e) $(3m^{-1})^2$ (f) $(3a^2)(5a^{-8})$
(g) $(m^2n)(m^5n^{-1})$ (h) $(2a^5b^{-3})^3$ (i) $(x^{-5}y^{-2})^{-1}$
(j) $(5xy^{-1})(7x^3y^{-1})$ (k) $(7m^3)(m^{-5}n^{-2})$ (l) $(5a^{-3})(3a^3)$

5. Simplify

(a) $a^{-3} \div a^{-5}$ (b) $12x^5 \div 4x^{10}$ (c) $24b^{-5} \div 6b^5$

(d) $m^2n \div m^5n^0$ (e) $a^{12} \div a^{15} \times a^3$ (f) $15a^4b^5 \div 5a^2b^7$

(g) $b^0 \div b^{-4}$ (h) $(y^4 \times y^2) \div y^{10}$ (i) $(4x^{-2})^2$

(j) $(3x^{-5})^{-1} \times 3x^2$ (k) $\dfrac{12b^2 \times 8b^{-4}}{6b^{-10}}$ (l) $(2a^3)^{-3} \times 4a^{-5}$

6. Evaluate

(a) $5^0 + 5^{-1}$ (b) $3^{-1} + 4^{-1}$ (c) $(5^{-1})^2$

(d) $(4^{-3} \times 4^2)^2$ (e) $(2^{-1})^{-1}$ (f) $(\frac{1}{4})^{-1}$

(g) $(\frac{2}{3})^0$ (h) $5^{-1} + 2^{-2}$ (i) $(\frac{1}{10})^{-1}$

(j) 10^{-2} (k) $(\frac{1}{3})^{-2}$ (l) $10^3 \times 10^{-5}$

$5^2 = 0 \quad 100 + 25 = 25$
$15^2 = 1 \quad 200 + 25 = 225$
$25^2 = 2 \quad 300 + 25 = 625$
$35^2 =$
$45^2 =$
$85^2 =$

4.4 EVALUATING POLYNOMIALS

Expressions such as $3x + 2y + 4$ are called polynomials.

EXAMPLE 1. *If $a = 2$, $b = 3$ and $c = 4$, evaluate*

$$3a + 8b^2 - 2ac$$

Solution

$$\begin{aligned} 3a + 8b^2 - 2ac &= 3(2) + 8(3)^2 - 2(2)(4) \\ &= 6 + 8(9) - 2(8) \\ &= 6 + 72 - 16 \\ &= 62 \end{aligned}$$

EXAMPLE 2. *If $x = 2$, $y = -1$, $m = -2$, evaluate*

$$2xy^2 - 3xym - m^2$$

Solution

$$\begin{aligned} 2xy^2 - 3xym - m^2 &= 2(2)(-1)^2 - 3(2)(-1)(-2) - (-2)^2 \\ &= 2(2)(1) - 3(2)(-1)(-2) - (4) \\ &= 4 - 12 - 4 \\ &= -12 \end{aligned}$$

EXERCISE 4-4

A

1. If $a = 2$, $b = 3$, $c = 1$, evaluate

(a) $2a + 3b$ (b) $2bc$

(c) $6a - 6c$ (d) $bc - a$

(e) $abc - a$ (f) $b^2 - c^2$

(g) $7a - 2b$ (h) $a^2 - 2$

2. If $x = 3$, $y = 0$, $b = 4$, evaluate

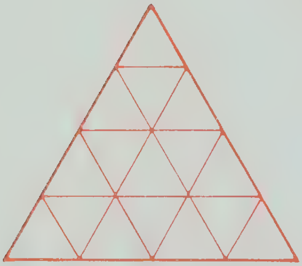
(a) $2x - 2y$ (b) $3b + x$

(c) $4xy$ (d) $2xb$

(e) $2xb - 1$ (f) $b^2 - y^2$

(g) $2x^2$ (h) $3xyb$

Four men can build four boats in four days. How long will it take one man to build a boat?



How many parallelograms do you see?

B **3.** If $a = 1$, $b = 2$, $c = 3$, evaluate

(a) $2a^2 + b^2$ (b) $3abc + 2c^2$
(c) $3ab^2 - 2$ (d) $6bc - a^2$
(e) $2a^2b^2 + c^2$ (f) $6abc - 2a^2 + b$

4. If $x = 2$, $y = -2$, $m = -1$, evaluate

(a) $2x + 3y - 2m$ (b) $3xy + 6m + 5$
(c) $x - y - m$ (d) $-2xm - 3ym + 2$
(e) $2xym - m + 7$ (f) $-2y - 3xm - m$

5. If $a = -1$, $b = -2$, $d = 3$, evaluate

(a) $2a^2 + b^2$ (b) $3b^2 - 2ad + 3$
(c) $7 - 3d^2 + 2a^2b^2$ (d) $2a^2b^2 - 3 + bd^2$
(e) $-2d^2 + b^2 - a^2$ (f) $-abd - ab^2 + 3$
(g) $4a^2 + b^2 - d^2$ (h) $-2a - b - 3b^2a + 1$

6. If $x = -2$, $m = -3$, $c = 0$, $d = -1$, evaluate

(a) $\dfrac{2xm}{d}$ (b) $\dfrac{3m - 2x}{2m}$

(c) $\dfrac{2x^2 + 2d^2 + m^2}{2x}$ (d) $\dfrac{3xmd - 2mc}{3}$

(e) $\dfrac{1}{2x - 3m}$ (f) $\dfrac{2xm^2c}{-4}$

4.5 ADDITION AND SUBTRACTION OF POLYNOMIALS

The distributive property may be used to simplify algebraic expressions of the following form:

$a(b + c)$
$= ab + ac$

EXAMPLE 1. *Expand and simplify $2(a + 2b) + 4(2a - 3b)$.*

Solution

$$\begin{aligned} &2(a + 2b) + 4(2a - 3b) \\ &= 2(a + 2b) + 4(2a - 3b) \\ &= 2a + 4b + 8a - 12b \\ &= 10a - 8b \end{aligned}$$

EXAMPLE 2. *Expand*

(a) $-2(3x + 2)$ *(b)* $-(3a - 2)$

$(+a)(+b) = ab$
$(-a)(-b) = ab$
$(-a)(+b) = -ab$
$(+a)(-b) = -ab$

Solution

(a) $-2(3x + 2)$
$= -2(3x + 2)$
$= -6x - 4$

(b) $-(3a - 2)$
$= -1(3a - 2)$
$= -1(3a - 2)$
$= -3a + 2$

EXAMPLE 3. *Expand the following and simplify*

(a) $2(x - 3) - 5(x + 7)$
(b) $2x(x - 3) - x(2x - 3)$

Solution

(a) $2(x-3)-5(x+7)$

$= 2(x-3)-5(x+7)$

$= 2x-6-5x-35$

$= -3x-41$

(b) $2x(x-3)-x(2x-3)$

$= 2x(x-3)-x(2x-3)$

$= 2x^2-6x-2x^2+3x$

$= -3x$

Test your skill
Divide $\frac{3}{4}$ by $5\frac{1}{2}$

EXERCISE 4-5

A Expand the following

1. (a) $2(x+3)$ (b) $3(2a+1)$
(c) $4(1+2b)$ (d) $5(3x-2)$
(e) $-2(x-5)$ (f) $-6(1-3a)$
(g) $-(x+6)$ (h) $-3(x-y)$

2. (a) $x(3x+1)$ (b) $2a(1+3a)$
(c) $-2b(b+3)$ (d) $-3m(1-2m)$
(e) $-a(2a-4)$ (f) $ab(1-c)$
(g) $2x(x^2-2x+1)$ (h) $-a(1-2a-a^2)$

B Expand and simplify

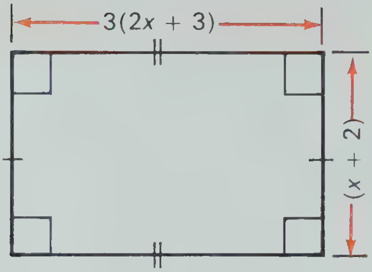
3. (a) $2(x+3)+3(x+5)$
(b) $4(2a-3)+3(1+5a)$
(c) $2(m+2)+3(m+5)+4(m-6)$
(d) $3(d-4)+2(5-3d)$
(e) $6(1-2x+x^2)+(2x^2-6x+5)$
(f) $5(a-3b)+6(2b-a)+2(a-b)$
(g) $4(a-2b+c)+3(2a+5b-6c)$
(h) $2(a^2-2a+2)+3+5(a^2-6a+1)$

4. (a) $3(x-5)-2(x+7)$
(b) $4(2x-3y)-2(x-y)+3(x-2y)$
(c) $5(3a-1)-4(2-3a)-(5a+1)$
(d) $-2(x^2-3x-1)-5(2x^2+x-3)$
(e) $-2(m-3)+7m-3(1-2m)+6$
(f) $3(a-2b+c)-3(2b-3a-c)+a$
(g) $5(4y-3)-(2y-7)+3(1-3y)$
(h) $3(1-2x+x^2)-2(3-x-x^2)+2x^2$
(i) $-(1-3m+a)-4(2-7m-3a)+6$
(j) $-3(3a-2b+c)-5(2b-c)+4(a+3c)$

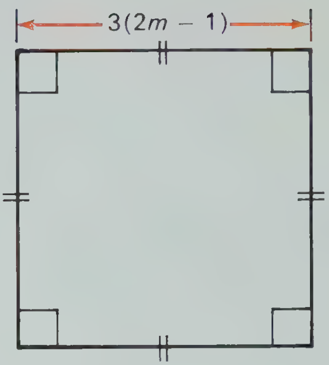
5. (a) $2x(x-7)-3x(2x+1)$
(b) $a(2a-3)-3a(1-2a)-a(3a-2)$
(c) $2x(x^2-2x+1)-3(x-2)-x(1-3x)$
(d) $3b(2b-7)-b(1+3b)-2b(b+3)$
(e) $2a(a-3)-4(a^2-2a-2)-a(1-3a)$
(f) $3b(a-b)-2a(2a-3b)-b(a-3b)$
(g) $5(x^2-2x+7)+3x(2x-1)-6x$
(h) $-(1-3a+2a^2)+a(a-7)-3(a^2+6)$
(i) $2m(3m-2n)-4n(3m-6n)$
(j) $2x(1-3x)-2x(4x+1)-6(x-3)$

6. Find the perimeters of the following

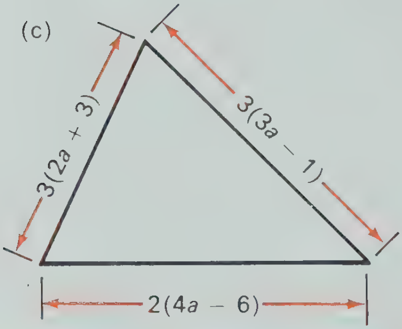
(a)



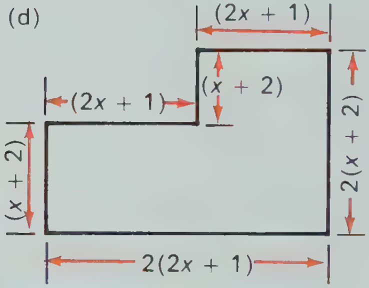
(b)



(c)



(d)



4.6 MULTIPLICATION OF POLYNOMIALS

We use the distributive property to find the product of two binomials.

EXAMPLE 1. *Expand* $(x + 2)(x + 3)$

Solution

$$
\begin{aligned}
&(x + 2)(x + 3)\\
&= (x + 2)(x + 3)\\
&= (x + 2)x + (x + 2)3\\
&= x^2 + 2x + 3x + 6\\
&= x^2 + 5x + 6
\end{aligned}
$$

	x	3
x	x^2	$3x$
2	$2x$	6

$(x + 2)(x + 3)$

$= x^2$	$+ 3x$	$+ 2x$	$+ 6$
F	O	I	L
product of First terms	product of Outside terms	product of Inside terms	product of Last terms

The product is found by multiplying each term of the first factor by each term of the second factor.

$$
\begin{aligned}
&(x + 2)(x + 3)\\
&= (x + 2)(x + 3)\\
&= x^2 + 3x + 2x + 6\\
&= x^2 + 5x + 6
\end{aligned}
$$

EXAMPLE 2. *Expand and simplify*

(a) $(3a - 2)(2a + 3)$ (b) $(2x - y)(x - y)$

Solution

(a) $(3a - 2)(2a + 3)$

$$= (3a - 2)(2a + 3)$$
$$= 6a^2 + 9a - 4a - 6$$
$$= 6a^2 + 5a - 6$$

(b) $(2x - y)(x - y)$

$$= (2x - y)(x - y)$$
$$= 2x^2 - 2xy - xy + y^2$$
$$= 2x^2 - 3xy + y^2$$

EXERCISE 4-6

B Expand and simplify

1. (a) $(x + 3)(x + 1)$ (b) $(a + 5)(a + 7)$
(c) $(m + 3)(m + 6)$ (d) $(t + 11)(t + 3)$
(e) $(d + 6)(d + 7)$ (f) $(4 + m)(2 + m)$
(g) $(r + 6)(r + 10)$ (h) $(1 + s)(s + 12)$
(i) $(a + 3)(a + 3)$ (j) $(3 + x)(3 + x)$

2. (a) $(m - 3)(m + 2)$ (b) $(x - 5)(x - 6)$
(c) $(a - 5)(a + 6)$ (d) $(b - 3)(b + 6)$
(e) $(y - 3)(y - 3)$ (f) $(m - 10)(m + 6)$
(g) $(d - 3)(d + 3)$ (h) $(s - 7)(s - 8)$
(i) $(6 - a)(3 - a)$ (j) $(t + 6)(5 - t)$

3. (a) $(2x + 1)(x + 6)$ (b) $(3a - 2)(2a - 1)$
(c) $(2m - 7)(m + 2)$ (d) $(3x - 1)(3x + 1)$
(e) $(2b - 3)(2b - 3)$ (f) $(d + 6)(2d - 5)$
(g) $(1 - 3r)(5 + 2r)$ (h) $(4r + 6)(r - 3)$
(i) $(3t - 5)(2t + 3)$ (j) $(1 - 3x)(5x - 7)$

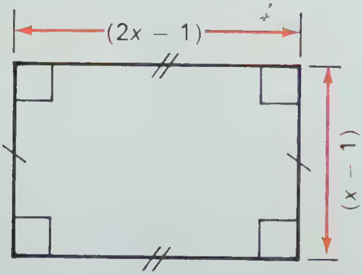
4. (a) $(7x - 5)(x - 2)$ (b) $(3m + 6)(m - 2)$
(c) $(1 - 5b)(1 + 5b)$ (d) $(3x - y)(3x + y)$
(e) $(2a + b)(3a + b)$ (f) $(a + b)(a + b)$
(g) $(3x - 2y)(x - y)$ (h) $(2b + c)(3b - 2c)$
(i) $(3m + 2n)(2m - n)$ (j) $(x - y)(x + y)$

Using standard Canadian coins, in how many different ways is it possible to make change for 50¢?

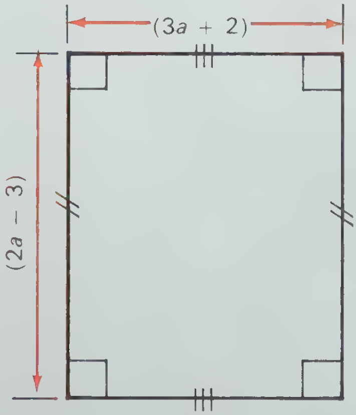
How could you make change for 49¢ using exactly 8 coins?

5. Find the area of the following:

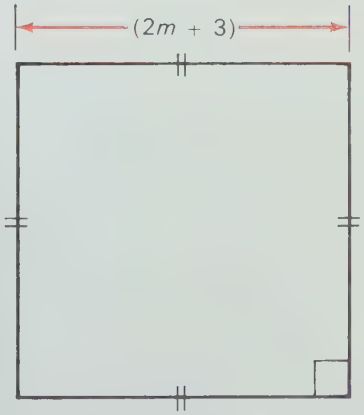
(a)



(b)



(c)



Test your skill:
$2\frac{1}{2}\%$ of 500 is -?

C **EXAMPLE 3.** *Expand and simplify* $(x + 3)(x^2 - 2x + 2)$

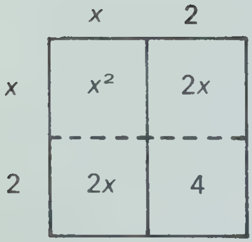
Solution

$$\begin{aligned}&(x + 3)(x^2 - 2x + 2)\\ &= (x + 3)(x^2 - 2x + 2)\\ &= x^3 - 2x^2 + 2x + 3x^2 - 6x + 6\\ &= x^3 + x^2 - 4x + 6\end{aligned}$$

6. Expand and simplify
(a) $(a + 3)(a^2 + 2a + 1)$
(b) $(x + 2)(x^2 + 3x + 3)$
(c) $(m - 2)(2m^2 - m - 1)$
(d) $(s - 3)(2s^2 + 3s - 3)$
(e) $(2x - 1)(x^2 - 3x - 4)$
(f) $(2b + 3)(2b^2 - 3b + 6)$
(g) $(2a^2 - 3a + 2)(3a + 5)$
(h) $(x^2 - 2x - 1)(x^2 + 3x + 1)$
(i) $(2m^2 - 3m + 1)(m^2 + m - 3)$
(j) $(y^2 + 2y - 3)(2y^2 - 3y - 1)$

4.7 SQUARE OF A BINOMIAL

The squaring of a binomial is a special case of polynomial multiplication.



EXAMPLE 1. *Expand and simplify*
(a) $(x + 2)^2$ (b) $(3a - 1)^2$ (c) $(x + 2)^2 + (x - 3)^2$

Solution

(a) $(x + 2)^2$
$= (x + 2)(x + 2)$
$= x^2 + 2x + 2x + 4$
$= x^2 + 4x + 4$

(b) $(3a - 1)^2$
$= (3a - 1)(3a - 1)$
$= 9a^2 - 3a - 3a + 1$
$= 9a^2 - 6a + 1$

(c) $(x + 2)^2 + (x - 3)^2$
$= (x + 2)(x + 2) + (x - 3)(x - 3)$
$= (x^2 + 2x + 2x + 4) + (x^2 - 3x - 3x + 9)$
$= (x^2 + 4x + 4) + (x^2 - 6x + 9)$
$= x^2 + 4x + 4 + x^2 - 6x + 9$
$= 2x^2 - 2x + 13$

EXERCISE 4-7

A **1.** Expand
(a) $(a + 1)^2$ (b) $(x + 3)^2$ (c) $(m + 2)^2$
(d) $(t + 5)^2$ (e) $(n - 6)^2$ (f) $(r - 2)^2$
(g) $(x - 4)^2$ (h) $(b - 3)^2$ (i) $(r - 7)^2$

2. Expand
(a) $(m + 6)^2$ (b) $(y - 1)^2$ (c) $(m + 8)^2$
(d) $(4 + x)^2$ (e) $(5 - m)^2$ (f) $(a + 7)^2$
(g) $(r + 8)^2$ (h) $(t + 2)^2$ (i) $(1 + m)^2$

Insert brackets to make
$3 + 2 - 5 - 3 + 7 = 10$
a true statement.

B **3.** Expand

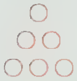
(a) $(2x + 1)^2$ (b) $(3m + 2)^2$ (c) $(5a + 1)^2$
(d) $(3t - 1)^2$ (e) $(3a - 2)^2$ (f) $(4b + 2)^2$
(g) $(5a + 3)^2$ (h) $(2x - 7)^2$ (i) $(6x - 1)^2$

4. Expand

(a) $(x - 6)^2$ (b) $(2y + 3)^2$ (c) $(5t - 3)^2$
(d) $(4b + 5)^2$ (e) $(6m + 1)^2$ (f) $(1 - 2r)^2$
(g) $(3 - 3t)^2$ (h) $(1 + 6a)^2$ (i) $(2s - 1)^2$

5. Expand and simplify

(a) $(x + 3)^2 + (x - 2)^2$
(b) $(a - 3)^2 + (a + 5)^2$
(c) $(2x + 1)^2 + (3x - 1)^2$
(d) $(m + 7)^2 + (2m - 3)^2$
(e) $(2b + 5)^2 + (3b - 1)^2$
(f) $(x + 3)^2 + (x + 1)^2 + (x + 2)^2$
(g) $(a - 2)^2 + (a - 3)^2 + (a + 1)^2$
(h) $(2x - 1)^2 + (x + 3)^2 + (3x - 2)^2$



Arrange the numbers 1, 2, 3, 4, 5, 6 in the six circles so that the sum of each side of the triangle is 9.

4.8 SIMPLIFICATION OF POLYNOMIAL EXPRESSIONS

EXAMPLE 1. *Simplify* $(x + 2)(x - 3) - 2(x - 7)$

Solution

$$\begin{aligned}
&(x + 2)(x - 3) - 2(x - 7)\\
&= (x + 2)(x - 3) - 2(x - 7)\\
&= (x^2 - 3x + 2x - 6) - 2x + 14\\
&= (x^2 - x - 6) - 2x + 14\\
&= x^2 - x - 6 - 2x + 14\\
&= x^2 - 3x + 8
\end{aligned}$$

EXAMPLE 2. *Simplify* $2(3a - 1)(a + 2) - (2a + 3)^2$

Solution

$$\begin{aligned}
&2(3a - 1)(a + 2) - (2a + 3)^2\\
&= 2(3a^2 + 6a - a - 2) - (4a^2 + 12a + 9)\\
&= 2(3a^2 + 5a - 2) - (4a^2 + 12a + 9)\\
&= 6a^2 + 10a - 4 - 4a^2 - 12a - 9\\
&= 2a^2 - 2a - 13
\end{aligned}$$

EXERCISE 4-8

A **1.** Simplify

(a) $2(x + 2)$ (b) $3(2x - 1)$ (c) $4(a - 5)$
(d) $(a - 7)$ (e) $2(1 - 3c)$ (f) $-3(1 + 2x)$
(g) $-4(2x - 1)$ (h) $-(x - 7)$ (i) $2(a + 2b - c)$
(j) $-3(2x + 4y)$

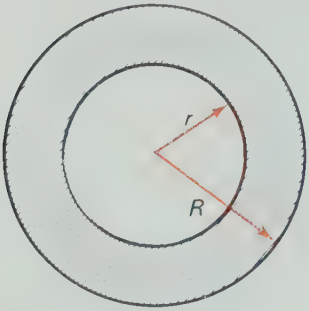
B **2.** Simplify
(a) $2(x - 3) + 3(x - 2)$
(b) $2(2a - 1) - 3(3a + 2)$
(c) $4(3m + 2) - (2m - 1)$
(d) $3(2b - 1) - (3b - 2) + 6$
(e) $4(3c - 6) - 2(1 - 3c) + 7c$
(f) $3(x + 1)(x + 2)$
(g) $2(2x - 1)(x - 3)$
(h) $4(x + 3)^2$
(i) $-2(x + 3)(x - 7)$

3. Simplify
(a) $(a + 7)(a - 2) + (a - 1)(a + 2)$
(b) $(x - 3)(x - 6) + (x + 2)^2$
(c) $(2m - 1)(m + 3) - (m - 4)(m + 2)$
(d) $3(2d - 7) + (2d + 3)(d - 7)$
(e) $(b - 4)^2 - (b + 3)^2$
(f) $2(m - 6) - (3m + 1)^2 + 6$

4. Simplify
(a) $2(x - 3)(x + 2) + 3(2x - 1)$
(b) $3(2a + 1)(a - 2) - 4(3a + 6)$
(c) $2(t + 5)^2 - 3(2t + 1)(t + 5)$
(d) $3(2b - 1)(3b + 2) - (3b - 1)^2$
(e) $2(x - 3)(x + 3) - 3(x - 7)^2$
(f) $2(3m - 1)(3m + 1) - (2m - 3)(m + 2)$

5. Simplify
(a) $2(x + 3)(2x - 1) - 3(2x - 4) + 7$
(b) $(2m - 1)^2 + 3(m - 2)(2m + 1) + 6$
(c) $(1 - 2x)(1 + 3x) - (1 - 2x)^2 + 3x$
(d) $4(3x - 1) - (2x + 1)^2 + (2x - 1)(2x + 1)$
(e) $2(3m^2 - 2m + 1) - (1 - 3m) + (m + 1)^2$
(f) $3(1 - 2b) + 2(b - 3)^2 - (2b - 1)(b + 3)$

Area of a ring:
$A = \pi(R + r)(R - r)$



Find the area of a ring with inside radius 2.4 cm, outside radius 5.6 cm.

4.9 DIVISION OF A POLYNOMIAL BY A MONOMIAL

$$\frac{3}{13} + \frac{5}{13} = \frac{3+5}{13} = \frac{8}{13}$$

EXAMPLE 1. *Simplify* (a) $\dfrac{4xy}{2x}$ (b) $\dfrac{-18m^2nt}{-6mn}$

Solution

(a) $$\frac{4xy}{2x} = 2y$$

(b) $$\frac{-18m^2nt}{-6mn} = 3mt$$

EXAMPLE 2. *Simplify* (a) $\dfrac{2a + 6}{2}$ (b) $\dfrac{3ab + 6bc}{3b}$

Solution

(a) $$\frac{2a + 6}{2} = \frac{2a}{2} + \frac{6}{2} = a + 3$$

(b) $$\frac{3ab + 6bc}{3b} = \frac{3ab}{3b} + \frac{6bc}{3b} = a + 2c$$

EXERCISE 4-9

A Simplify

1. (a) $\frac{4ab}{2}$ (b) $\frac{6xyt}{6}$ (c) $\frac{10ab}{a}$

(d) $\frac{12abc}{6a}$ (e) $\frac{7mnt}{7m}$ (f) $\frac{6dem}{-3}$

(g) $\frac{-24bc}{-6b}$ (h) $\frac{-21abc}{7abc}$ (i) $\frac{-18mnt}{-6mn}$

2. (a) $\frac{6x^2y}{6x}$ (b) $\frac{3a^2b}{a^2}$ (c) $\frac{6x^2y}{3xy}$

(d) $\frac{-12abc^2}{3abc}$ (e) $\frac{3m^2n^2}{3mn}$ (f) $\frac{x^6}{x^2}$

(g) $\frac{18x^2m^2n}{-3x^2m}$ (h) $\frac{12x^6y^4}{3x^2y^3}$ (i) $\frac{21a^4b^2c}{-7a^2bc}$

B Simplify

3. (a) $\frac{3x + 6}{3}$ (b) $\frac{2ab + 6b}{2}$ (c) $\frac{10a - 5b}{5}$

(d) $\frac{4mn + 6mt}{2m}$ (e) $\frac{6x^2 - 4x}{2x}$ (f) $\frac{4x^3 - 6x^2 + 8x}{2x}$

(g) $\frac{3ab - 6ab^2 + 9a^2b}{3ab}$

4. (a) $\frac{4x^2 - 8x}{-2x}$ (b) $\frac{6a^2b - 9ab^2 - 12ab}{-3ab}$

(c) $\frac{16 + 4a - 12a^2}{-4}$ (d) $\frac{-7t + 14t^2 - 21t^3}{-7t}$

(e) $\frac{-27abc^2}{-9abc}$ (f) $\frac{6x^2y - 3xy + 3xy^2}{-3xy}$

4.10 DIVISION OF A POLYNOMIAL BY A BINOMIAL

EXAMPLE 1. *Divide $-2x^2 + 2x^3 - 5x + 2$ by $x + 2$*

Solution

(a) Arrange the terms of the divisor and dividend so that the exponents are in descending or ascending order:

$$x + 2\overline{)2x^3 - 2x^2 - 5x + 2}$$

Divide 146 by 3

$3\overline{)146}$

(b) Divide x of the divisor into $2x^3$ of the dividend to obtain $2x^2$ in the quotient:

$$\begin{array}{r} 2x^2 \\ x + 2\overline{)2x^3 - 2x^2 - 5x + 2} \end{array}$$

$$\begin{array}{r} 4 \\ 3\overline{)146} \end{array}$$

$$\begin{array}{r} 4 \\ 3\overline{)146} \\ 12 \end{array}$$

(c) Multiply the divisor $x + 2$ by $2x^2$ to obtain $2x^3 + 4x^2$:

$$\begin{array}{rrrrr}
 & & 2x^2 & & \\ \cline{2-5}
x+2\big) & 2x^3 & -\,2x^2 & -\,5x & +\,2 \\
 & 2x^3 & +\,4x^2 & &
\end{array}$$

$$\begin{array}{r} 4 \\ 3\overline{)146} \\ \underline{12} \\ 26 \end{array}$$

(d) Subtract $2x^3 + 4x^2$ from the dividend:

$$\begin{array}{rrrrr}
 & & 2x^2 & & \\ \cline{2-5}
x+2\big) & 2x^3 & -\,2x^2 & -\,5x & +\,2 \\
 & 2x^3 & +\,4x^2 & & \\ \cline{2-3}
 & & -\,6x^2 & -\,5x &
\end{array}$$

$$\begin{array}{r} 48 \\ 3\overline{)146} \\ \underline{12} \\ 26 \\ \underline{24} \\ 2 \end{array}$$

(e) Repeat the procedure by dividing x of the divisor into $-6x^2$. Continue the process until the remainder is a constant.

$$\begin{array}{lrrrrrl}
 & & & & & & \text{QUOTIENT} \\
 & & & 2x^2 & -\,6x & +\,7 & \\ \cline{3-6}
 & x+2\big) & 2x^3 & -\,2x^2 & -\,5x & +\,2 & \text{DIVIDEND} \\
\text{DIVISOR} & & 2x^3 & +\,4x^2 & & & \\ \cline{3-4}
 & & & -\,6x^2 & -\,5x & & \\
 & & & -\,6x^2 & -\,12x & & \\ \cline{4-5}
 & & & & 7x & +\,2 & \\
 & & & & 7x & +\,14 & \\ \cline{5-6}
\text{REMAINDER} & & & & & -\,12 &
\end{array}$$

EXAMPLE 2. *Divide $4x^3 + 9x + 5$ by $2x + 1$*

Solution A placeholder for the term containing x^2 must be placed in the dividend:

$$2x+1\overline{)4x^3 + 0x^2 + 9x + 5}$$

The division procedure is now followed:

$$\begin{array}{rrrrr}
 & & 2x^2 & -\,x & +\,5 \\ \cline{2-5}
2x+1\big) & 4x^3 & +\,0x^2 & +\,9x & +\,5 \\
 & 4x^3 & +\,2x^2 & & \\ \cline{2-3}
 & & -\,2x^2 & +\,9x & \\
 & & -\,2x^2 & -\,x & \\ \cline{3-4}
 & & & 10x & +\,5 \\
 & & & 10x & +\,5 \\ \cline{4-5}
 & & & & 0
\end{array}$$

EXERCISE 4-10

A Arrange the following in descending powers of the variable

1. (a) $2x^2 + 3x^3 - 2x + 1$
  (b) $5a - 3a^2 + 2 + 6a^3$
  (c) $5m^4 + 2m - 3m^3 + m^2 + 7$

(d) $1 - 4b^5 + 3b^3 - b^2$
(e) $7 - 6x^6 + 2x^2 - 3x^4$

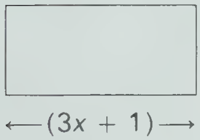
B Divide the following

2. (a) $(x^2 + 4x + 3) \div (x + 1)$
(b) $(a^2 - a - 12) \div (a + 3)$
(c) $(5m + m^2 - 14) \div (m - 2)$
(d) $(2x^2 + 5x - 12) \div (x + 4)$
(e) $(6a^2 - a - 2) \div (3a - 2)$
(f) $(6m^2 - 7m + 6) \div (2m - 3)$

3. (a) $(x^3 + x^2 - 3x - 2) \div (x + 2)$
(b) $(2a^3 - 7a^2 + 7a - 2) \div (a - 2)$
(c) $(3x + 6x^3 - 13x^2 + 1) \div (2x - 1)$
(d) $(19m^3 + 15m^4 + 7m + 21m^2 - 2) \div (3m + 2)$
(e) $(10b^2 - 5b - 2 + 12b^4 - 5b^3) \div (1 + 4b)$
(f) $(2x^3 - x^2 - 4x + 8) \div (2x + 3)$

4. (a) $(9m^3 - 7m - 2) \div (3m + 2)$
(b) $(6x^3 - 7x^2 + 1) \div (2x - 1)$
(c) $(12t^2 + 6t^3 + 6t) \div (2t + 2)$
(d) $(6a^2 + 12a^4 - 4a^3 + 2 - 8a) \div (3a - 1)$
(e) $(x^3 - 1) \div (x - 1)$
(f) $(2x^4 - x^3 - 6x + 7) \div (2x - 1)$

5.



If the area of the given rectangle is $6x^2 - 7x - 3$ square units, find the width.

6. If $I = \frac{E}{R}$, find I if $R = 2a + 3$ and $E = 6a^3 + 5a^2 - 4a + 3$.

7. $P = I^2 \times R$ where P is power, I is amperage and R is resistance. If $P = 2x^3 + 11x^2 + 12x - 9$ and $I^2 = x^2 + 6x + 9$, find R.

4.11 FACTORING

If you had $1 000 000 and you gave away $10 a minute, how long would it take you to give away all your money?

Many actions or events are reversible. For the sake of explanation we could list them as "doing" and "undoing" operations.

Doing	Undoing
tying your shoe	untying your shoe
open the door	close the door
start the car	stop the car
(5 × 3 = 15)	(15 = 5 × 3)
multiplying	factoring

Factoring is the reverse operation of multiplying. When factoring an expression we find two or more factors whose product is the given expression. 24 may be factored in many ways.

$24 = 24 \times 1$
$24 = 12 \times 2$
$24 = 8 \times 3$
$24 = 6 \times 4$
$24 = 6 \times 2 \times 2$
$24 = 3 \times 2 \times 2 \times 2$

A factor of an expression divides into the expression with no remainder.

8 is a factor of 24 since $\frac{24}{8} = 3$
3 is a factor of 24 since $\frac{24}{3} = 8$
4 is a factor of 24 since $\frac{24}{4} = 6$

24, 12, 8, 6, 4, 3, 2 and 1 are all factors of 24 since they divide into 24 exactly. What are the prime factors of 24?

EXERCISE 4-11 I

State the missing factor for each of the following:

1. (a) $24 =$ ▨6
(b) $36 = 9$▨
(c) $3x = 3$▨
(d) $4x^2 =$ ▨x
(e) $5ab = 5$▨
(f) $18x^2 =$ ▨$3x$
(g) $12xy = 3y$▨
(h) $10x^2y^2 =$ ▨$5x$
(i) $20a^2b = ab$▨
(j) $100ab^2c = 10ab$▨
(k) $4x^2yb =$ ▨$4xy$
(l) $4a + 2b =$ ▨$(2a + b)$

THE COMMON FACTOR

The factoring of algebraic expressions can be considered the opposite of expanding.

Three people are going to sit in the front seat of a car. How many possible seating arrangements are there?

$$\xleftarrow{\text{EXPANDING}}$$
$$4x + 6y = 2(2x + 3y)$$
$$\xrightarrow[\text{FACTORING}]{}$$

When the terms of a polynomial have a common factor, the polynomial can be factored using the distributive property.

EXAMPLE 1. *Factor the following*

(a) $2x + 6y$ (b) $3x^2 + 6x$ (c) $3a^2 - 6a + 3$

Solution

(a) $2x + 6y = 2(x + 3y)$
(b) $3x^2 + 6x = 3x(x + 2)$
(c) $3a^2 - 6a + 3 = 3(a^2 - 2a + 1)$

Common factors

Factoring the Common Factor

1. Find the common factor(s) by inspection.
2. Obtain the remaining factor by division.

EXAMPLE 2. *Factor the following*

(a) $3a + 6$ (b) $2x + 8y$ (c) $2a^2 - 6a$
(d) $10x^2y - 5xy^2$ (e) $3x + 9x^2 + 6x^3$

Solution

(a) $$3a + 6 = 3\left(\frac{3a+6}{3}\right) = 3(a+2)$$

(b) $$2x + 8y = 2\left(\frac{2x+8y}{2}\right) = 2(x+4y)$$

(c) $$2a^2 + 6a = 2a\left(\frac{2a^2+6a}{2a}\right) = 2a(a+3)$$

(d) $$10x^2y - 5xy^2 = 5xy\left(\frac{10x^2y - 5xy^2}{5xy}\right) = 5xy(2x-y)$$

(e) $$3x + 9x^2 + 6x^3 = 3x\left(\frac{3x+9x^2+6x^3}{3x}\right) = 3x(1+3x+2x^2)$$

Common Factors found by inspection

Remaining Factor found by division

EXERCISE 4-11 II

Factor the following

1. (a) $2x + 6$ (b) $3a - 9$ (c) $12 - 6b$
(d) $4a - 2b$ (e) $3a + 6b - 9c$ (f) $5a^2 - 10b + 15$
(g) $4b^2 + 4$ (h) $3x^2 + 6x - 3$ (i) $20xy - 10b$
(j) $2t - 4m + 6r$

2. (a) $4x^2 + 6x$ (b) $5ab - 6ac$
(c) $6xy + 3xt + 9ax$ (d) $2a^2 - 4a$
(e) $12a^2 - 6ab$ (f) $10x - 5x^2$
(g) $8b^3 - 4b^2 + 6b$ (h) $3m - 6m^2 - 9m^3$
(i) $6b^2 - 2b + 10b^3$ (j) $3m^4 - 6m^3 + 12m^2$

3. (a) $3xy + 7xyt$ (b) $7mn + 6m^2n$
(c) $12a^2b - 6ab^2 + ab$ (d) $2rt^2 + 6rt^3 - 4r^2t$
(e) $10xym - 5xyt$ (f) $14pqr - 7p^2qr + pqrt$
(g) $4ab + 2b(x + y)$ (h) $3(x + y) - a(x + y)$
(i) $x(x + 2) + 3(x + 2)$ (j) $a(a - 3) - 2(a - 3)$
(k) $m(m - 1) - 2(m - 1)$ (l) $t(t + 5) + 6(t + 5)$

Test your skill:
$3 + 2\frac{1}{2} - 1.25$

4.12 FACTORING THE DIFFERENCE OF SQUARES

Expanding $(a + b)(a - b) = a^2 - b^2$.
Factoring $a^2 - b^2 = (a + b)(a - b)$.

EXAMPLE 1. *Factor (a)* $x^2 - 25$ *(b)* $9a^2 - 4$

Solution

(a) $x^2 - 25 = x^2 - 5^2 = (x + 5)(x - 5)$

(b) $9a^2 - 4 = (3a)^2 - 2^2 = (3a + 2)(3a - 2)$

Insert brackets to make
$3 - 4 + 2 - 5 - 1 - 7 = 4$
a true statement.

EXERCISE 4-12

A 1. Find the remaining factors

(a) $x^2 - 4 = (x + 2)(▨)$
(b) $a^2 - 9 = (a - 3)(▨)$
(c) $m^2 - 36 = (m + 6)(▨)$
(d) $m^2 - 25 = (▨)(m - 5)$
(e) $4x^2 - 1 = (▨)(2x + 1)$
(f) $16 - n^2 = (4 - n)(▨)$

B 2. Factor

(a) $x^2 - 49$
(b) $m^2 - 16$
(c) $a^2 - 144$
(d) $b^2 - 121$
(e) $c^2 - 100$
(f) $d^2 - 64$

3. Factor

(a) $4x^2 - 16$
(b) $1 - y^2$
(c) $16m^2 - 1$
(d) $9a^2 - 25$
(e) $100 - a^2$
(f) $9t^2 - 36$
(g) $4m^2 - 49$
(h) $1 - 36x^2$
(i) $49 - 9a^2$

4. Factor

(a) $4a^2 - 9b^2$
(b) $16x^2 - 25y^2$
(c) $a^2 - 16b^2$
(d) $144m^2 - 16b^2$
(e) $25b^2 - 49x^2$
(f) $1 - 400t^2$
(g) $25a^2 - 16b^2$
(h) $144b^2 - 36t^2$
(i) $225x^2 - 1$

5. Factor and find the value of

(a) $9^2 - 4^2$
(b) $6^2 - 3^2$
(c) $2^2 - 1$
(d) $7^2 - 2^2$
(e) $10^2 - 8^2$
(f) $12^2 - 2^2$
(g) $99^2 - 98^2$
(h) $75^2 - 73^2$
(i) $27^2 - 20^2$

A bottle and its cork cost a total of \$1.50. The bottle costs one dollar more than the cork. How much does each cost?

4.13 FACTORING A TRINOMIAL

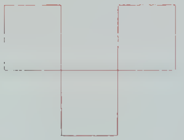
Certain trinomials can be factored into the product of two binomials. Observe the following examples:

This number is the sum of these numbers

$$x^2 + 5x + 6 = (x + 3)(x + 2)$$
$$x^2 + 3x + 2 = (x + 1)(x + 2)$$
$$x^2 - x - 12 = (x - 4)(x + 3)$$
$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

This number is the product of these numbers

The three unit squares in the arrangement shown form a figure with a perimeter of 12 units. What is the smallest perimeter you can get by rearranging the squares?



EXERCISE 4-13

A 1. Complete the following statements

(a) $x^2 + 7x + 12 = (x + 3)(x + ▨)$
(b) $x^2 + 8x + 15 = (x + ▨)(x + 3)$
(c) $x^2 - x - 6 = (x - ▨)(x + 2)$
(d) $x^2 - 7x + 12 = (x - 3)(x - ▨)$
(e) $a^2 + 2a - 15 = (a + ▨)(a - 3)$
(f) $m^2 - 6m + 9 = (▨ - 3)(m - 3)$
(g) $b^2 + 2b + 1 = (b + 1)(▨ + 1)$

(h) $c^2 - 10c + 25 = (c - 5)(c - ▨)$
(i) $t^2 + 3t - 28 = (t + 7)(▨ - 4)$
(j) $d^2 - 8d - 9 = (d - ▨)(d + 1)$

2. Complete the following statements
(a) $x^2 + 5x + 6 = (▨ + 3)(x + ▨)$
(b) $x^2 + 9x + 14 = (x + ▨)(▨ + 2)$
(c) $a^2 - 2a - 15 = (a - ▨)(▨ + 3)$
(d) $m^2 - 8m + 16 = (▨ - 4)(m - ▨)$
(e) $b^2 - 10b + 21 = (b - ▨)(▨ - 7)$
(f) $t^2 + 4t + 4 = (t + ▨)(▨ + 2)$
(g) $d^2 - 3d - 18 = (▨ - 6)(d + ▨)$
(h) $x^2 + 13x + 22 = (x + ▨)(▨ + 2)$
(i) $n^2 + 8n - 20 = (n + ▨)(▨ - 2)$
(j) $a^2 + 2a + 1 = (▨ + 1)(a + ▨)$

In general $x^2 + (a + b)x + ab = (x + a)(x + b)$

EXAMPLE. *Factor* $x^2 + 7x + 12$.

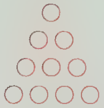
Solution

For $x^2 + 7x + 12$,
$a + b = 7$
$ab = 12$ ("a" and "b" are 4 and 3)
$x^2 + 7x + 12 = (x + 4)(x + 3)$

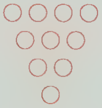
3. Complete the following table

Trinomial	$a + b$	ab	a, b	Factors
(a) $x^2 + 5x + 6$	5	6	3, 2	$(x + 3)(x + 2)$
(b) $a^2 - a - 12$	−1	−12	−4, 3	$(a - 4)(a + 3)$
(c) $m^2 - 6m + 9$	−6	9	−3, −3	
(d) $r^2 - 3r - 18$	−3	−18		
(e) $n^2 - 10n + 25$	−10			
(f) $b^2 + 2b - 15$				
(g) $t^2 + 5t + 4$				
(h) $x^2 + 9x + 14$				
(i) $c^2 - 4t - 21$				
(j) $y^2 - 8y + 16$				

4. State the remaining factors
(a) $x^2 + 7x + 10 = (x + 2)(▨)$
(b) $a^2 - a - 12 = (a - 4)(▨)$
(c) $b^2 + 8b + 7 = (▨)(b + 7)$



Move 3 circles in the arrangement above to make it look like the arrangement below.



(d) $m^2 - 6m + 5 = (m - 1)(▨)$
(e) $x^2 + x - 20 = (▨)(x - 4)$
(f) $r^2 + 4r + 4 = (▨)(r + 2)$
(g) $t^2 + 2t - 8 = (t - 2)(▨)$
(h) $b^2 - 3b - 18 = (b - 6)(▨)$
(i) $c^2 + 10c + 25 = (c + 5)(▨)$
(j) $x^2 + 2x + 1 = (▨)(x + 1)$

B 5. Factor
(a) $x^2 + 3x + 2$
(b) $a^2 + 8a + 15$
(c) $t^2 + 6t + 8$
(d) $x^2 + 10x + 25$
(e) $m^2 + 7m + 6$
(f) $b^2 - 2b - 8$
(g) $r^2 + 3r - 18$
(h) $d^2 - 7d + 12$
(i) $n^2 - 6n + 5$
(j) $a^2 - 2a - 15$
(k) $r^2 - 4r + 4$
(l) $h^2 - 7h - 18$

6. Factor
(a) $x^2 - 9x + 20$
(b) $a^2 + 14a + 24$
(c) $m^2 - 9m - 22$
(d) $t^2 + 13t + 36$
(e) $d^2 + 20d + 100$
(f) $n^2 + 4n - 21$
(g) $x^2 + 2x - 3$
(h) $a^2 - 3a - 88$
(i) $m^2 + m - 20$
(j) $t^2 - t - 20$
(k) $h^2 - 11h + 30$
(l) $a^2 + 13a + 30$

7. Factor
(a) $x^2 - 3x - 70$
(b) $a^2 - 14a + 49$
(c) $r^2 + 24r + 144$
(d) $m^2 - 11m + 24$
(e) $b^2 + 8b + 16$
(f) $t^2 - 2t - 24$
(g) $d^2 + 2d - 35$
(h) $m^2 - 8m - 33$
(i) $x^2 + 6x - 40$
(j) $h^2 - 15h + 50$
(k) $r^2 + r - 56$
(l) $t^2 + t - 6$

C 8. Factor fully (remove common factor first)
(a) $2x^2 + 10x + 12$
(b) $3m^2 + 21m + 36$
(c) $5a^2 - 10a - 40$
(d) $2t^2 - 12t + 18$
(e) $10x^2 + 20x - 150$
(f) $3b^2 + 30b + 75$
(g) $7x^2 - 14x - 21$
(h) $5x^2 - 5x - 60$
(i) $4r^2 - 16r + 16$
(j) $2m^2 + 10m + 12$

REVIEW EXERCISE

A 1. Evaluate
(a) 5^3
(b) 3^4
(c) 2^5
(d) 15^0
(e) 2^{-1}
(f) 3^{-1}
(g) $(-7)^0$
(h) 10^{-1}
(i) $(\frac{1}{2})^2$
(j) $(\frac{2}{3})^3$
(k) $3^8 \div 3^6$
(l) $5^{-3} \times 5^4$
(m) $7^{-5} \times 7^5$
(n) $2^0 \div 2$

2. Express the following using exponents.
(a) $a \times a \times a \times a$
(b) $x \times x \times x \times x \times x \times x$
(c) $x \times x \times y \times y \times y$
(d) $w \times w \times w \times y \times y \times y \times z \times z$
(e) $5 \times 5 \times 5$
(f) $3 \times 3 \times 2 \times 2 \times 2$

3. Simplify
(a) $x^3 \times x^5$
(b) $a \times a^3$
(c) $y \times y \times y^5$
(d) $b^0 \times b^5$
(e) $b^{-3} \times b^8$
(f) $x^{17} \times x^3$
(g) $w^{-3} \times w^{-2}$
(h) $x^2 \times x^5 \times y^2 \times y^8$
(i) $x^0 \times x^{-6}$
(j) $x \times x^{-1}$
(k) $a^3 \times a^{-7}$
(l) $x \times x^{-1} \times x^{-3}$

4. Simplify

(a) $x^{12} \div x^3$
(b) $a^5 \div a^4$
(c) $b^8 \div b$
(d) $w^3 \div w^5$
(e) $b^6 \div b^0$
(f) $x^8 \div x^{13}$
(g) $y^0 \div y^5$
(h) $k^{-3} \div k^2$
(i) $z \div z^4$
(j) $a^{-7} \div a^{10}$
(k) $y^{-5} \div y$
(l) $x^3 \div x^{-2}$

5. If $a = 2$, $b = 3$, $c = 0$, evaluate

(a) $2ab$
(b) $a^2 + b^2$
(c) $2bc + b$
(d) $-3ab$
(e) $4b^2$
(f) $b^2 - a^2$

6. Expand the following

(a) $3(x + 4)$
(b) $2(2a - 1)$
(c) $-3(m + 3)$
(d) $-a(2a + 3)$
(e) $y(1 - x)$
(f) $-(x - 3)$
(g) $2(x^2 - x + 1)$
(h) $-4(m^2 - 2m)$
(i) $-(x^2 - x - 7)$

7. Simplify the following

(a) $\dfrac{12abc}{6}$
(b) $\dfrac{-10xy}{-5x}$
(c) $\dfrac{m^5}{m^2}$
(d) $\dfrac{-2abc^2}{-2abc}$
(e) $\dfrac{35trm}{-7tr}$
(f) $\dfrac{0}{-3abc}$
(g) $\dfrac{-24mnp}{24mnp}$
(h) $\dfrac{6a^2b^3c}{-2ab^2c}$
(i) $\dfrac{-21b^3d^3c}{-7b^2d^3}$

8. Simplify

(a) $3a^2 \times 5a^5$
(b) $4y^2 \times 7y$
(c) $(a^2b^3)(a^5b)$
(d) $(6x^3)(-2x^5)$
(e) $(-7a^3)(-5a^5)$
(f) $(3a)(-5a)$
(g) $(a^7)(7ab)$
(h) $(3a^2b)(-5ab^4)$
(i) $(15a^5)(3a^{-3})$
(j) $(4xy)(-5xy^2)$
(k) $(5a^{-7})(3a^{-2})$
(l) $(6ab^{-2})(-3a^4b^3)$

9. Simplify

(a) $12b^3 \div 4b$
(b) $45x^7 \div 9x$
(c) $18a^2b^5 \div 2a^2$
(d) $24x^3y^3 \div 8x^3y$
(e) $12x^5 \div 3x^{-2}$
(f) $(-14a^2) \div (7a)$
(g) $54ab^3 \div 6ab$
(h) $12x^3 \div 2x^{-2}$
(i) $(-35a^5) \div (-7a^3)$
(j) $51xy^5 \div 3xy$
(k) $(56w^3) \div (-7w^{-2})$

10. Simplify

(a) $\dfrac{15x^3 \times 3x^5}{9x^4}$
(b) $\dfrac{12a^2 \times 5a^3}{4a}$
(c) $(x^3y^5)^2$
(d) $\dfrac{(3x^2)^3}{9x^4}$
(e) $(8x^{-3})(2x^7) \div 4x$
(f) $(3x^5)^2 \times (2x^{-1})^3$
(g) $\left(\dfrac{15a^2b}{3ab}\right)^2$
(h) $(5m^5n)(m^2n)^{-2}$

11. Evaluate

(a) 3^{-2}
(b) $(\frac{1}{4})^{-1}$
(c) $(\frac{3}{4})^{-1}$
(d) $4^0 + 2^{-1}$
(e) $2^{-1} + 3^{-1}$
(f) $(\frac{1}{2})^{-2}$
(g) $\dfrac{1}{3^{-1}}$
(h) $2^{-2} + 8^{-1}$

12. By estimating the answer, locate the decimal point.

(a) $21.3 \times 5.2 \doteq 1108$

(b) $1.85 \times 17.4 \doteq 322$

(c) $0.52 \times 36.9 \doteq 1919$

(d) $0.92 \times 582 \doteq 5354$

(e) $12.8 \div 2.3 \doteq 556$

(f) $5.93 \div 2.09 \doteq 284$

(g) $182.04 \div 8.735 \doteq 2084$

(h) $40.2 \div 81.1 \doteq 496$

(i) $\dfrac{5.3 \times 7.89}{12.02} \doteq 3479$

(j) $\dfrac{0.48 \times 593}{29.3} \doteq 9714$

13. Expand

(a) $(x + 2)(x + 3)$

(b) $(m + 3)^2$

(c) $(b - 2)(b - 4)$

(d) $(t - 3)(t + 3)$

(e) $(r - 2)^2$

(f) $(1 + m)^2$

(g) $(b - 4)(b + 3)$

(h) $(d - 4)(d - 5)$

(i) $(x - 1)^2$

(j) $(a - 5)(a + 5)$

14. Factor the following

(a) $2x + 6$

(b) $2x^2 - 4xy$

(c) $5mn - 10m$

(d) $x^2 - 16$

(e) $m^2 - 121$

(f) $d^2 - 64$

(g) $a^2 - 9$

(h) $1 - b^2$

(i) $c^2 - 49$

B **15.** If $a = -1$, $b = 3$, $c = -2$ and $d = 0$, evaluate

(a) $2b^2 + 3ac$

(b) $2a^2 - c$

(c) $-6abd$

(d) $2a^2b - 3b^2c$

(e) $4a^2 - 3 + 6abd$

(f) $1 - 3abc + 2c^2$

(g) $\dfrac{6ab - 2}{-5}$

(h) $\dfrac{a^2 + b^2 + c^2}{-2}$

16. Simplify

(a) $2(x - 3) - 3(1 - 2x) - (x - 4)$

(b) $2a(1 - a) - 3(2a^2 - 2) + 3(2a - 1)$

(c) $2(2a - 3b + c) - (3a - b - c) - 2(a - 2b)$

(d) $m(3m - 1) - 2m(1 - m) + 6$

(e) $2(3b^2 - 2b - 1) - b(1 + 3b) - 5$

(f) $2x(1 - 2x) - x(3x - 2) + 7x$

(g) $5 - 2c(3c - 4) + 6(c^2 - 3c + 2)$

17. Expand and simplify

(a) $(2a - 3)(a + 6)$

(b) $3(2x - 1)(x + 3)$

(c) $2(m + 7)^2$

(d) $-2(1 - 3x)(1 + 5x)$

(e) $4(2b - 1)(b - 3) + (b + 5)^2$

(f) $2(x - 3)(x + 3) - (x - 1)^2 + 5x$

(g) $3(1 + 3y)(2 - y) - 2(2y + 1)(y + 3)$

(h) $2(a - 3) - (a + 3) + 2(a - 1)(a - 5) - 3(a + 3)^2$

(i) $3(2x^2 - x - 3) - (x + 1)^2 - (2x - 1)(3 - 2x)$

(j) $2(d - 7)^2 - (2d + 3)(d - 2) + 3(d - 6)(2d + 1)$

(k) $(m - 7)(m + 7) - 2(m + 1)^2 - 3(m - 2)(m + 2)$

18. Divide

(a) $(x^2 - 6x - 7) \div (x + 1)$

(b) $(a^2 - 8a + 15) \div (a - 3)$

(c) $(2x^2 + 5x - 3) \div (2x - 1)$

(d) $(8b + b^3 - 5b^2 - 4) \div (b - 2)$

(e) $(2c^2 + 3 - 4c + 4c^3) \div (2c + 3)$

(f) $(6x^3 - x^2 + 16) \div (3x + 4)$

(g) $(8a^3 - 1) \div (2a - 1)$

(h) $(3x^3 + 2x^4 - 5x - 4x^2 + 3) \div (2x - 1)$

(i) $(6 + 5b - 2b^2 + 6b^3) \div (2 + 3b)$

19. Factor the following

(a) $4x - 12$ (b) $3a^2 + 9a$ (c) $20a - 10a^2$

(d) $2b^4 - 6b^3 + 2b^2$ (e) $3xy - 6xy^2 + 12x^2y$

(f) $12a^2bc - 6ab^2c + 3abc$ (g) $3(x + 2) - x(x + 2)$

(h) $m(x - y) - n(x - y)$ (i) $3mn(x + y) - 2m(x + y)$

20. Factor

(a) $m^2 - 121$ (b) $x^2 + 9x + 18$

(c) $b^2 - 12b + 36$ (d) $4r^2 - 49$

(e) $c^2 - 6c - 27$ (f) $a^2 + 4a - 12$

(h) $1 - 8d + 12d^2$ (g) $1 - 49a^2b^2$

(j) $y^2 + 21y + 80$ (i) $b^2 + 10b - 56$

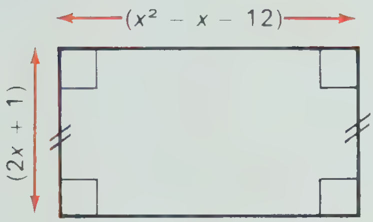
(l) $16x^4 - y^2$ (k) $36a^2 - 121b^2$

(n) $a^2 + 19a - 92$ (m) $x^2 - x - 90$

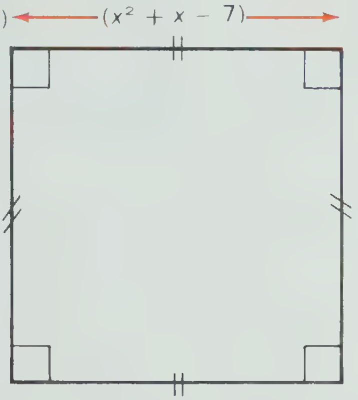
(o) $a^2 + 40a + 400$

21. Find the area of the following figures.

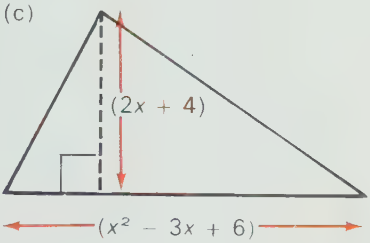
(a)



(b)

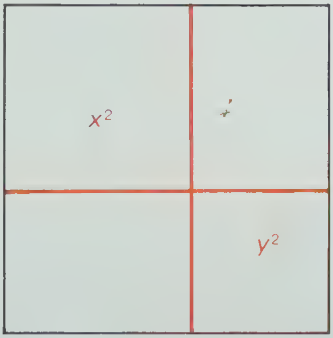


(c)

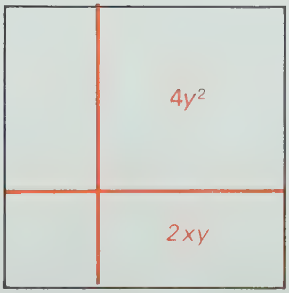


22. Give the dimensions and total area of each square if the expressions represent the area of the section they are in.

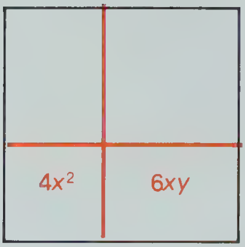
(a)



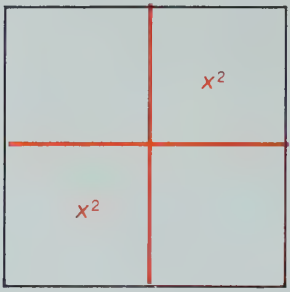
(b)



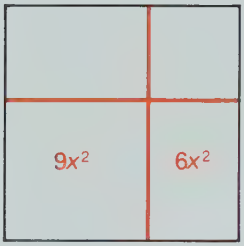
(c)



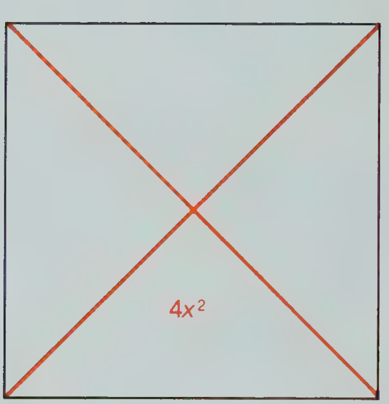
(d)



(e)



(f)



REVIEW AND PREVIEW TO CHAPTER 5

EXERCISE 1

If $E(x) = 2x + 1$
then, $E(2) = 2(2) + 1 = 4 + 1 = 5$
and $E(a) = 2(a) + 1 = 2a + 1$

1. If $E(x) = 3x$, evaluate
(a) $E(2)$ (b) $E(3)$ (c) $E(0)$ (d) $E(-2)$
(e) $E(-3)$ (f) $E(-5)$ (g) $E(a)$

2. If $E(x) = 2x - 3$, evaluate
(a) $E(4)$ (b) $E(7)$ (c) $E(10)$ (d) $E(-4)$
(e) $E(-3)$ (f) $E(0)$ (g) $E(a)$

3. If $E(a) = 2a^2 - 1$, evaluate
(a) $E(1)$ (b) $E(3)$ (c) $E(-2)$ (d) $E(0)$
(e) $E(-1)$ (f) $E(10)$ (g) $E(-10)$ (h) $E(x)$

EXERCISE 2

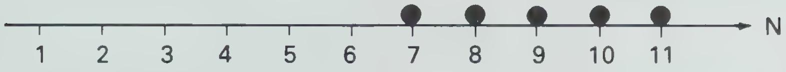
Test your skill: $10^{2.5} \div 10^{1.5}$

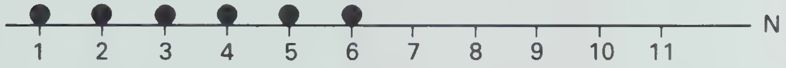
1. Graph the following on a number line
(a) $\{x \mid x > 2, x \in N\}$
(b) $\{x \mid x < 3, x \in I\}$
(c) $\{x \mid x \geqq -1, x \in I\}$
(d) $\{x \mid x \leqq 5, x \in N\}$
(e) $\{x \mid x > 3, x \in R\}$
(f) $\{x \mid x \leqq -3, x \in R\}$
(g) $\{x \mid x \geqq 0, x \in R\}$

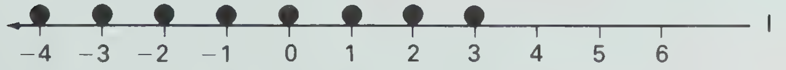
2. Graph the following on the R-line
(a) $-2 < x < 2$
(b) $-4 < x \leqq 3$
(c) $-6 \leqq x \leqq -2$
(d) $5 \leqq x < 11$

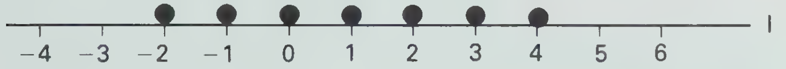
EXERCISE 3

Use set notation to describe each of the following sets.

1.


2.


3.


4.


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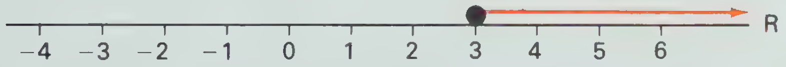
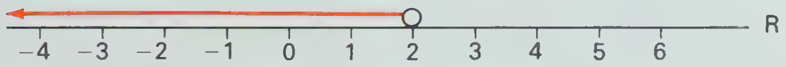
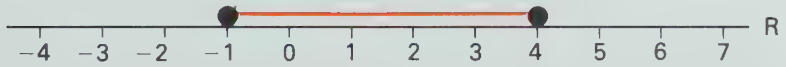
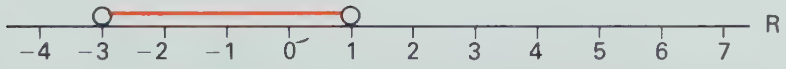
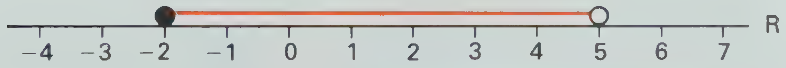
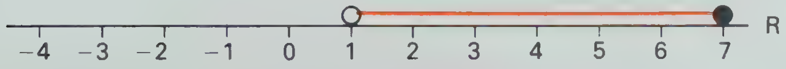


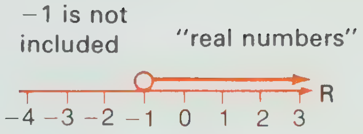
6.


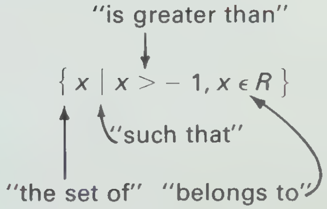
7.


8.


9.


10.



−1 is not included
"real numbers"



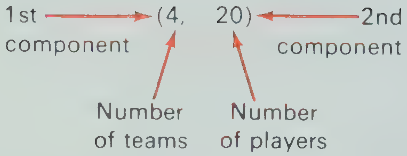
CHAPTER 5

Functions and Relations

5.1 ORDERED PAIRS

The study of how numbers relate to each other is the basis of mathematics. The table below displays the relationship between the number of basketball teams and the minimum number of players needed.

Number of Basketball Teams	Minimum Number of Players Needed
1	5
2	10
3	15
4	20
5	25
6	30



The set of ordered pairs {(1,5), (2,10), (3,15), (4,20), (5,25),}, (6,30) is called a binary relation. The first component represents the number of teams and the second component the number of players. What does (8,40) mean? Does (10,2) have any meaning in this example?

The pairs of numbers are called ordered pairs since the order of the components is important. We have decided to write the number of teams first and the number of players second. (3,15) means something entirely different than (15,3). The set of first components of a relation is called the domain of the relation and the set of second components the range of the relation. In our example the domain is the set {1,2,3,4,5,6} and the range is the set {5,10,15,20,25,30}.

EXAMPLE 1. *State the domain and range of the following:*
(a) (2,3), (4,6), (8,10), (11,13)
(b) (2,3), (2,7), (2,8)

Solution
(a) Domain is the set {2,4,8,11}
Range is the set {3,6,10,13}
(b) Domain is the set {2}
Range is the set {3,7,8}

EXERCISE 5-1

A 1. A triangle is a three-sided plane figure.
(a) Complete the following table:

Number of Triangles	Total Number of Sides
1	
2	
3	
.	
.	
.	
100	

Find 3 positive, consecutive numbers whose sum is 96.

(b) State the domain of the relation.
(c) State the range of the relation.
(d) What does (30,90) mean?
(e) Complete the following for the given relation: (17,▨), (▨,27).
(f) What meaning, if any, do the following have: (0,0), (5,35), ($2\frac{1}{3}$,7)?

B 2. The number of kilometres a car travels and the number of litres of gasoline used are related. A car travels approximately 7 km on one litre of gasoline (consumes about 14.3 L/100 km).

Litres of Gas	Number of Kilometres
3	21
5	
6	
	140
	210

(b) List the relation as a set of ordered pairs in the form (litres of gas, number of kilometres).
(c) List the elements of the domain of the relation.
(d) List the elements of the range of the relation.
(e) What does (10,70) mean?
(f) Complete the following for the given relation

(2,▨), (▨,315)

3. There is a relationship between your distance from a storm and the time between the flash and the thunderclap. Count the number of seconds from when you see the flash to when you hear the thunder. For each second you are approximately 330 m from the storm.
(a) Complete the following table in your notebook.

Time between Flash and Thunder (Seconds)	**Distance from Storm (Metres)**
2	
3	
5	
	9900
	132 00

(b) List the relation as a set of ordered pairs in the form (time, distance).
(c) List the elements of the domain.
(d) List the elements of the range.
(e) What is the second component of ($3\frac{1}{2}$,▨)?
(f) What is the first component of (▨,1650)?
(e) What does ($5\frac{1}{2}$,1815) mean?

4. It is possible to determine the temperature by counting the number of chirps per minute of a house cricket. The relationship is defined by

$$T = \frac{5\,(N + 32)}{36}$$

where N is the number of chirps per minute and T is the temperature.
(a) Complete the following table in your notebook.

Chirps per Minute	Temperature °C
56	
80	
92	
96	
100	
40	

The relationship, if the cricket is a tree cricket, is defined as

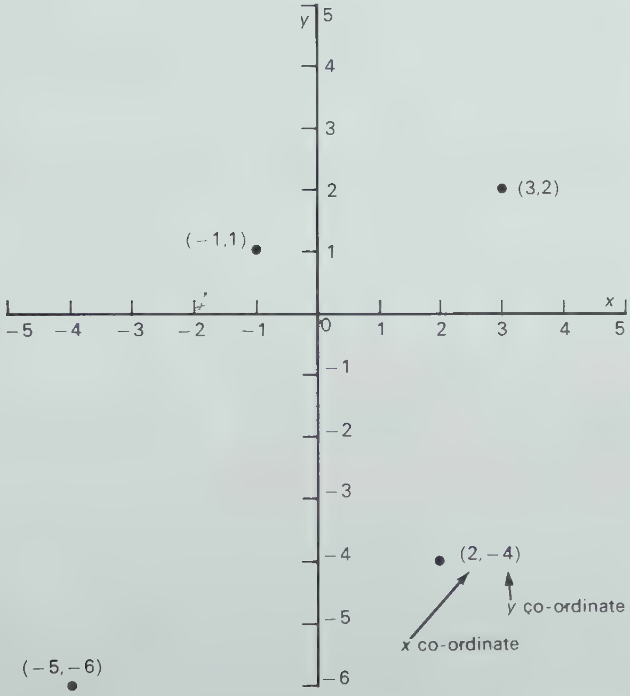
$$T = \frac{5N - 67}{40}$$

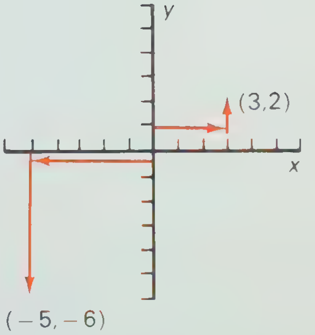
where T is the temperature and N the number of chirps per minute.

(b) List the relation as ordered pairs in the form (chirps, temperature).
(c) List the elements of the domain.
(d) List the elements of the range.

5.2 GRAPHING SETS OF ORDERED PAIRS

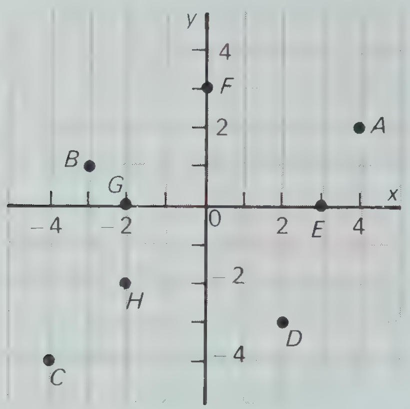
In order to study relations more effectively, a pictorial method of illustrating ordered pairs was devised.





EXERCISE 5-2

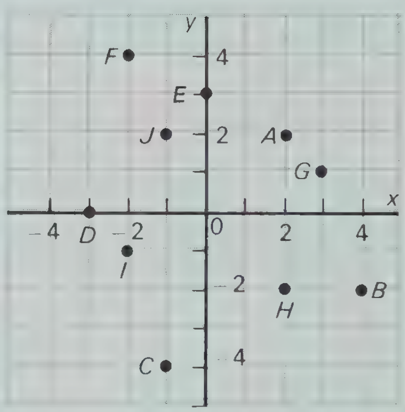
A **1.**



State the co-ordinates of the points shown in the diagram.

2. What points have the following co-ordinates?

(i) (−1,−4) (ii) (−3,0)
(iii) (2,−2) (iv) (−1,2)
(v) (3,1) (vi) (2,2)
(vii) (0,3) (viii) (−2,4)



B **3.** Plot the following points on a pair of axes. Join the points in the order *A–B–C–D–E*. Identify the resulting geometric figure.

(a) $A(-4,4)$, $B(-6,-2)$, $C(3,-2)$, $D(5,4)$
(b) $A(0,6)$, $B(-3,-4)$, $C(5,-3)$
(c) $A(-2,3)$, $B(-4,3)$, $C(-4,1)$, $D(-2,1)$
(d) $A(0,0)$, $B(0,-6)$, $C(4,-6)$, $D(4,0)$
(e) $A(3,-5)$, $B(2,3)$, $C(-3,3)$, $D(-8,-5)$
(f) $A(2,-2)$, $B(-2,2)$, $C(-4,1)$, $D(0,5)$, $E(4,1)$

4. (2,4) is an ordered pair where the second component is twice the first. List 3 other pairs with this characteristic. Graph this set of ordered pairs and notice the pattern.

5. (3,1) is an ordered pair where the first element is 3 times the second. List 3 other ordered pairs with this characteristic. Graph this set of ordered pairs and notice the pattern.

6. (5,7) is an ordered pair where the second element is 2 more than the first. List 3 other ordered pairs with this characteristic. Graph the set and notice the pattern.

7. (3,2) is an ordered pair where the first element is one more than the second. List 3 other ordered pairs with this characteristic. Graph the set and notice the pattern.

8. List a set of 4 ordered pairs where the first element equals the second. Graph the set and notice the pattern.

9. List a set of 4 ordered pairs where the first element is 4 less than the second. Graph the set and notice the pattern.

10. (2,4), (5,7), (8,10). What relation exists between the ordered pairs?

How long will it take you to count to 2 billion if you can count one number per second?

11. List a set of 4 ordered pairs such that the sum of the components is 6. Graph the set and notice the pattern.

12. List a set of 4 ordered pairs such that the first component minus the second is 2. Graph the set and notice the pattern.

13. List a set of 4 ordered pairs where the first component is 1 more than twice the second. Graph the set and notice the pattern.

C **14.** Plot the following points and join them in order.
$(0,6), (4,1), (0,0), (-4,1), (0,6), (0,0), (-6,0), (-5,-2), (5,-2), (6,0), (0,0)$

5.3 GRAPHING RELATIONS *Part I*

Previously relations have been expressed in three ways:
(a) as a word statement
(b) in table form
(c) as ordered pairs
In this section relations will be expressed graphically.

EXAMPLE 1. *The toll for crossing a certain bridge is a dollar for the car and driver plus 50 cents for each passenger. The relation may be expressed as follows*

$$t = 100 + 50n,$$

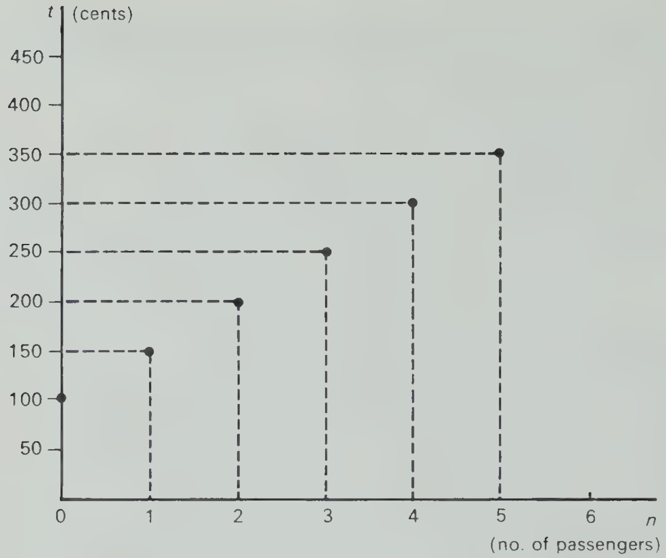
where t is the toll (in cents) and n the number of passengers.
(a) Set up a table of values for $n \in \{0,1,2,3,4,5\}$ and illustrate the relation graphically.
(b) State the domain and range of the relation.

Solution
(a) $t = 100 + 50n$

n	t
0	100
1	150
2	200
3	250
4	300
5	350

$$t = 100 + 50n$$
when $n = 0$, $t = 100 + 50(0) = 100$
when $n = 1$, $t = 100 + 50(1) = 150$

Test your skill: $(0.003)^3$



(b) The domain is $\{0,1,2,3,4,5\}$.
The range is $\{100,150,200,250,300,350\}$.

EXAMPLE 2.

(a) Graph $\{(x,y) \mid y = 2x + 1,\ x,y \in I\}$
(b) State the domain and range of the relation.

$\{(x,y) \mid y = 2x + 1,\ x,y \in I\}$

the set of all ordered pairs (x,y)

such that

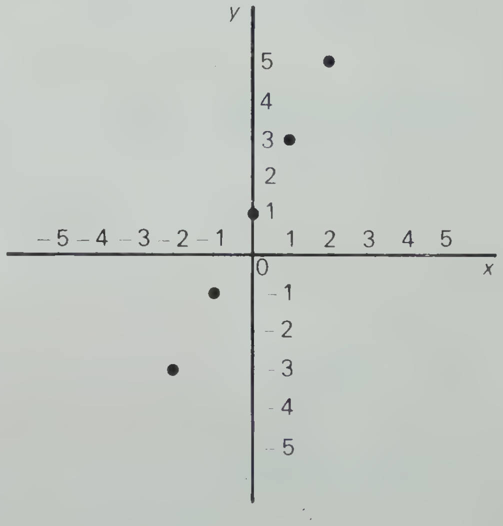
x,y belong to I

Solution

(a) Set up a table of values

x	y
2	5
1	3
0	1
-1	-1
-2	-3

$$y = 2x + 1$$
when $x = 2$, $y = 2(2) + 1$
$= 4 + 1 = 5$



The graph is only a partial graph of the relation since x can have values greater than 2 and less than -2.

(b) The domain is $\{\ldots,-2,-1,0,1,2,\ldots\}$.
The range is $\{\ldots,-3,-1,1,3,5,\ldots\}$.

EXERCISE 5-3

A 1. State the domain and range of the following relations
(a) $\{(3,2),(4,7)\ (5,12)\ (6,17)\}$
(b) $\{(6,2),(8,9),(10,12),(15,14),(17,20)\}$
(c) $\{(2,3),(2,4),(2,5),(2,6)\}$
(d) $\{(3,1),(4,1),(5,1),(6,1)\}$
(e) $\{(a,b),(b,a),(c,d),(d,c)\}$

Test your skill: write $\frac{1}{81}$ as a repeating decimal.

B 2. List 4 ordered pairs for each of the following relations
(a) $\{(x,y) \mid y = x + 1,\ x,y \in I\}$
(b) $\{(x,y) \mid y = 2x - 1,\ x,y \in I\}$
(c) $\{(x,y) \mid y = 3x + 2,\ x,y \in I\}$
(d) $\{(x,y) \mid y = 3x,\ x,y \in N\}$
(e) $\{(x,y) \mid y = x - 7,\ x,y \in I\}$
(f) $\{(x,y) \mid x + y = 6,\ x,y \in N\}$

3. A 70-kg man decides to go on a diet for 6 weeks. If he loses 1 kg a week, his mass at the end of any week may be expressed as

$$m = 70 - n$$

where m is his mass and n the number of weeks into the diet.
(a) illustrate this relation graphically
(b) state the domain and range of the relation

4. The height of a 25-cm candle depends upon the length of time it burns.

$$h = 25 - 5t$$

where h is the height in centimetres at the end of each hour and t is the time in hours.
(a) illustrate this relation graphically
(b) state the domain and range of the relation

5. Graph the following relations. Use four ordered pairs in each case.
(a) $\{(x,y) \mid y = x + 1,\ x,y \in I\}$
(b) $\{(x,y) \mid y = 2x - 2,\ x,y \in I\}$
(c) $\{(x,y) \mid y = 3x + 1,\ x,y \in I\}$
(d) $y = 4x - 5,\ x \in \{0,2,4\}$
(e) $y = \frac{1}{2}x + 3,\ x \in \{-2,0,2,4\}$
(f) $y = \dfrac{x + 1}{3},\ x \in \{-4,-1,2,5\}$
(g) $\{(x,y) \mid x + y = 8,\ x,y \in I\}$
(h) $y = x^2 + 1,\ x \in \{-2,-1,0,1,2\}$

6. State a rule of correspondence for the following relations

(a)

x	y
2	3
5	6
7	8
9	10

(b)

x	y
7	3
9	5
11	7
−1	−5

(c)

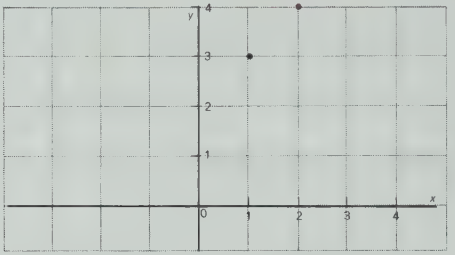
x	y
3	6
4	8
5	10
6	12

(d)

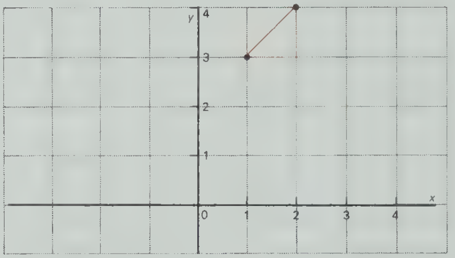
x	y
6	−3
10	−5
12	−6
14	−7

5.4 GRAPHING RELATIONS *Part II*

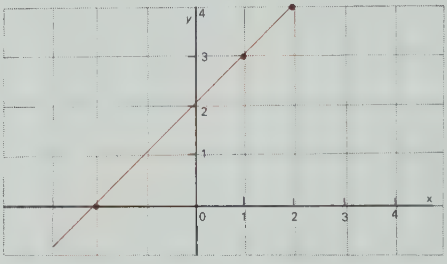
The graph of $y = x + 2$, $x \in \{1,2\}$ is



If we let $1 \leqq x \leqq 2$, $x \in R$ the graph of the relation would be



If we let x take on all real values, i.e. $x \in R$, the graph of the relation is



Addition:

ham
eggs
mash

EXAMPLE 1. *Graph the following relations and state the domain and range of each. Use at least 4 ordered pairs in each case.*

(a) $\{(x,y) \mid y = 2x - 1,\ x,y \in R\}$

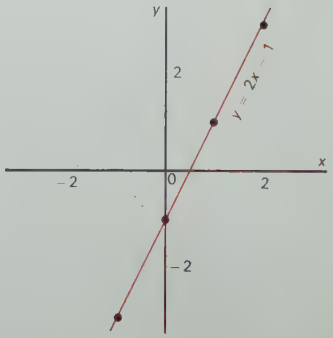
(b) $\left\{(x,y) \mid y = \dfrac{x-1}{2},\ x,y \in R\right\}$

(c) $\{(x,y) \mid y = 2,\ x,y \in R\}$

Solution

(a)

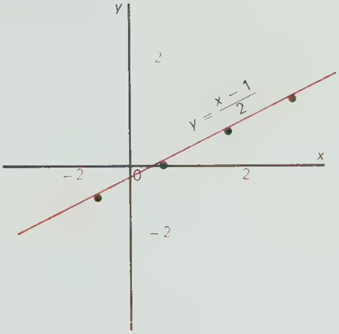
x	y
2	3
1	1
0	−1
−1	−3



Domain $x \in R$
Range $y \in R$

(b) To simplify calculations, select values for x so that the numerator is divisible evenly by 2.

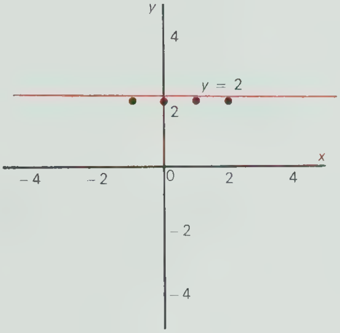
x	y
5	2
3	1
1	0
−1	−1



Domain $x \in R$
Range $y \in R$

(c)

x	y
2	2
1	2
0	2
−1	2

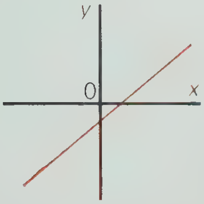


Domain $x \in R$
Range $y = 2$

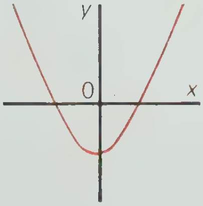
EXERCISE 5-4

A 1. Use the vertical line test to determine which of the following are graphs of a function:

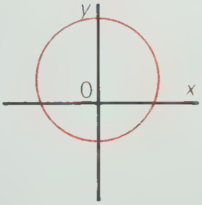
(a)



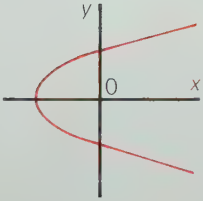
(b)



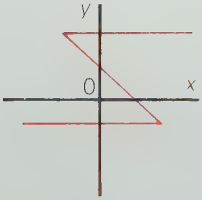
(c)



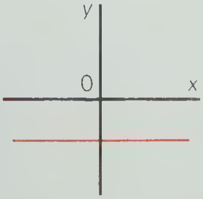
(d)



(e)



(f)



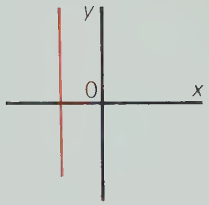
Functions

Relations where for every first element in the ordered pair there is only one second element are called functions. The graph of a function can be identified quickly using the vertical line test. *If any vertical line will cut the graph at most once, the relation is a function.*

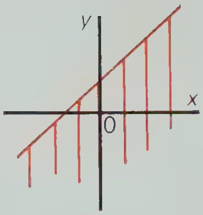
Test your skill:
if $a = 2$, $b = -3$, $c = 4$, evaluate

$$3abc - 4bc$$

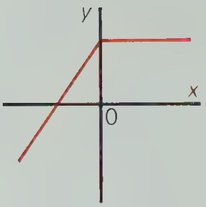
(g)



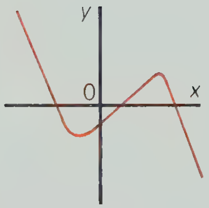
(h)



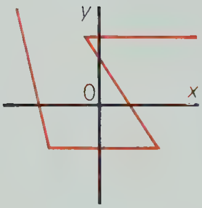
(i)



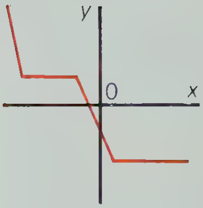
(j)



(k)



(l)



2. Graph the following relations. Use at least 4 ordered pairs in each case.

(a) $\{(x,y) \mid y = x + 1,\ x,y \in R\}$
(b) $\{(x,y) \mid y = x - 3,\ x,y \in R\}$
(c) $\{(x,y) \mid y = x + 6,\ x,y \in R\}$
(d) $\{(x,y) \mid y = x,\ x,y \in R\}$

3. Graph the following relations. All variables represent real numbers.

(a) $y = 2x + 1$
(b) $y = 2x - 3$
(c) $y = 3x + 2$
(d) $y = 4x - 1$
(e) $y = 3x$

4. Graph the following relations:

(a) $\{(x,y) \mid y = -2x + 1,\ x,y \in R\}$
(b) $\{(x,y) \mid y = -3x - 2,\ x,y \in R\}$
(c) $\{(x,y) \mid y = 1 - 4x,\ x,y \in R\}$
(d) $\{(x,y) \mid y = -3 + 2x,\ x,y \in R\}$

5. Graph the following relations. All variables represent real numbers.

(a) $y = \frac{1}{2}x + 3$ (HINT: use even numbers for x)

(b) $y = -\frac{1}{3}x - 1$ (HINT: use numbers divisible by 3 for x)

(c) $y = \frac{x + 1}{2}$

(d) $y = \frac{x - 3}{3}$

(e) $y = \frac{2x + 1}{2}$

(f) $y = \frac{2}{3}x - 2$

$$\begin{aligned} 1 &= 1 \\ 1+2+1 &= 4 \\ 1+2+3+2+1 &= 9 \\ 1+2+3+4+3+2+1 &= 16 \end{aligned}$$

Use the pattern to find this sum:

$1 + 2 + 3 \ldots + 19 + 20 + 19 \ldots + 3 + 2 + 1 =$ ▨

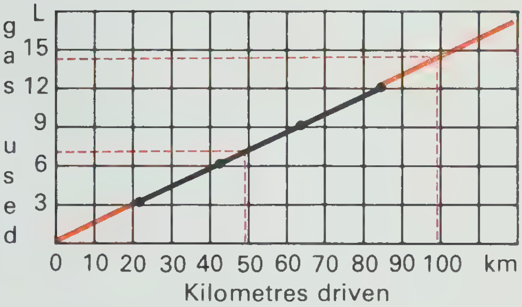
6. Graph the following relations and state the domain and range of each.

(a) $\{(x,y) \mid y = 3,\ x,y \in R\}$
(b) $\{(x,y) \mid x = -2,\ x,y \in R\}$
(c) $\{(x,y) \mid y = -2,\ x,y \in R\}$
(d) $\{(x,y) \mid x = 1,\ x,y \in R\}$
(e) $\{(x,y) \mid y = 0,\ x,y \in R\}$
(f) $\{(x,y) \mid x = 0,\ x,y \in R\}$
(g) $\{(x,y) \mid y + 3 = 0,\ x,y \in R\}$
(h) $\{(x,y) \mid x + 4 = 0,\ x,y \in R\}$

5.5 INTERPOLATING AND EXTRAPOLATING

This table and graph show the relation between kilometres driven and litres of gas consumed for a small car.

km driven	L of gas
21	3
42	6
63	9
84	12



The graph shows four pairs of known values actually plotted and a straight line drawn through the points and extended beyond them. (Why was a straight line drawn?)

Interpolation

When we assume that the graph shows values between the known values we are *interpolating.*

EXAMPLE 1.

How many litres of gas will be used in driving the above mentioned car 49 km?

Solution From the graph it is seen that 7 L of gas will be required to drive 49 km.

Extrapolation

When we assume that the graph shows values beyond the known values, we are *extrapolating.*

EXAMPLE 2.

How many litres of gas will be used in driving the above mentioned car 100 km?

Solution From the graph it is seen that approximately 14 L of gas will be used.

There is a risk involved in interpolation, and especially in extrapolation when we do not have the relation defined by a mathematical statement.

EXAMPLE 3.

Plot the points defined by the following values and construct a graph to find the values required to complete the table.

x	0	2	4	6	8	10	12	14	16	18
y	0	7.2		16.8		20				

Solution

fig A

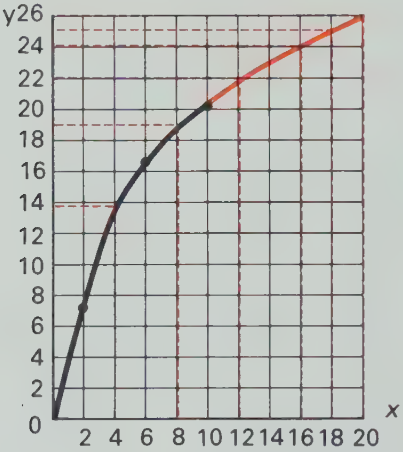
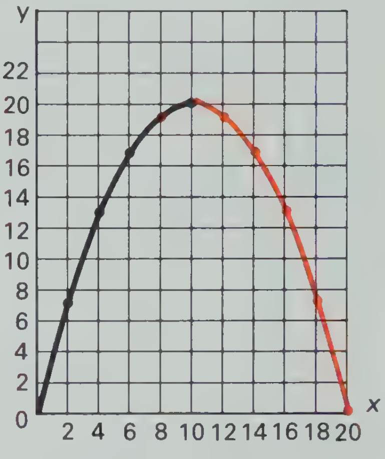


fig B



From the known values it might appear that the graph would continue as shown (Fig. A) and from it the following values would be interpolated and extrapolated.

x	0	2	4	6	8	10	12	14	16	18	20
y	0	7.2	13.8	16.8	18.8	20.0	22.0	23.2	24.4	25.2	26.0

In fact the known values were taken from the equation $y = 4x - \frac{x^2}{5}$ (Fig. B), which describes the path of a ball thrown with a vertical velocity of 20 m/s and a horizontal velocity of 5 m/s. The true values are

x	0	2	4	6	8	10	12	14	16	18	20
y	0	7.2	12.8	16.8	19.2	20.0	19.2	16.8	12.8	7.2	0

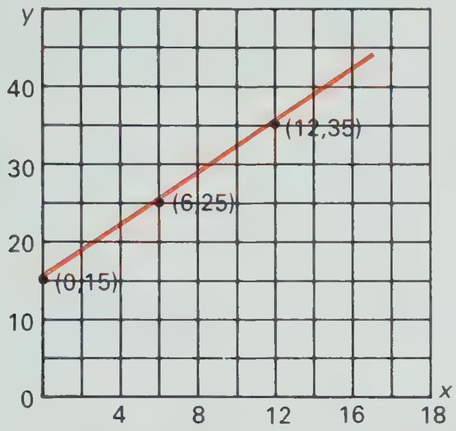
Notice that the values arrived at through interpolation are close to the actual values, while the farther we extrapolate the farther the values differ from the actual values.

Interpolation and extrapolation can be valuable tools when used with caution.

EXERCISE 5-5

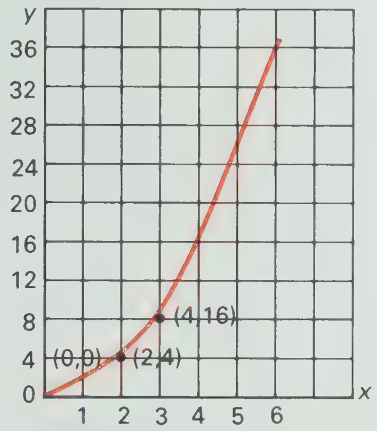
A **1.** From the given graphs, find the values missing in the tables.

(a)



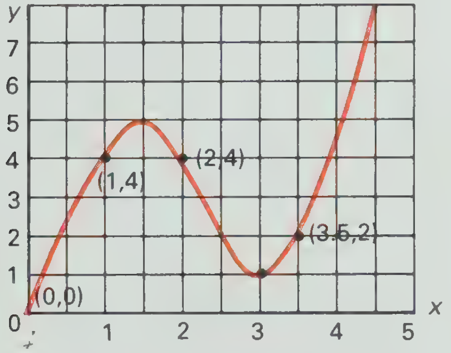
x	2	10	14	16	18
y					

(b)



x	1.0	3.0	3.5	5.5	6
y					

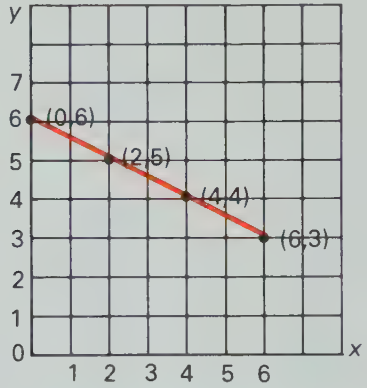
(c)



x	0.5	1.5	2.5	4.0	4.5
y					

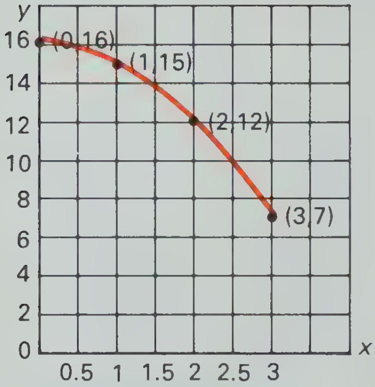
B 2. Copy the following graphs and continue them to allow you to extrapolate and find the missing values.

(a)



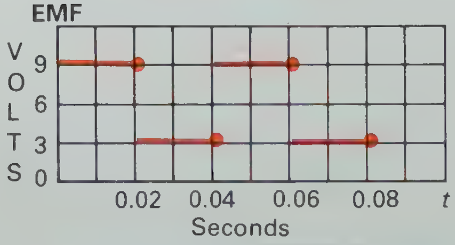
x	3	5	6.5	
y				0

(b)



x	1.5	2.5	3.5	
y				0

(c)



t	0.01	0.035	0.06	0.09	0.11	0.30
EMF						

3. Set up axes and plot points to represent the following ordered pairs. Join the points with a smooth curve and extend to interpolate and extrapolate the required values.

(a) (0,2), (1,▨), (2,8), (3,▨), (4,14), (5,▨), (5.5,▨)
(b) (2,0), (4,▨), (6,2), (7,▨), (8,3), (10,▨), (14,▨)
(c) (0,2), (1,▨), (2,6), (3,11), (3.5,▨), (4,▨)
(d) (0,9), (1,▨), (2,5), (2.5,▨), (3,0), (4,▨)

4. The diagram in the margin shows a sequence of small congruent triangles fitted together to make similar triangles.

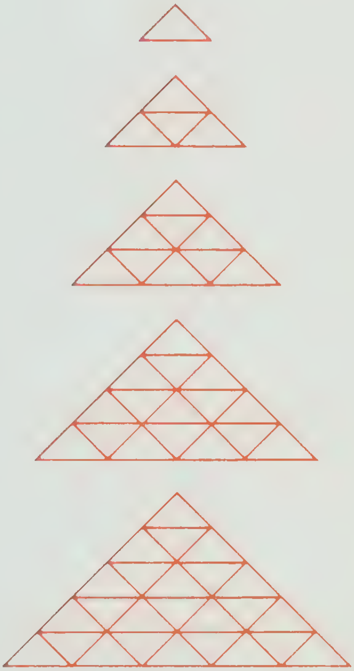
On the same set of axes, graph the number of sides on the horizontal axis, and the number of congruent triangles and the number of similar triangles on the vertical axis. Use a large sheet of graph paper.

(a) Interpolate to find the number of congruent triangles and the number of similar triangles when there are 30 sides. Count to see if you are correct.

(b) Extrapolate for the same quantities when the number of sides is 63 and 84.

(c) Interpolate for the number of triangles when there are 24 sides. Does a partial row affect the accuracy of the interpolation?

number of sides	number △'s	number △'s
3	1	1
9	4	5
18	9	13
30	▨	▨
45	25	48
63	▨	▨
84	▨	▨



5.6 $y = mx + b$

The relation $2x - 3y + 7 = 0$ can be written in the form $y = \frac{2}{3}x + \frac{7}{3}$. The relation is now expressed in the general form $y = mx + b$ where:

m, the coefficient of x, is $\frac{2}{3}$
b, the constant term, is $\frac{7}{3}$

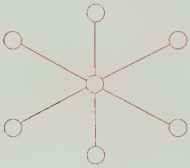
$2x - 3y + 7 = 0$
$-3y = -2x - 7$
$y = \frac{2}{3}x + \frac{7}{3}$
note that the coefficient of y is 1

EXAMPLE 1. *Write the relation $2x - 6y = 7$ in the form $y = mx + b$. State the values of m and b.*

Solution

$$2x - 6y = 7$$
$$-6y = -2x + 7$$
$$\frac{-6y}{-6} = \frac{-2x}{-6} + \frac{7}{-6} \quad \text{Divide by } -6$$
$$y = \frac{1}{3}x - \frac{7}{6}$$
$$m = \frac{1}{3} \quad \text{and} \quad b = -\frac{7}{6}$$

Place the numbers 11, 12, 13, 14, 15, 16, 17 in the circles so that any three circles in a line have the same sum.



EXERCISE 5-6

Express each of the following in the form $y = mx + b$; state the values of m and b in each case.

A 1. (a) $y = 2x + 3$ (b) $y = -3x - 2$
(c) $y = 4x + 6$ (d) $y = -\frac{1}{2}x$
(e) $y + 3x = 8$ (f) $y - 2x + 7 = 0$

B 2. (a) $y + 3x - 7 = 0$ (b) $y - 4x = 6$
(c) $3x + y = 9$ (d) $4x + 6 = y$
(e) $5x = 3 + y$ (f) $2 = -3x + y$

3. (a) $-y = 3x + 2$ (b) $-y = -4x - 7$
(c) $-y - 2x = 6$ (d) $3 - y = 8x$
(e) $3x - y = 12$ (f) $5x = 7 - y$

4. (a) $2y = 4x + 6$ (b) $3y = -15x + 12$
(c) $4y + 6x = 8$ (d) $2y + 7x = 12$
(e) $5y - 10x = 15$ (f) $6x + 2y = 8$

5. (a) $3x - 2y = 8$ (b) $4x + 5y = 7$
(c) $-3y + 6 = 8x$ (d) $5y - 3x = 6$
(e) $4x + 6 = -5y$ (f) $3x + 2y = 17$

If $2! = 2 \times 1 = 2$
and $3! = 3 \times 2 \times 1 = 6$
and $4! = 4 \times 3 \times 2 \times 1$
$= 24$
find 7! and 12!

6. (a) $4x + 2y - 8 = 0$ (b) $3x - 2y - 6 = 0$
(c) $7x - 3y - 6 = 0$ (d) $-3x - 4y + 6 = 0$
(e) $-2 + 3y = 8x$ (f) $-4y + 3x = 12$

7. Express the following in the form $y = mx + b$, and then sketch the graph. Use at least three ordered pairs for each.
(a) $3x + y = 6$ (b) $2y + x = 6$
(c) $2y - x = 3$ (d) $2x - y = 4$
(e) $x - 3 = 2y$ (f) $3x + 2y = 1$
(g) $x - 3y = 2$ (h) $x + 3y = 0$
(i) $4y + 3 = x$ (j) $x - y + 2 = 0$

If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
evaluate
$\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix}$

INVESTIGATION 5.1

1. On the same set of axes sketch the graphs of the following. Use at least 3 ordered pairs from each relation. (Domain $-3 \leqq x \leqq 3$)
(a) $y = 2x + 1$
(b) $y = 2x + 5$
(c) $y = 2x - 3$
(d) $y = 2x$
(e) $y = 2x + 4$

x	$y = 2x + 1$	y
0	$y = 2(0) + 1$	1
1	$y = 2(1) + 1$	3
2	$y = 2(2) + 1$	5

These relations are in the form $y = mx + b$, where $m = 2$ and $b \in \{-3,0,1,4,5\}$.

2. On the same set of axes sketch the graphs of the following. Use at least 3 ordered pairs from each relation. (Domain $-3 \leqq x \leqq 3$)
(a) $y = -2x + 3$
(b) $y = -2x + 6$
(c) $y = -2x - 3$
(d) $y = -2x - 1$
(e) $y = -2x$

x	$y = -2x + 3$	y
0	$y = -2(0) + 3$	3
1	$y = -2(1) + 3$	1
2	$y = -2(2) + 3$	-1

3. Using the graphs in questions 1 and 2, discuss the role of b in $y = mx + b$.

4. On the same set of axes sketch the graphs of the following. Use at least 3 ordered pairs from each relation. (Domain $-3 \leqq x \leqq 3$)
(a) $y = 2x + 3$
(b) $y = x + 3$
(c) $y = -2x + 3$
(d) $y = -\frac{1}{2}x + 3$
(e) $y = 3x + 3$

x	$y = 2x + 3$	y
0	$y = 2(0) + 3$	3
1	$y = 2(1) + 3$	5
2	$y = 2(2) + 3$	7

These relations are in the form $y = mx + b$ where $b = 3$ and $m \in \{-2,-\frac{1}{2},1,2,3\}$.

5. On the same set of axes sketch the graphs of the following. Use at least 3 ordered pairs from each relation. (Domain $-3 \leqq x \leqq 3$)

(a) $y = 2x - 2$
(b) $y = -2x - 2$
(c) $y = x - 2$
(d) $y = 3x - 2$
(e) $y = -\frac{1}{2}x - 2$

x	$y = 2x - 2$	y
0	$y = 2(0) - 2$	-2
1	$y = 2(1) - 2$	0
2	$y = 2(2) - 2$	2

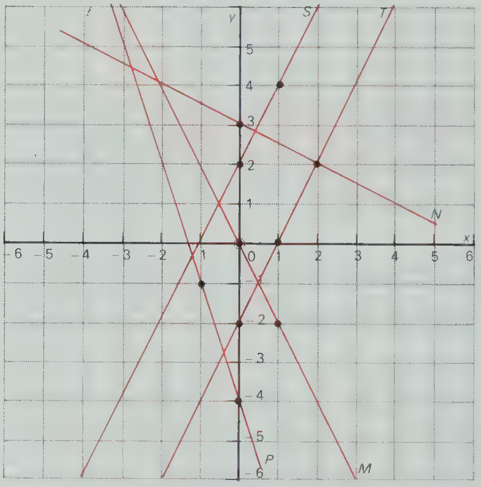
6. (a) Using the graphs in questions 4 and 5, discuss the role of m in $y = mx + b$.

(b) For what values does the line rise
 (i) upwards to the right?
 (ii) upwards to the left?

(c) How does the value of m affect the steepness of the graph?

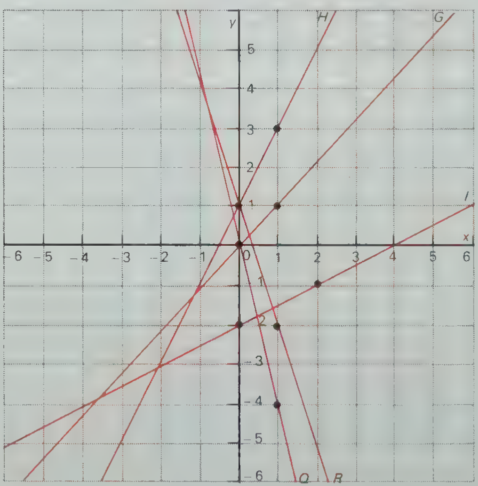
7. Match the equations with the appropriate graphs.

A $y = 2x - 2$
B $y = -2x$
C $y = 2x + 2$
D $y = -\frac{1}{2}x + 3$
E $y = -3x - 4$



8. Match the equations with the appropriate graphs.

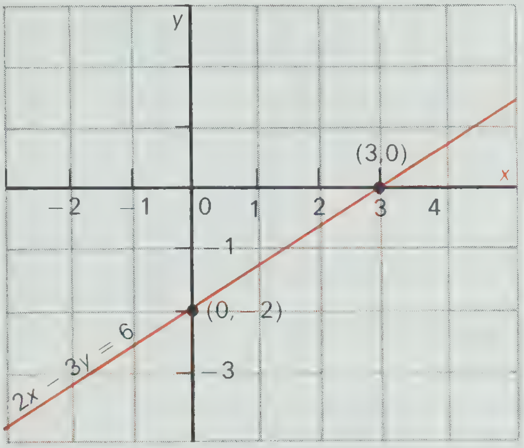
A $y = -3x + 1$
B $y = x$
C $y = 2x + 1$
D $y = -4x$
E $y = \frac{1}{2}x - 2$



5.7 GRAPHING RELATIONS USING THE INTERCEPTS

It is often convenient to graph a relation using the intercepts.

$2x - 3y = 6$
To find the x
intercept, let $y = 0$
$2x - 3(0) = 6$
$2x = 6$
$x = 3$
To find the y
intercept, let $x = 0$
$2(0) - 3y = 6$
$-3y = 6$
$y = -2$



3 is called the **x intercept** of the line, and -2 the **y intercept**.

EXERCISE 5-7

A **1.** State the x and y intercepts of the following:

(a) $3x + 4y = 12$ (b) $2x + 3y = 6$
(c) $5x + 2y = 10$ (d) $3x + 5y = 15$
(e) $x + 2y = 4$ (f) $3x + y = 6$
(g) $2x - y = 6$ (h) $3x - 2y = 6$
(i) $4x - 3y = 12$ (j) $x - 3y = 9$

B **2.** Graph the lines which have intercepts as follows:

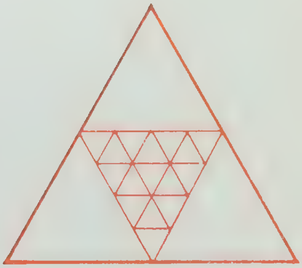
(a) x intercept 2
y intercept 3
(b) x intercept 3
y intercept -1
(c) x intercept -4
y intercept 5
(d) x intercept -2
y intercept -3

3. Graph the following relations, using the x and y intercepts.

(a) $x + y = 6$ (b) $x - y = 2$
(c) $2x + y = 6$ (d) $x - 2y = 4$
(e) $2x - y = 4$ (f) $2x + 3y = 6$
(g) $2x - 3y = 12$ (h) $3x - 5y = 15$
(i) $-3x + 2y = 6$ (j) $-4x + 3y = -12$
(k) $3x + y = -9$ (l) $-x - y = -1$

5.8 POSSIBLE VALUES OF A RELATION

In previous sections, ordered pairs that satisfied a relation were determined by replacing x with numerical values and then finding the corresponding value of y.



How many equilateral triangles can you find?

For the relation $y = 2x - 1$,
When $x = 2$, $y = 2(2) - 1$
$= 4 - 1$
$= 3$
Hence $(2,3)$ satisfies the relation $y = 2x - 1$.

$y = 2x - 1$
Left side
L.S. $= y$
Right side
R.S. $= 2x - 1$

EXAMPLE 1. *Does (3,5) satisfy $y = 2x - 1$?*

Solution
For the ordered pair $(3,5)$
When $y = 5$, L.S. $= 5$
When $x = 3$, R.S. $= 2(3) - 1$
$= 6 - 1$
$= 5$

L.S. = R.S.

Since the left side equals the right side, $(3,5)$ satisfies $y = 2x - 1$.

(3,3)
(1,3)
(2,5)
$y = 2x + 1$
(3,7)
.
.
.

EXAMPLE 2. *Does (3,3) satisfy $y = 2x + 1$?*

Solution
When $y = 3$, L.S. $= 3$
When $x = 3$, R.S. $= 2(3) + 1$
$= 6 + 1$
$= 7$
L.S. $\neq$ R.S.
$(3,3)$ does not satisfy $y = 2x + 1$.

EXAMPLE 3. *Does $(-1,-2)$ satisfy $y \leqq 3x - 2$?*

Solution
When $y = -2$, L.S. $= -2$
When $x = -1$, R.S. $= 3(-1) - 2$
$= -3 - 2$
$= -5$
For $(-1,-2)$, L.S. $>$ R.S.
Since in $y \leqq 3x - 2$, L.S. $\leqq$ R.S.
$(-1,-2)$ does not satisfy $y \leqq 3x - 2$.

EXERCISE 5-8

1. Which of the following ordered pairs satisfy $y = 2x - 2$?
(a) $(1,0)$ (b) $(-2,-2)$ (c) $(3,3)$
(d) $(0,-2)$ (e) $(-1,-4)$ (f) $(5,6)$

2. Which of the following ordered pairs satisfy $y = -3x + 1$?
(a) $(-1,-2)$ (b) $(2,-5)$ (c) $(0,1)$
(d) $(-3,10)$ (e) $(1,-3)$ (f) $(-2,5)$

3. Which of the following ordered pairs satisfy $y \geqq 2x + 3$?
(a) $(1,8)$ (b) $(4,-2)$ (c) $(-2,2)$
(d) $(-1,0)$ (e) $(3,9)$ (f) $(2,10)$

4. Which of the following ordered pairs satisfy $y < -2x - 2$?
(a) $(2,-5)$ (b) $(-1,-1)$ (c) $(-2,1)$
(d) $(0,3)$ (e) $(1,-6)$ (f) $(3,-8)$

Test your skill:
$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

5. Test to find whether the ordered pair satisfies the relation.

(a) $y \leqq 2x - 3$; (2,2) (b) $y > x + 6$; (2,9)
(c) $y = 3x - 2$; (1,2) (d) $y < -x - 3$; (−2,−3)
(e) $y \geqq 2x + 4$; (−1,3) (f) $y \leqq -3x + 6$; (−4,10)
(g) $y = -x - 2$; (2,0) (h) $y > 3x - 4$; (1,−2)

6. Test to find whether the ordered pair satisfies the relation.

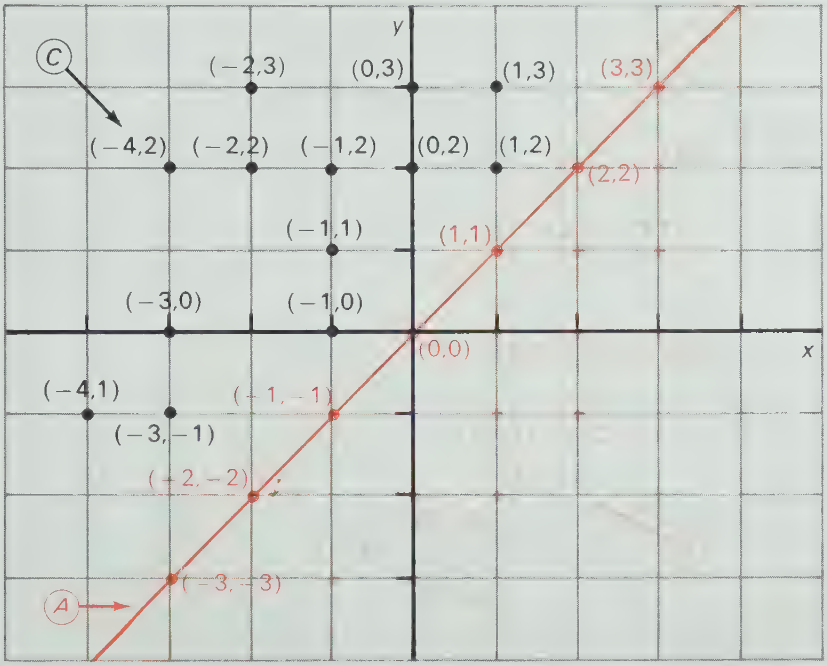
(a) $y < -2x + 4$; (−1,5) (b) $y \geqq 2x$; (2,4)
(c) $y = -3x - 3$; (−1,1) (d) $y < -3x$; (−1,3)
(e) $y \leqq 5x - 10$; (1,−4) (f) $y > -3x - 4$; (−2,3)
(g) $y > x$; (−3,−4) (h) $y < -x$; (2,−1)

7. Find five ordered pairs that satisfy each of the following relations.

(a) $y = 3x - 2$ (b) $2x + y = 3$
(c) $y = \frac{1}{2}x + 3$ (d) $3x - y = 0$
(e) $y < 2x - 5$ (f) $y \geqq 3x - 2$

5.9 GRAPHING INEQUALITIES *Part I*

A straight line divides a plane into 3 distinct regions. Consider the relation $y = x$, $x, y \in R$.



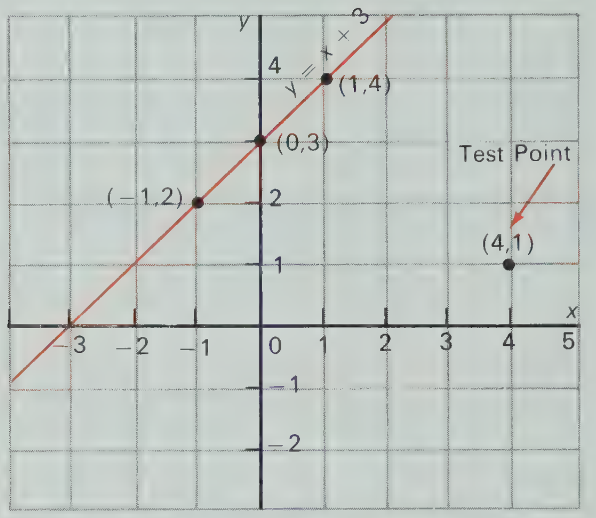
Region *A*: For every point here, $y = x$.
Region *B*: For every point here, $y < x$.
Region *C*: For every point here, $y > x$.

Find the sum of all numbers between 100 and 900 which are divisible by 19.

EXAMPLE 1. *Sketch the graph of $y = x + 3$, and indicate on the graph the regions*

(a) $y = x + 3$
(b) $y > x + 3$
(c) $y < x + 3$

Solution

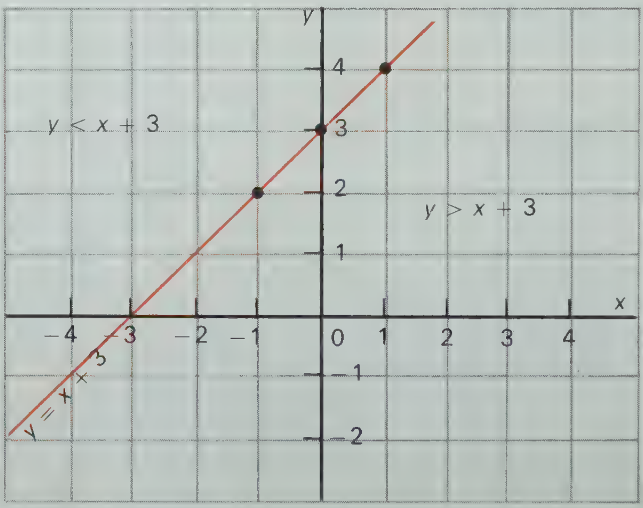


Show how to make change for a dollar using exactly 50 standard coins.

To decide where $y > x + 3$ and $y < x + 3$, take a test point not on the line. In the area of the test point (4,1)

when $y = 1$, L.S. $= 1$
when $x = 4$, R.S. $= x + 3$
$= 4 + 3$
$= 7$
Since $1 < 7$
L.S. $<$ R.S.
$y < x + 3$

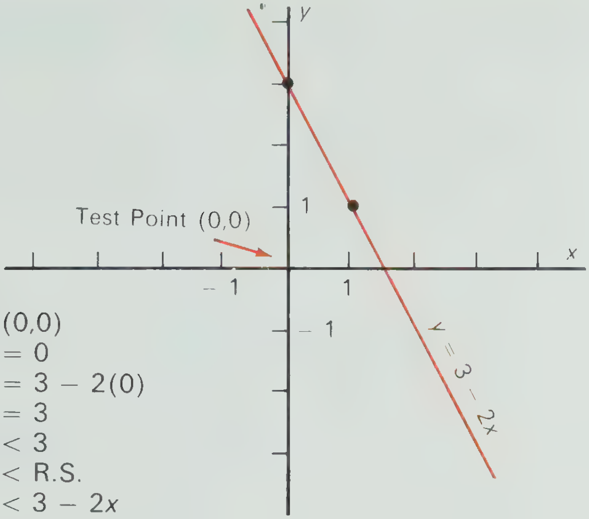
It is left to the student to show that on the other side of the line $y > x + 3$.



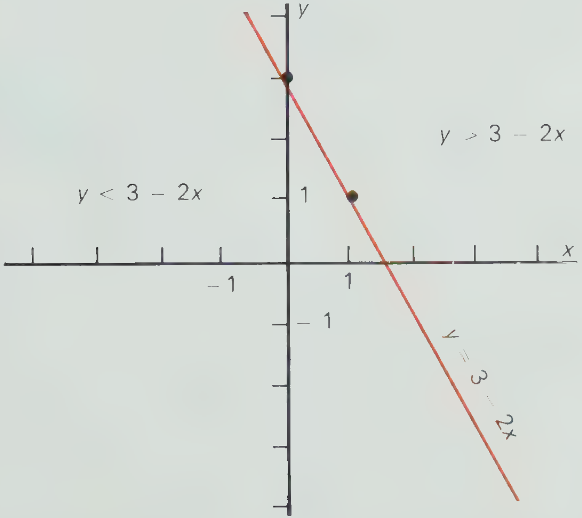
EXAMPLE 2. *Sketch the graph of $y = 3 - 2x$ and indicate the 3 regions*

(a) $y = 3 - 2x$
(b) $y > 3 - 2x$
(c) $y < 3 - 2x$

Solution



For the test point (0,0)
when $y = 0$, L.S. $= 0$
when $x = 0$, R.S. $= 3 - 2(0)$
$= 3$
Since $0 < 3$
L.S. < R.S.
$y < 3 - 2x$



When the boundary line does not pass through the origin, it is convenient to choose (0,0) as the test point.

EXERCISE 5-9

For each of the following relations sketch a graph, clearly indicating the three regions associated with each.

1. $y = x + 1$
2. $y = x - 2$
3. $y = 2x - 1$
4. $y = 3x + 2$
5. $y = -x - 3$
6. $y = 4 - 3x$
7. $y = \frac{x + 1}{2}$
8. $2x + y = 3$
9. $3x - y = 5$

Addition:
send
more
money

Test your skill:
$3\frac{3}{4} \div 3\frac{1}{4}$

5.10 GRAPHING INEQUALITIES *Part II*

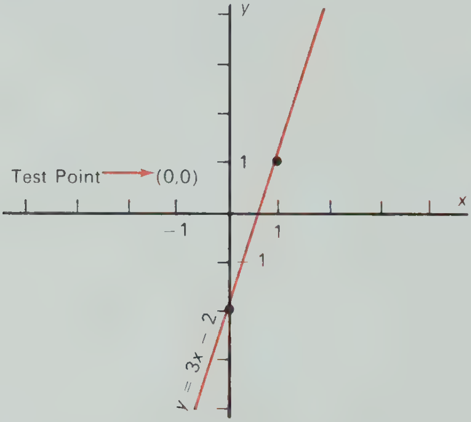
In the previous section we found that a line divides the plane into 3 regions. In this section you will be asked to graph 1 or 2 of these regions.

EXAMPLE 1. *Sketch the graph of $y \leqq 3x - 2$.*

Test your skill:
3.14×0.1^2

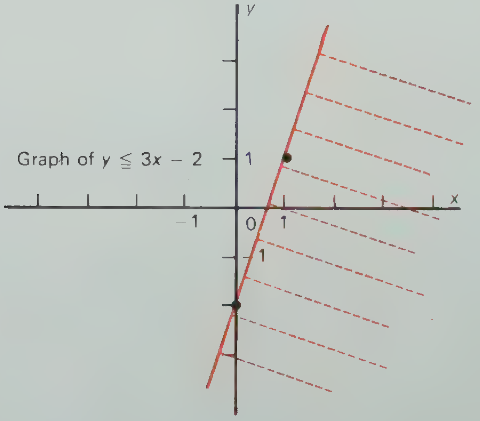
Solution

First draw the boundary line ($y = 3x - 2$)



Secondly, use a test point to locate the required region.

For the test point (0,0)
when $y = 0$, L.S. $= 0$
when $x = 0$, R.S. $= 3(0) - 2$
$= -2$
Since $0 > -2$
L.S. $>$ R.S.
In the region of the test point, $y > 3x - 2$
$\therefore \quad y < 3x - 2$ is on the other side of the line

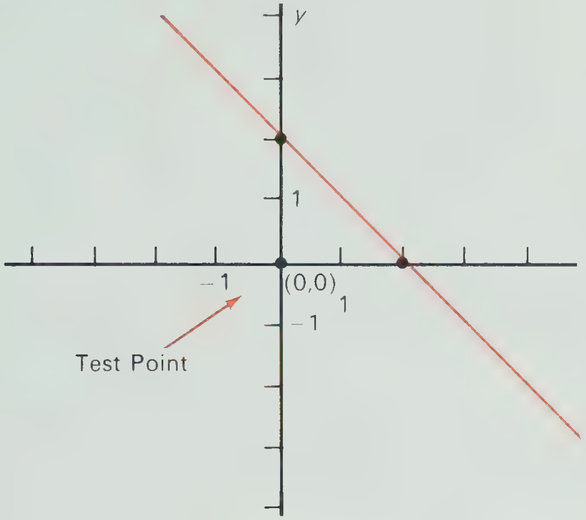


Sue had 79 hits in 423 times at bat, Find her batting average, correct to 3 decimal places.

EXAMPLE 2. *Sketch the graph of $y < 2 - x$.*

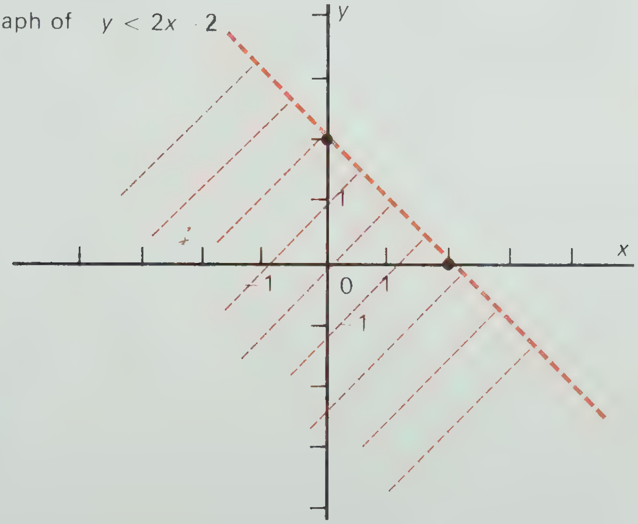
Solution

Draw the boundary line $y = 2 - x$



For the test point (0,0)
when $y = 0$, L.S. $= 0$
when $x = 0$, R.S. $= 2 - 0$
$= 2$
Since $0 < 2$
L.S. $<$ R.S.
In the region of the test point $y < 2 - x$

Test your skill:
$0.002 \div 20$


Graph of $y < 2x - 2$

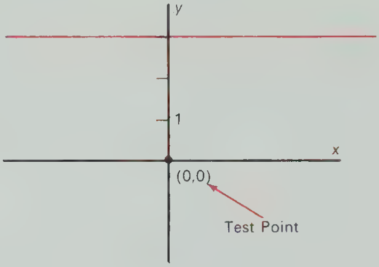
Why is the boundary line a broken line?

Insert signs to make the statement true:
5 ▨ 3 ▨ 6 ▨ 4 = 19

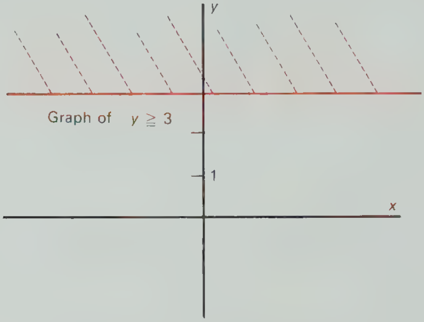
EXAMPLE 3. *Sketch the graph of* $y \geqq 3$.

Solution

Draw boundary line $y = 3$



For the test point (0,0), $y < 3$.
The region where $y > 3$ is on the other side of the boundary line.



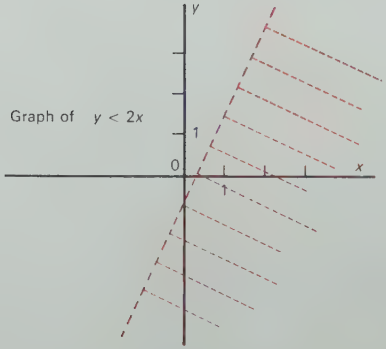
EXAMPLE 4. *Sketch the graph of* $y < 2x$.

Solution: Draw the boundary line $y = 2x$
Since (0,0) is on the line, select (1,0)

when	$y = 0$		L.S. $= 0$
	$x = 1$		R.S. $= 2$
	$0 < 2$	$\therefore$	L.S. $<$ R.S.

In the region of the test point, $y < 2x$

If it takes 30 s to make one cut, how long will it take to cut a 30 m board into 30 parts?



EXERCISE 5-10

Sketch the graph of each of the following, $x, y \in R$.

1. (a) $y \geqq x + 4$ (b) $y \leqq 2x + 1$
(c) $y \leqq x - 4$ (d) $y > 3x + 1$
(e) $y < 2x - 3$ (f) $y \geqq 3 + x$

2. (a) $y \geqq 4 - x$ (b) $y < -2x + 1$
(c) $y > x + 6$ (d) $y \leqq 3 - 3x$
(e) $y < -1 + x$ (f) $y \geqq 5 - 4x$

3. (a) $x \leqq 2$ (b) $y > -1$
(c) $y \leqq 4$ (d) $x > -3$
(e) $y \geqq 0$ (f) $x < 0$

4. (a) $y \geqq \dfrac{x - 1}{2}$ (b) $y < \dfrac{2 + x}{3}$
(c) $y + 2x < 0$ (d) $y - 3 > -4x$

5.11 GRAPHING NON-LINEAR RELATIONS

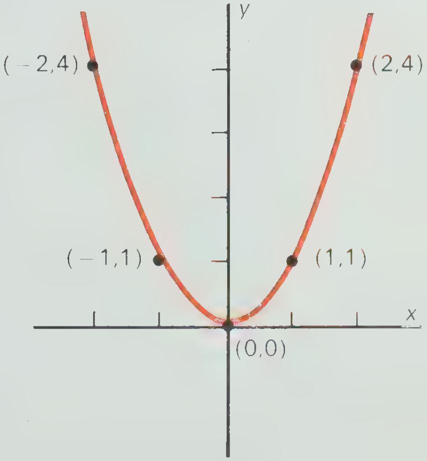
In this section, relations that are not necessarily linear will be graphed.

EXAMPLE 1. *Graph the relation $y = x^2$.*

Solution

when $x = 0$, $y = (0)^2 = 0$
when $x = 1$, $y = (1)^2 = 1$
when $x = -1$, $y = (-1)^2 = 1$
when $x = 2$, $y = (2)^2 = 4$
when $x = -2$, $y = (-2)^2 = 4$

x	y
0	0
1	1
−1	1
2	4
−2	4



EXAMPLE 2. *Graph the relation $x^2 + y^2 = 25$.*

Solution

When $x = 0$, $(0)^2 + y^2 = 25$
$y^2 = 25$
$y = +5$, or -5

When $y = 0$, $x^2 + (0)^2 = 25$
$x^2 = 25$
$x = +5$, or -5

When $x = 3$, $(3)^2 + y^2 = 25$

$9 + y^2 = 25$

$y^2 = 16$

$y = +4$, or -4

When $x = -3$, $(-3)^2 + y^2 = 25$

$9 + y^2 = 25$

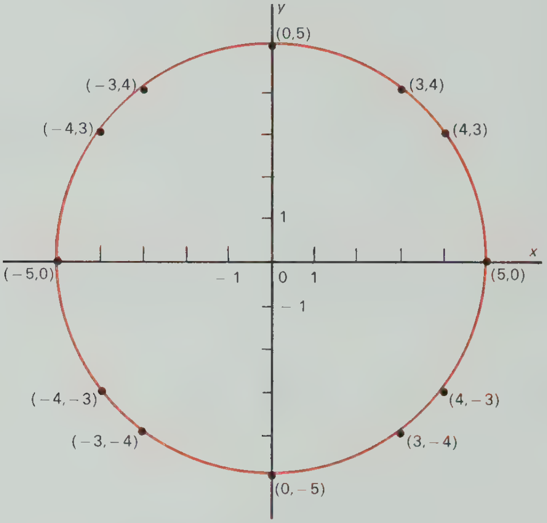
$y = +4$, or -4

When $x = 4$, $y = +3$ or -3

When $x = -4$, $y = +3$ or -3

Is this relation a function?

x	y
0	5
0	−5
5	0
−5	0
3	4
3	−4
−3	4
−3	−4
4	3
4	−3
−4	3
−4	−3



EXAMPLE 3. *Graph the relation $x = y^2 + 3$.*

Solution

When $y = 0$, $x = (0)^2 + 3 = 3$

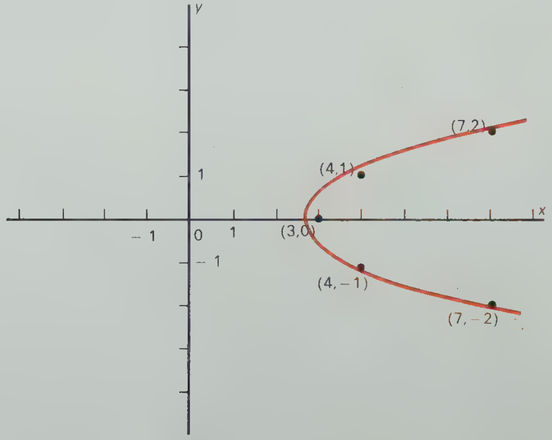
When $y = 1$, $x = (1)^2 + 3 = 4$

When $y = -1$, $x = (-1)^2 + 3 = 4$

When $y = 2$, $x = (2)^2 + 3 = 7$

When $y = -2$, $x = (-2)^2 + 3 = 7$

x	y
3	0
4	1
4	−1
7	2
7	−2



Is this relation a function?

EXERCISE 5-11

Addition
dog
cat
ado

1. Draw the graphs of the following relations. Use the vertical line test to determine those that are functions.

(a) $y = x^2 + 2$ (b) $y = x^2 - 3$
(c) $x = y^2 + 2$ (d) $x = y^2 - 2$
(e) $x^2 + y^2 = 36$ (f) $y = 2x^2$
(g) $x = 2y^2 + 1$ (h) $x^2 + y^2 = 16$
(i) $y = x^2 - 4x$ [use $x \in \{-1,0,1,2,3,4,5\}$]

REVIEW EXERCISE

A **1.** For each of the following relations, give the domain, range, and determine the rule used to form the relation.

(a) $\{(1,2), (2,4), (3,6), (4,8)\}$ (b) $\{(1,1), (2,4), (3,9), (4,16)\}$
(c) $\{(1,3), (2,4), (3,5), (4,6)\}$ (d) $\{(1,3), (2,5), (3,7), (4,9)\}$
(e) $\{(1,1), (2,4), (3,7), (4,10)\}$ (f) $\{(1,2), (2,5), (3,10), (4,17)\}$

State the domain and range for each of the above.

2. State the x and y intercepts of the following

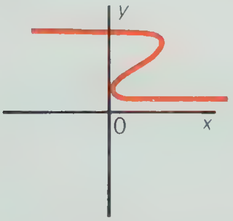
(a) $2x + 7y = 14$ (b) $x - 5y = 10$
(c) $2x - y = 0$ (d) $3y - 7x = -21$
(e) $5y - 4x = 20$ (f) $2x - 6y - 6 = 0$
(g) $y = -2x + 4$ (h) $y = 3x - 9$

3. Test whether or not the ordered pair satisfies the relation

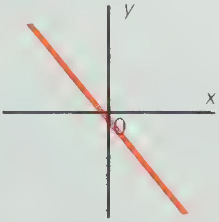
(a) $y = 2x + 7$; $(1,9)$ (b) $2x + 3y = 12$; $(1,4)$
(c) $y < 3x + 1$; $(1,5)$ (d) $y \geqq 2x - 3$; $(0,0)$
(e) $y > -2x + 1$; $(1,5)$ (f) $y = 3$; $(3,1)$
(g) $y \leqq -3x - 4$; $(-1,-6)$ (h) $2x + y = 0$; $(1,-2)$
(i) $y < 3x$; $(-2,-7)$ (j) $y \geqq 2x + 6$; $(1,7)$

4. Use the vertical line test to determine which of the following are graphs of functions.

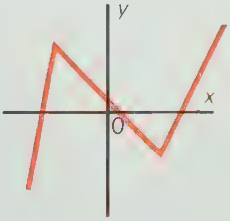
(a)



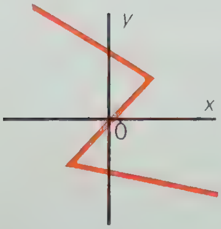
(b)



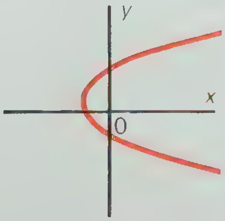
(c)



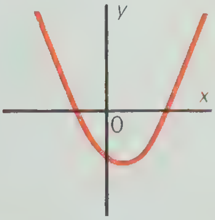
(d)



(e)



(f)



B 5. Graph the following relations, using the x and y intercepts.

(a) $x + y = 6$ (b) $x - 2y = 8$
(c) $2x - y = 4$ (d) $2x - 7y = 14$
(e) $3x - 2y = -6$ (f) $-x + y = -2$
(g) $4x + 3y - 12 = 0$ (h) $5x = 10 - 2y$
(i) $2y = 6x - 6$ (j) $0 = 2x - 3y - 18$

6. Express the following in the form $y = mx + b$, and then sketch the graph. Use at least three ordered pairs for each.

(a) $2x + y = 4$ (b) $y - 3x - 2 = 0$
(c) $2x - y = 1$ (d) $x - 2y = 4$
(e) $x + 2y = 0$ (f) $x - y - 3 = 0$
(g) $3x + 2y = 6$ (h) $2y + 3 = 4x$

7. Graph the following relations. Use four ordered pairs in each case.

(a) $\{(x,y) \mid y = 3x - 2,\ x,y \in R\}$
(b) $\{(x,y) \mid y = 1 - 2x,\ x,y \in R\}$
(c) $\{(x,y) \mid y = -3,\ x,y \in R\}$
(d) $\{(x,y) \mid y = 4x + 1,\ x,y \in R\}$
(e) $\{(x,y) \mid x = -4,\ x,y \in R\}$
(f) $\left\{(x,y) \mid y = \dfrac{x-1}{2},\ x,y \in R\right\}$
(g) $\{(x,y) \mid y = 3 - \tfrac{1}{2}x,\ x,y \in R\}$

8. Sketch the graph of each of the following, $x, y \in R$.

(a) $y \leqq x + 3$ (b) $y > 4 - x$
(c) $y < 2x - 5$ (d) $y \geqq -3x + 5$
(e) $y < \dfrac{x+1}{2}$ (f) $y \leqq \dfrac{2x-1}{2}$
(g) $y \leqq 5$ (h) $x \geqq -1$

9. Draw the graphs of the following relations. Use the vertical line test to determine those that are functions.

(a) $y = x^2$ (b) $x = y^2$
(c) $y = x^2 - 1$ (d) $x = y^2 + 3$
(e) $x^2 + y^2 = 25$ (f) $y = 2x^2 - 1$
(g) $x = -2y^2$ (h) $y = 2 - 2x^2$

10. If $100 per year was placed in an account paying 8% per year, the following table gives the amount in the account after n years. Plot the points and graph the relation. Interpolate and extrapolate to find the missing values.

n	2	4	6	8	10	12	14	16
Amount	$208	$450		$1064		$1898		

11. (a) Plot the points represented by the following ordered pairs. (0,8), (3,10), (6,8), (9,2).
(b) Join the points with a smooth curve and extend it to find the missing values. (2,▨), (5,▨), (7,▨), (▨,0), (10,▨).

REVIEW AND PREVIEW TO CHAPTER 6

EXERCISE 1

Expand and simplify the following:

1. (a) $2(x-3)-2(x-4)+5$
(b) $3(a-5)+2(2a-3)$
(c) $5(2t-7)-3(1-3t)+7$
(d) $2(3m-1)-(2-2m)+6$
(e) $3x-(2-2x)-3(2x-4)$

2. (a) $(x-3)(x-2)+(x+7)(x-6)$
(b) $2(2m-1)(m+3)-3(m-5)(m+4)$
(c) $4(3b-1)(2b+2)+2(b+1)^2+6$
(d) $2(y-3)-(y+2)(y-1)-3(y-2)^2$
(e) $3(2a-3)-4(a-1)^2+2(a-1)(2a-3)$

Addition:
beaver
tiger
rabbit

EXERCISE 2

Solve the following equations by inspection:

1. (a) $x+3=12$ (b) $y+7=10$
(c) $t+4=8$ (d) $2+m=16$
(e) $b-3=4$ (f) $x-7=12$
(g) $y+3=-6$ (h) $t+5=-8$
(i) $m-2=-8$ (j) $-x+3=+6$

2. (a) $3x=15$ (b) $4a=20$
(c) $7t=28$ (d) $-2t=8$
(e) $-3x=-21$ (f) $-4m=-20$
(g) $5a=-25$ (h) $4y=0$
(i) $-6x=18$ (j) $4x=-4$

EXERCISE 3

Solve the following inequalities by inspection. The first equation has been done for you.
The domain is the set of integers.

1. (a) $x+2>5$ ans: ss $=\{4,5,6,\ldots\}$
(b) $x+3>7$ (c) $x\times 3>5$ (d) $x-4<2$ (e) $x+2<9$
(f) $x+7\geqslant 12$ (g) $x-6\leqslant 15$ (h) $x-2\geqslant 0$ (i) $x+4\leqslant 0$
(j) $x+3\leqslant -2$ (k) $x-2\geqslant -6$

2.
(a) $3x>12$ (b) $4x<16$ (c) $2x\geqslant 4$ (d) $8x\leqslant 24$
(e) $6x\leqslant -12$ (f) $5x>-10$ (g) $\frac{1}{2}x>3$ (h) $\frac{1}{3}x<12$
(i) $\frac{1}{4}x<-2$ (j) $\frac{1}{5}x\geqslant 0$

CHAPTER 6

Equations

6.1 EQUATIONS IN ONE VARIABLE

Part I

The solving of equations is a very important branch of mathematics. It is an essential skill for many other subjects. In this section three of the four rules for solving equations will be reviewed.

Rule I	If $8 = 8$ then, $8 + 3 = 8 + 3$	If $a = b$ then, $a + c = b + c$	Equality is preserved if the same number is added to each side of an equation
Rule II	If $9 = 9$ then, $9 - 2 = 9 - 2$	If $a = b$ then, $a - c = b - c$	Equality is preserved if the same number is subtracted from each side of an equation
Rule III	If $8 = 8$ then, $\frac{8}{2} = \frac{8}{2}$	If $a = b$ then, $\frac{a}{c} = \frac{b}{c}$ $(c \neq 0)$	Equality is preserved if both sides of an equation are divided by the same non-zero number

EXAMPLE 1. *Solve and check $2(2x - 1) - (x + 3) = x + 1$, $x \in R$.*

Solution

$$\begin{aligned}
2(2x - 1) - (x + 3) &= x + 1 \\
4x - 2 - x - 3 &= x + 1 && \text{(removing brackets)} \\
3x - 5 &= x + 1 && \text{(simplifying)} \\
3x - 5 + 5 &= x + 1 + 5 && \text{(rule I)} \\
3x &= x + 6 && \text{(simplifying)} \\
3x - x &= x - x + 6 && \text{(rule II)} \\
2x &= 6 && \text{(simplifying)} \\
\frac{2x}{2} &= \frac{6}{2} && \text{(rule III)} \\
x &= 3 && \text{(simplifying)}
\end{aligned}$$

Check

$$\begin{aligned}
\text{L.S.} &= 2(2x - 1) - (x + 3) \\
&= 2(6 - 1) - (3 + 3) \\
&= 10 - 6 \\
&= 4
\end{aligned}
\qquad
\begin{aligned}
\text{R.S.} &= x + 1 \\
&= 3 + 1 \\
&= 4
\end{aligned}$$

$\therefore$ 3 is the root of the equation.

EXAMPLE 2. *Solve and check* $2(1-3a)-2(a+1)=-16$, $a \in R$

Solution

$$\begin{aligned} 2(1-3a)-2(a+1) &= -16 & \\ 2-6a-2a-2 &= -16 & \text{(removing brackets)} \\ -8a &= -16 & \text{(simplifying)} \\ \frac{-8a}{-8} &= \frac{-16}{-8} & \text{(rule III)} \\ a &= 2 & \text{(simplifying)} \end{aligned}$$

Check L.S. $= 2(1-3a)-2(a+1)$ R.S. $= -16$

$$\begin{aligned} &= 2(1-6)-2(2+1) \\ &= 2(-5)-2(3) \\ &= -10-6 \\ &= -16 \end{aligned}$$

∴ 2 is the root of the equation.

What is the minimum length of board required to cut 5 pieces, each one 10 cm long, allowing 2 mm waste for each cut?

EXERCISE 6-1

Solve the following equations. Variables have domain R.

A

1. (a) $x+3=7$ (b) $a+2=5$
(c) $m-3=6$ (d) $t-5=12$
(e) $3+r=11$ (f) $x-4=-11$
(g) $b+3=-4$ (h) $-2+m=0$

2. (a) $2x=6$ (b) $3m=27$
(c) $4a=-32$ (d) $-2a=6$
(e) $-3a=-18$ (f) $5d=0$
(g) $-10t=-70$ (h) $-x=3$

B

3. (a) $2x+3=7$
(b) $3a+7=2a-4$
(c) $5b=7b-4$
(d) $4t-7t+3-8+2t=0$
(e) $5m-7m+3=4m+6$

4. (a) $2(x-1)=x+6$
(b) $3(a+1)+4=2(a-1)$
(c) $2(m+3)+3(m-1)=3(m-2)+1$
(d) $3(2b-1)-4+2(b+3)=7$
(e) $5+2(t-3)-(t-7)+6=11$

5. (a) $3+2(2x-1)+6=5(x-3)$
(b) $7-2(1-3a)+16=8a+11$
(c) $10(2m-7)-3(m+2)=20m-70$
(d) $(2t+3)-(1-3t)-4t=3t$
(e) $-2(2b-3)-(1-b)=-1$

Using standard coins, in how many ways is it possible to make change for 25¢?

6. (a) $3x-2(x-3)+6=4x-(x-1)$
(b) $4a-3=2(a-3)-4(2a-1)$
(c) $2m-6=-(1-3m)+m$
(d) $7t-3(2t-4)=-3t+6(1-4t)-4$
(e) $4b-(3b-1)-3+6(b-2)=0$

6.2 EQUATIONS IN ONE VARIABLE

Part II

In this section the fourth rule for solving equations will be reviewed.

Rule IV	If $\frac{1}{2} = \frac{1}{2}$ then, $4 \times \frac{1}{2} = 4 \times \frac{1}{2}$	If $a = b$ then, $c \times a = c \times b$	Equality is preserved if both sides of the equation are multiplied by the same number

EXAMPLE 1. *Solve and check* $\frac{x}{4} = 3$, $x \in R$.

Solution

$$\frac{x}{4} = 3$$

$$\overset{1}{\cancel{4}} \times \left[\frac{x}{\cancel{4}_1}\right] = 4 \times 3 \qquad \text{(rule IV)}$$

$$x = 12 \qquad \text{(simplifying)}$$

Check

$$\text{L.S.} = \frac{x}{4} \qquad \text{R.S.} = 3$$

$$= \frac{12}{4}$$

$$= 3$$

$\therefore$ 3 is the root of the equation.

EXAMPLE 2. *Solve and check* $\frac{x}{2} + \frac{x}{3} = 5$, $x \in R$.

Solution

$$\frac{x}{2} + \frac{x}{3} = 5$$

L.C.D. of 2 and 3 is 6

$$6\left[\frac{x}{2} + \frac{x}{3}\right] = 6 \times 5 \qquad \text{(rule IV)}$$

$$3x + 2x = 30 \qquad \text{(simplifying)}$$

$$5x = 30$$

$$\frac{5x}{5} = \frac{30}{5} \qquad \text{(rule III)}$$

$$x = 6 \qquad \text{(simplifying)}$$

Check

$$\begin{aligned} \text{L.S.} &= \frac{x}{2} + \frac{x}{3} \qquad \text{R.S.} = 5 \\ &= \frac{6}{2} + \frac{6}{3} \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

$\therefore$ 6 is the root of the equation.

EXAMPLE 3. *Solve and check* $\dfrac{4x-2}{5} - \dfrac{3x-1}{2} = 5,\ x \in R.$

Test your skill: $(0.3)^2$

Solution

$$\frac{4x-2}{5} - \frac{3x-1}{2} = 5$$

L.C.D. of 5 and 2 is 10

$$\begin{aligned} 10\left[\frac{(4x-2)}{5} - \frac{(3x-1)}{2}\right] &= 10 \times 5 && \text{(rule IV)} \\ 2(4x-2) - 5(3x-1) &= 50 && \text{(simplifying)} \\ 8x - 4 - 15x + 5 &= 50 && \text{(removing brackets)} \\ -7x + 1 &= 50 && \text{(simplifying)} \\ -7x + 1 - 1 &= 50 - 1 && \text{(rule II)} \\ -7x &= 49 && \text{(simplifying)} \\ \frac{-7x}{-7} &= \frac{49}{-7} && \text{(rule III)} \\ x &= -7 && \text{(simplifying)} \end{aligned}$$

Check

$$\begin{aligned} \text{L.S.} &= \frac{4x-2}{5} - \frac{3x-1}{2} \qquad \text{R.S.} = 5 \\ &= \frac{4(-7)-2}{5} - \frac{3(-7)-1}{2} \\ &= \frac{-28-2}{5} - \frac{-21-1}{2} \\ &= \frac{-30}{5} - \frac{-22}{2} \\ &= -6 - (-11) \\ &= -6 + 11 \\ &= 5 \end{aligned}$$

$\therefore$ -7 is the root of the equation.

EXERCISE 6-2

Solve the following equations. Variables have domain R.

A **1.** (a) $\frac{x}{2} = 6$ (b) $\frac{x}{-3} = 5$

(c) $\frac{m}{-2} = -5$ (d) $\frac{1}{4}t = 4$

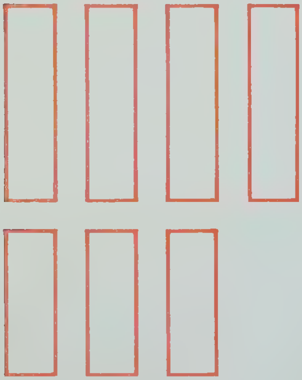
(e) $-\frac{1}{3}b = 7$ (f) $-\frac{1}{2}a = -6$

B **2.** (a) $\frac{x}{2} = \frac{5}{3}$ (b) $\frac{2a}{3} = \frac{3}{2}$

(c) $\frac{2m}{3} + \frac{1}{2} = \frac{m}{2}$ (d) $\frac{b}{2} + 1 = \frac{b}{3}$

(e) $\frac{x+1}{2} = 4$

A carpenter has 7 pieces of lumber 10 cm wide. If four pieces are 40 cm long and three pieces are 30 cm long, can you fit them all together, without cutting any of them, to form a square?



3. (a) $\frac{2m-1}{3} = 5$

(b) $\frac{x+3}{2} - 1 = 0$

(c) $\frac{a+2}{2} = \frac{a-1}{3}$

(d) $\frac{t+7}{4} = \frac{2t-1}{3}$

(e) $\frac{b}{3} = \frac{2b-1}{2}$

4. (a) $\frac{x+1}{2} + \frac{x+1}{3} = 5$

(b) $\frac{2m+1}{3} - \frac{m+1}{4} = 3$

(c) $\frac{1-3b}{4} - \frac{2+b}{6} = \frac{1}{3}$

(d) $\frac{4t-3}{2} - \frac{1}{4} = \frac{t}{8}$

(e) $\frac{1-3a}{4} - \frac{a-1}{3} = -a$

6.3 EQUATIONS INVOLVING POLYNOMIALS

Equations arising from the product of polynomials will be solved in this section.

EXAMPLE 1. *Solve and check* $3(x + 1) - 2(x + 3) = 7$.

Solution

$$\begin{aligned} 3(x + 1) - 2(x + 3) &= 7 \\ 3x + 3 - 2x - 6 &= 7 && \text{(removing brackets)} \\ x - 3 &= 7 && \text{(simplifying)} \\ x - 3 + 3 &= 7 + 3 && \text{(rule I)} \\ x &= 10 && \text{(simplifying)} \end{aligned}$$

Check

$$\begin{array}{ll} \text{L.S.} = 3(x + 1) - 2(x + 3) & \text{R.S.} = 7 \\ \quad = 3(10 + 1) - 2(10 + 3) & \\ \quad = 3(11) - 2(13) & \\ \quad = 33 - 26 & \\ \quad = 7 & \end{array}$$

$\therefore$ 10 is the root of the equation.

EXAMPLE 2. *Solve and check*

$$2(a - 3)(a + 1) - 3 = (2a - 1)(a + 2).$$

Solution

Find the fifth root of 243

$$\begin{aligned} 2(a - 3)(a + 1) - 3 &= (2a - 1)(a + 2) \\ 2(a^2 + a - 3a - 3) - 3 &= (2a^2 + 4a - a - 2) && \text{(multiplying)} \\ 2(a^2 - 2a - 3) - 3 &= (2a^2 + 3a - 2) && \text{(simplifying)} \\ 2a^2 - 4a - 6 - 3 &= 2a^2 + 3a - 2 && \text{(removing brackets)} \\ 2a^2 - 4a - 9 &= 2a^2 + 3a - 2 && \text{(simplifying)} \\ 2a^2 - 2a^2 - 4a - 3a - 9 + 9 &= 2a^2 - 2a^2 + 3a - 3a - 2 + 9 \\ -7a &= 7 \\ -\frac{7a}{-7} &= \frac{7}{-7} \\ a &= -1 \end{aligned}$$

Check

$$\begin{array}{ll} \text{L.S.} = 2(a - 3)(a + 1) - 3 & \text{R.S.} = (2a - 1)(a + 2) \\ \quad = 2(-1 - 3)(-1 + 1) - 3 & \quad = (2[-1] - 1)(-1 + 2) \\ \quad = 2(-4)(0) - 3 & \quad = (-2 - 1)(1) \\ \quad = 0 - 3 & \quad = (-3)(1) \\ \quad = -3 & \quad = -3 \end{array}$$

$\therefore$ -1 is the root of the equation.

EXERCISE 6-3

B **1.** Solve and check
(a) $(x - 3) + 2 + (x + 1) = 6$
(b) $2(x - 1) = 6$
(c) $3(x - 1) - 2 = 2(x - 2)$
(d) $2(x - 3) + 3(x + 2) = 10$
(e) $3(2x - 4) - 2(2x + 2) = 0$
(f) $3(m - 1) + 2(2m - 3) = 5(m - 4) + 1$

2. Solve and check
(a) $2(x^2 + 2x + 1) = 2(x^2 + x + 4)$
(b) $(a + 1)(a + 4) = (a + 2)(a + 1)$
(c) $(m + 2)(m + 1) - (2m - 3) = m^2 + 6$
(d) $(b - 3)(2b + 1) = 2(b - 1)(b + 1) + 9$
(e) $2(x + 3)(x + 2) - 2(x - 6) = 2x^2$
(f) $(2m - 3)(m - 2) - 2(m^2 - 1) = -6$

3. Solve
(a) $(a - 3)(a + 2) - (a + 3)(a + 1) = 1$
(b) $(2m - 1)(m + 2) - 2(m + 1)(m + 1) = m$
(c) $2(y - 3)(y + 1) - 2y^2 = 2(y - 3)$
(d) $2(x - 3)(2x - 1) - 3(x - 1)(x + 2) = (x - 1)(x - 1) + 26$
(e) $(a - 3)(a + 4) - (a + 2)(a + 1) = -16$
(f) $(2m - 3)(m - 1) - 2(m + 2)(m + 3) = 6$

Test your skill:
0.25×0.1

4. Solve
(a) $4a^2 - (2a - 1)(2a + 1) = a$
(b) $2(2x - 3) + (x - 4)(2x + 1) = 2x^2 - 1$
(c) $(m - 3)(2m + 1) - 2m^2 = 2$
(d) $2(a + 1)(a + 2) - (2a - 1)(a + 3) = 6$
(e) $2(x - 1)(2x - 2) - 4(x + 1)(x + 2) = 16$
(f) $3(2b - 3)(b + 2) = (3b - 1)(2b - 1) + 5$

6.4 SOLVING EQUATIONS IN TWO VARIABLES GRAPHICALLY

Sentences such as $x + 2 = 5$ and $2x - 1 = 7$ are called equations in one variable. An equation is **solved** when you can state the value of the variable that makes the sentence true. The values $x = 3$ and $x = 4$ are the respective solutions of the above equations. In this section we will consider the solution of equations in two variables.

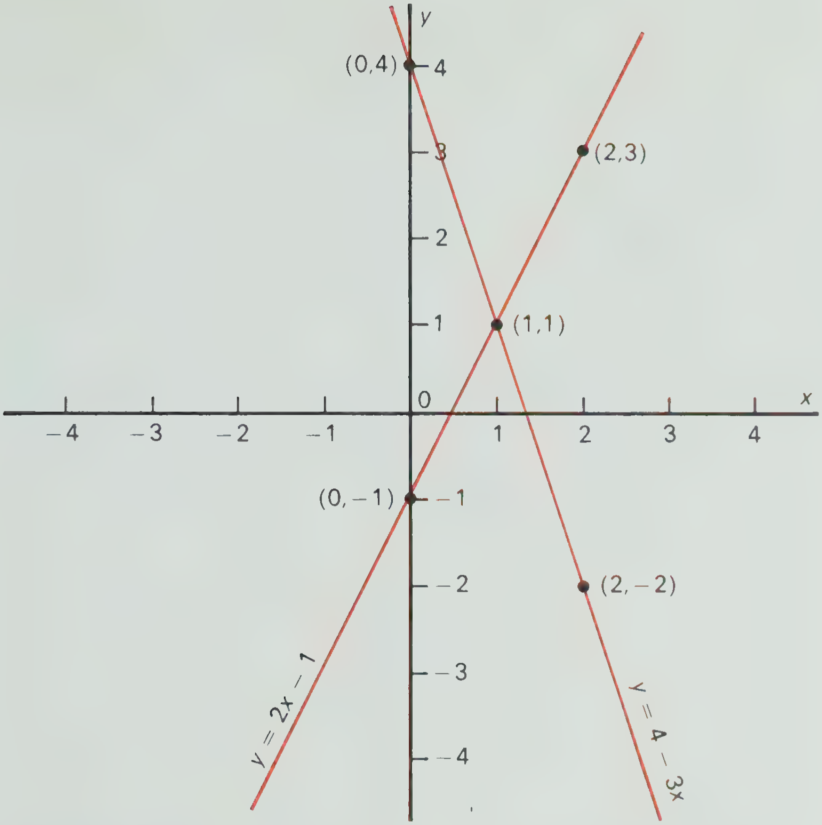
EXAMPLE 1. *Sketch the graphs of $y = 2x - 1$ and $y = 4x - 3$*

$y = 2x - 1$

x	y
0	−1
1	1
2	3

$y = 4 - 3x$

x	y
0	4
1	1
2	−2



There are many ordered pairs that satisfy the relation $y = 2x - 1$ and similarly there are many different ordered pairs that satisfy $y = 4 - 3x$. There is one ordered pair that satisfies both relations. In this case it is the ordered pair (1,1). Graphically, it is the point of intersection of the graphs of the relations $y = 2x - 1$ and $y = 4 - 3x$. Finding the co-ordinates of this point of intersection is called solving the equations.

EXAMPLE 2. *Solve the following graphically. (These equations are called simultaneous equations.)*

$$y + 2x = -7$$
$$y - 3x = 3$$

Solution

Set up a table of values and graph the relations

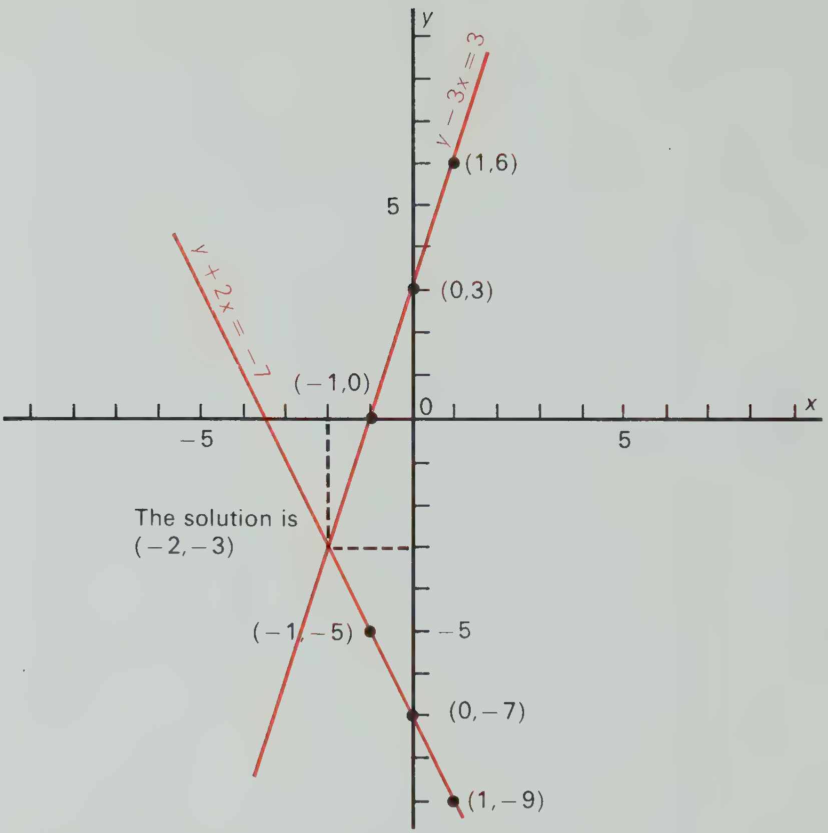
$$y + 2x = -7$$
$$y = -2x - 7$$

x	y
−1	−5
0	−7
1	−9

$$y - 3x = 3$$
$$y = 3x + 3$$

x	y
−1	0
0	3
1	6

Test your skill: $(-1)^{305}$



EXAMPLE 3. *Solve the following graphically. (In this example it is convenient to use the intercept method to draw the graphs.)*

$$\begin{aligned} 3x - 2y &= 12 \\ 2y - x &= -8 \end{aligned}$$

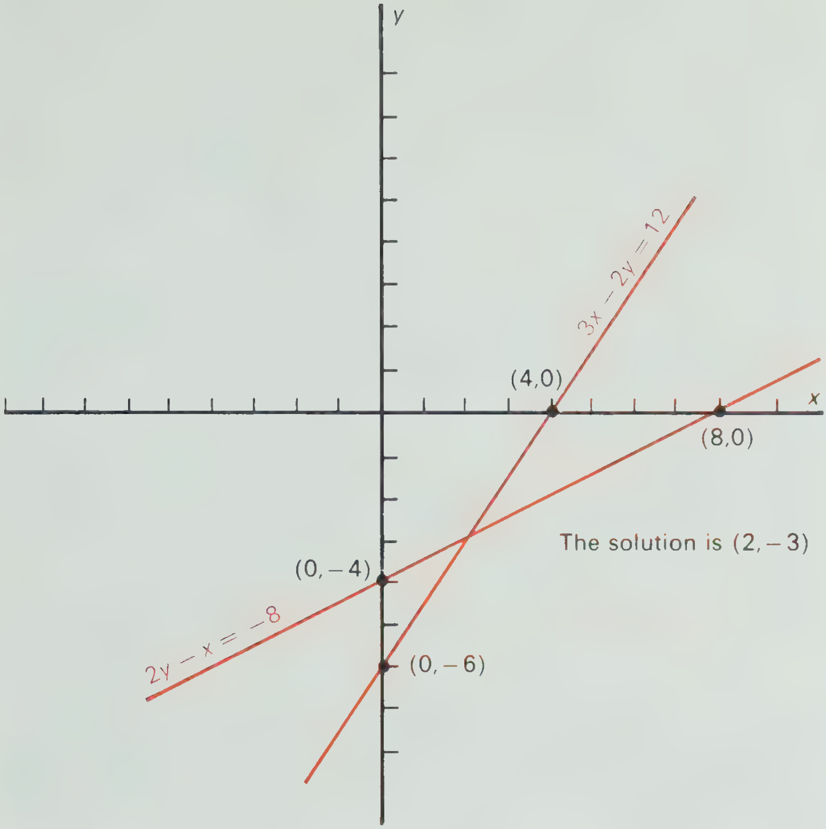
Solution

For $3x - 2y = 12$, when $x = 0$, $y = -6$
when $y = 0$, $x = 4$

$\therefore$ Two ordered pairs are $(0,-6)$ and $(4,0)$

For $2y - x = -8$, when $x = 0$, $y = -4$
when $y = 0$, $x = 8$

$\therefore$ Two ordered pairs are $(0,-4)$ and $(8,0)$



EXERCISE 6-4

Test your skill:
0.073 × 1.38

B **1.** Solve the following graphically

(a) $y = x + 3$
$y = 5 - x$

(b) $y = 2x + 1$
$y = 4 - x$

(c) $y = 2x - 3$
$y = 5 - 2x$

(d) $y = 5 + 3x$
$y = 1 + x$

(e) $y = x + 8$
$y = -3x$

(f) $y = 2x$
$y = 3 - x$

2. Solve the following graphically

(a) $x + 2y = 4$
$x - y = 1$

(b) $x + y = 6$
$3x - y = 6$

(c) $3x - 2y = 18$
$2x + y = 12$

(d) $5x - 2y = 10$
$x + y = 9$

(e) $2x + 3y = -12$
$2x - y = -4$

(f) $x + 2y = 2$
$x - y = -4$

Use the intercept method to draw the graphs.

3. Solve the following graphically

(a) $y = 2x + 3$
$y = 5 - 2x$

(b) $y = 1 - x$
$y = \dfrac{4x - 1}{2}$

(c) $y = 1 - 2x$
$y = 4x + 4$

(d) $y = 2 - 2x$
$y = 4x - 13$

(e) $y = \dfrac{2x}{3}$
$y = \dfrac{2 - 2x}{3}$

(f) $y = \dfrac{3x}{5}$
$y = \dfrac{2 - 3x}{5}$

C **4.** Solve the following graphically and answer the question below

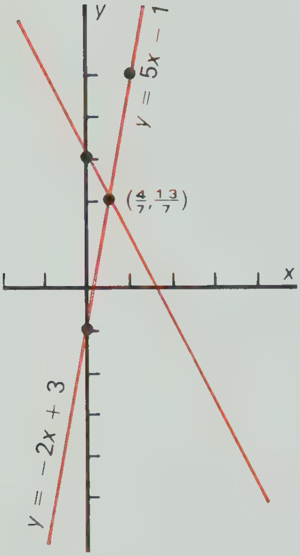
(a) $y = x + 5$
$y = x - 1$

(b) $y = 3x + 2$
$y = 3x - 1$

(c) $y = -2x + 2$
$y = -2x - 3$

(d) $2x + 3y = 6$
$4x + 6y = 12$

How many points of intersection are there for each pair of equations?



Addition
moon
men
can
reach

6.5 SOLVING EQUATIONS IN TWO VARIABLES BY COMPARISON

The graphical method of solving simultaneous equations becomes difficult when the co-ordinates of the ordered pair of the solution set are not integers and cannot be read easily from the graph. An algebraic method of solving simultaneous equations will now be introduced.

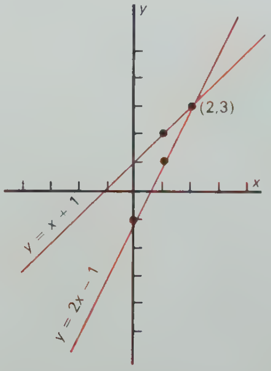
if $a = b$
and $a = c$
then $b = c$

We will use this fact to solve simultaneous equations.

EXAMPLE 1. *Solve $y = 2x - 1$ and $y = x + 1$.*

Solution

Graphically:



$y = 2x - 1$

x	y
1	1
2	3
3	5

$y = x + 1$

x	y
1	2
2	3
3	4

or

$$y = y$$

if $y = 2x - 1 \qquad y = x + 1$

then $$2x - 1 = x + 1$$
$$2x - x = 1 + 1$$
$$x = 2$$

Substituting $x = 2$ in $y = 2x - 1$

$$\begin{aligned} y &= 2x - 1 \\ y &= 2(2) - 1 \\ y &= 4 - 1 \\ y &= 3 \end{aligned}$$

Checking in $y = x + 1$

$$\begin{aligned} \text{L.S.} &= y & \text{R.S.} &= x + 1 \\ &= 3 & &= 2 + 1 \\ & & &= 3 \end{aligned}$$

The solution is (2,3).

EXAMPLE 2. *Solve* $3x - 2y = 1$ ①
$2x - 3y = -1$ ②

(Numbering the equations ① and ② simplifies identification.)

Solution
Express each equation in the form $y = mx + b$

$$\begin{aligned} 3x - 2y &= 1 \\ -2y &= 1 - 3x \\ y &= -\frac{1}{2} + \frac{3}{2}x \\ y &= \frac{3x - 1}{2} \end{aligned} \qquad\qquad \begin{aligned} 2x - 3y &= -1 \\ -3y &= -1 - 2x \\ y &= \frac{1}{3} + \frac{2x}{3} \\ y &= \frac{2x + 1}{3} \end{aligned}$$

Since $y = y$

$$\therefore \frac{3x - 1}{2} = \frac{2x + 1}{3}$$

$$6\left[\frac{3x - 1}{2}\right] = 6\left[\frac{2x + 1}{3}\right]$$

L.C.D. of 2 and 3 is 6 (Rule IV)

$$3(3x - 1) = 2(2x + 1)$$
$$9x - 3 = 4x + 2$$

$$9x - 4x = 2 + 3$$
$$5x = 5$$
$$x = 1$$

Substitute $x = 1$ in ①

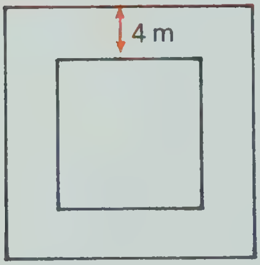
$$\begin{aligned} 3x - 2y &= 1 \\ 3(1) - 2y &= 1 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$

Check in ②

$$\begin{aligned} \text{L.S.} &= 2x - 3y \\ &= 2(1) - 3(1) \\ &= 2 - 3 \\ &= -1 \\ \text{R.S.} &= -1 \end{aligned}$$

$\therefore$ the solution is (1,1)

How can you use 2 planks each 4 m long to cross a moat 4 m wide?



EXERCISE 6-5

A 1. Express each of the following in the form $y = mx + b$

(a) $2x + y = 7$
(b) $y - 3x = 2$
(c) $-4x + y = 5$
(d) $4x + y - 2 = 0$
(e) $y - 4x + 2 = 0$
(f) $3x = 2 + y$
(g) $4x + 2y = 5$
(h) $5x + 3y = 12$
(i) $4y - 2x = 7$
(j) $5y - x = 1$
(k) $3x - y = 2$
(l) $3x - 2y = 6$

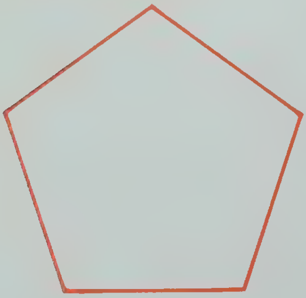
B Solve and check.

2. (a) $\begin{array}{l} y = 2x + 1 \\ y = x + 3 \end{array}$
(b) $\begin{array}{l} y = 3x - 1 \\ y = x + 5 \end{array}$
(c) $\begin{array}{l} y = 4x - 5 \\ y = x + 7 \end{array}$
(d) $\begin{array}{l} y = -2x + 3 \\ y = x + 6 \end{array}$
(e) $\begin{array}{l} y = -4x + 3 \\ y = -2x - 7 \end{array}$
(f) $\begin{array}{l} y = 4x + 3 \\ y = -x - 7 \end{array}$
(g) $\begin{array}{l} y = 2x + 3 \\ y = 6 - 2x \end{array}$
(h) $\begin{array}{l} y = x + 3 \\ y = 8 - 9x \end{array}$

3. (a) $\begin{array}{l} y = x + 2 \\ y = \frac{1}{2}x + 3 \end{array}$
(b) $\begin{array}{l} y = \frac{1}{2}x - 2 \\ y = \frac{1}{4}x + 1 \end{array}$
(c) $\begin{array}{l} y = \dfrac{x - 1}{2} \\ y = \dfrac{x + 2}{3} \end{array}$
(d) $\begin{array}{l} y = \dfrac{5x + 2}{3} \\ y = 2x + 1 \end{array}$
(e) $\begin{array}{l} y = \dfrac{1 - 3x}{4} \\ y = \dfrac{2x - 3}{2} \end{array}$
(f) $\begin{array}{l} y = \frac{1}{2}x + 3 \\ y = \dfrac{2x + 1}{3} \end{array}$

4. (a) $\begin{array}{l} 2x + 3y = 5 \\ 4x + y = 5 \end{array}$
(b) $\begin{array}{l} 3x + 2y = 12 \\ 2x + 3y = 13 \end{array}$
(c) $\begin{array}{l} 3x + 4y = 2 \\ 4x - y = 9 \end{array}$
(d) $\begin{array}{l} x + 3y = -5 \\ 3x - 2y = 7 \end{array}$
(e) $\begin{array}{l} 2x + 5y = 4 \\ 6x - 2y = -5 \end{array}$
(f) $\begin{array}{l} 2x + 3y = -2 \\ 2x - 3y = 0 \end{array}$
(g) $\begin{array}{l} 3x - 4y = -4 \\ 2y - 6x = 5 \end{array}$
(h) $\begin{array}{l} 5x - 8y = 8 \\ 10x + 4y = 1 \end{array}$

How many diagonals has a pentagon?



6.6 SOLVING EQUATIONS IN TWO VARIABLES BY ELIMINATION

Another algebraic method of solving two simultaneous equations is based on the elimination of one of the variables.

EXAMPLE 1. *Solve* $x + y = 7$ ①
$x - y = 1$ ②

Solution
By adding the equations we can eliminate one of the variables, resulting in a linear equation in one variable, which can easily be solved.

$$\begin{array}{lrcl}
 & x + y &=& 7 \quad ① \\
 & x - y &=& 1 \quad ② \\ \cline{2-4}
\text{Adding} & 2x &=& 8 \\
 & x &=& 4
\end{array}$$

(Note that the y terms are opposites)

Substitute $x = 4$ in ①

$$\begin{aligned}
x + y &= 7 \\
4 + y &= 7 \\
y &= 3
\end{aligned}$$

Check in ②

$$\begin{aligned}
\text{L.S.} &= x - y \\
&= 4 - 3 \\
&= 1 \\
\text{R.S.} &= 1
\end{aligned}$$

The solution is (4,3)

EXAMPLE 2. *Solve* $2x + y = 7$ ①
$x + y = 5$ ②

Solution

$$\begin{array}{lrcl}
 & 2x + y &=& 7 \quad ① \\
 & x + y &=& 5 \quad ② \\ \cline{2-4}
\text{Adding} & 3x + 2y &=& 12
\end{array}$$

Into how many regions can a circle be divided using five chords?

Adding does not eliminate one of the variables when the signs are the same. However, subtracting will do so.

$$\begin{array}{rrcl}
\text{Subtracting} & 2x + y &=& 7 \quad ① \\
\text{(add the} & x + y &=& 5 \quad ② \\ \cline{2-4}
\text{opposites)} & x &=& 2
\end{array}$$

(Note that the y terms are identical)

Substituting $x = 2$ in ①

$$\begin{aligned}
2x + y &= 7 \\
2(2) + y &= 7 \\
4 + y &= 7 \\
y &= 3
\end{aligned}$$

Check in ②

$$\begin{aligned} \text{L.S.} &= x + y \\ &= 2 + 3 \\ &= 5 \\ \text{R.S.} &= 5 \end{aligned}$$

The solution is (2,3)

EXAMPLE 3. *Solve* $2x + 3y = 16$ ①
$5x - 2y = 2$ ②

Neither addition or subtraction eliminates one of the variables.

Adding

$$\begin{array}{rl} 2x + 3y = 16 & \textcircled{1} \\ 5x - 2y = 2 & \textcircled{2} \\ \hline 7x + y = 18 & \end{array}$$

Subtracting

$$\begin{array}{rl} 2x + 3y = 16 & \textcircled{1} \\ 5x - 2y = 2 & \textcircled{2} \\ \hline -3x + 5y = 14 & \end{array}$$

Elimination will not occur until either the x terms or y terms are identical or opposites.

Solution 1
(eliminating y)

$$\begin{array}{lrl} & 2x + 3y = 16 & \textcircled{1} \\ & 5x - 2y = 2 & \textcircled{2} \\ \textcircled{1} \times 2 & 4x + 6y = 32 & \textcircled{1} \\ \textcircled{2} \times 3 & 15x - 6y = 6 & \textcircled{2} \\ \hline \text{adding} & 19x = 38 & \\ & x = 2 & \end{array}$$

Substituting in ①

$$\begin{aligned} 2x + 3y &= 16 \\ 2(2) + 3y &= 16 \\ 4 + 3y &= 16 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

Check in ②

$$\begin{aligned} \text{L.S.} &= 5x - 2y \\ &= 5(2) - 2(4) \\ &= 10 - 8 \\ &= 2 \\ \text{R.S.} &= 2 \end{aligned}$$

Solution 2
(eliminating x)

$$\begin{array}{lrl} & 2x + 3y = 16 & \textcircled{1} \\ & 5x - 2y = 2 & \textcircled{2} \\ \textcircled{1} \times 5 & 10x + 15y = 80 & \textcircled{1} \\ \textcircled{2} \times 2 & 10x - 4y = 4 & \\ \hline \text{subtracting} & 19y = 76 & \\ & y = 4 & \end{array}$$

Substituting in ②

$$\begin{aligned} 5x - 2y &= 2 \\ 5x - 2(4) &= 2 \\ 5x - 8 &= 2 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Check in ①

$$\begin{aligned} \text{L.S.} &= 2x + 3y \\ &= 2(2) + 3(4) \\ &= 4 + 12 \\ &= 16 \\ \text{R.S.} &= 16 \end{aligned}$$

∴ The solution is (2,4)

Test your skill:
$3\frac{1}{2} \times 4\frac{1}{4}$

EXERCISE 6-6

A Solve for x and y

1. (a) $x - y = 2$
$x + y = 4$

(b) $x + y = 10$
$x - y = 2$

(c) $x - y = 0$
$x + y = 4$

(d) $2x - y = 3$
$x + y = 6$

(e) $2x + y = 4$
$3x - y = 6$

(f) $-x + y = 0$
$x + y = 2$

(g) $-x + 2y = 1$
$x + y = 2$

(h) $2x + y = 5$
$x + y = 4$

B Solve and check.

2. (a) $2x + y = 10$
$3x - 2y = 8$

(b) $a + 2b = 7$
$a - b = 1$

(c) $3b + c = 12$
$2b + 5c = 21$

(d) $m + 3n = 10$
$3m + 2n = 16$

(e) $r - 3s = 1$
$3r - 2s = 17$

(f) $2x + 5y = 19$
$3x - y = 3$

(g) $5x - 3y = 4$
$4x - y = 6$

(h) $4a - b = 7$
$6a + 5b = 17$

Test your skill:
$2\frac{1}{2} \div \frac{5}{8}$

3. (a) $3a - 2b = 5$
$2a + 3b = 12$

(b) $2x + 3y = 8$
$5x + 2y = 9$

(c) $4m + 3n = 17$
$5m + 2n = 16$

(d) $5x - 2y = 1$
$3x + 4y = 11$

(e) $4r - 3s = 2$
$3r + 5s = 16$

(f) $7t - 3m = 11$
$3t + 4m = 10$

(g) $3b - 4c = 11$
$2b - 3c = 7$

(h) $2c - 3d = 2$
$5c - 2d = 16$

4. (a) $x - 2y = 3$
$2x - 3y = 4$

(b) $3a - 2b = -8$
$4a + 3b = -5$

(c) $2x + 3y = 11$
$3x - 2y = -16$

(d) $5x + 2y = 5$
$2x + 3y = 13$

(e) $5m + 3n = -19$
$2m - 5n = 11$

(f) $3b - 4c = 5$
$5b + 3c = -11$

(g) $3r - 4t = 10$
$5r - 12t = 6$

(h) $4c - 3d = -3$
$5c - 2d = -9$

C **5.** (a) $\frac{x}{2} + \frac{y}{3} = 2$
$\frac{x}{2} - \frac{y}{3} = 0$

(b) $\frac{x}{2} + \frac{y}{5} = 2$
$\frac{3x}{2} - \frac{y}{5} = 2$

Clear the fractions before attempting solution.

(c) $x - y = 6$
$\frac{2x}{3} + \frac{y}{3} = 1$

(d) $\frac{1}{2}x + y = -4$
$\frac{x}{2} - \frac{3y}{2} = 1$

(e) $\frac{x}{3} + \frac{y}{2} = 2$
$\frac{3x + 1}{2} + \frac{y + 1}{3} = 6$

(f) $\frac{1}{2}x - \frac{1}{3}y = 1$
$\frac{x - 1}{3} - \frac{y - 1}{4} = \frac{1}{2}$

6.7 PROBLEM SOLVING

We solve problems using mathematical methods. In this section, problems resulting in simultaneous equations are studied.

EXAMPLE 1. *Find two numbers such that twice the larger plus 3 times the smaller is 17 and 5 times the larger less twice the smaller is 14.*

Solution
Let the larger number be represented by x.
Let the smaller number be represented by y.

$$\begin{array}{ccccc} \text{twice the larger} & & \text{3 times the smaller} & & \\ 2x & + & 3y & = 17 & \text{①} \\ 5x & - & 2y & = 14 & \text{②} \\ \text{5 times the larger} & & \text{twice the smaller} & & \end{array}$$

Now the equations are solved using either the comparison method or the elimination method.

$$\begin{array}{lrl} & 2x + 3y = 17 & \text{①} \\ & 5x - 2y = 14 & \text{②} \\ \text{①} \times 2 & 4x + 6y = 34 & \text{①} \\ \text{②} \times 3 & 15x - 6y = 42 & \text{②} \end{array} \qquad \text{Solving by elimination}$$

$$\begin{array}{lr} \text{Adding} & 19x = 76 \\ & x = 4 \end{array}$$

$$\begin{array}{lr} \text{Substitute } x = 4 \text{ in ①} & 2x + 3y = 17 \\ & 2(4) + 3y = 17 \\ & 8 + 3y = 17 \\ & 3y = 9 \\ & y = 3 \end{array}$$

Check in the original statement of the problem:

$$\begin{array}{cccl} \text{Twice the larger} & & \text{3 times the smaller} & \\ 2(4) & + & 3(3) & = 8 + 9 = 17 \text{ (checks)} \\ & \text{plus} & & \\ \text{5 times the larger} & & \text{twice the smaller} & \\ 5(4) & - & 2(3) & = 20 - 6 = 14 \text{ (checks)} \\ & \text{less} & & \end{array}$$

Therefore the larger number is 4 and the smaller is 3.

EXAMPLE 2. *A garage ordered 4 more alternators than fuel pumps. Twice the number of alternators plus 3 times the number of fuel pumps is 48. How many fuel pumps were ordered?*

Solution

Let the number of alternators be represented by x.
Let the number of fuel pumps be represented by y.

$x - y = 4$ ① The difference between alternators
$2x + 3y = 48$ ② and fuel pumps is 4.

①× 3 $3x - 3y = 12$ ① Solving by elimination
$2x + 3y = 48$ ②

Adding $5x = 60$
$x = 12$

Substitute $x = 12$ in ①

$$x - y = 4$$
$$12 - y = 4$$
$$-y = -8$$
$$y = 8$$

Check in the original statement of the problem:

If $x = 12$ and $y = 8$, there are 4 more alternators than fuel pumps;

$$\overset{\text{twice the number of alternators}}{2(12)} \underset{\text{plus}}{+} \overset{\text{three times the fuel pumps}}{3(8)} = 24 + 24 = 48$$

Therefore the garage ordered 8 fuel pumps.

Find 4 positive consecutive numbers whose sum is 322.

EXERCISE 6-7

A 1. Express each of the following as an algebraic expression.
(a) Three times a number.
(b) One half a number.
(c) 7 more than x.
(d) 12 more than y.
(e) b increased by 12.
(f) Twice m.
(g) m diminished by 4.
(h) 8 less than x.
(i) 4 times a number, increased by 6.
(j) Twice a number diminished by 7.
(k) 7 more than b.
(l) 3 times a number less 6.

2. Express each of the following as an algebraic equation in two variables.
(a) The sum of two numbers is 12.
(b) The sum of two numbers is 26.
(c) The difference between two numbers is 6.

(d) There are 28 boys and girls in the class.
(e) The store sold a total of 156 switches and batteries.
(f) There were 4 more cars than trucks bought.
(g) Twice a number plus 6 times another number is 88.
(h) 5 times a number less 3 times another number is 18.
(i) The difference in two numbers is 56.
(j) The store sold a total of 77 basketballs and footballs.

B Solve the following problems using two equations in two variables.

3. The sum of two numbers is 131 and their difference is 25. Find the numbers.

4. A department store employed 120 people. The full-time staff outnumbered the part-time staff by 66. How many people worked part time?

5. On a busy Saturday a bakery sold 55 more cakes than pies. If the total number of cakes and pies sold was 151, how many pies were sold?

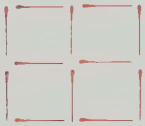
6. The sum of two numbers is 24. Twice the smaller plus 3 times the larger is 62. Find the numbers.

7. A sporting goods store sold 3 more footballs than basketballs. Three times the number of basketballs sold plus 4 times the number of footballs is 96. How many footballs were sold?

8. The difference in two numbers is 6. Three times the larger minus twice the smaller is 21. Find the numbers.

9. In David's math class there are 30 students. There are 2 more boys than girls. How many girls are in the class?

Remove only two matchsticks to leave two squares.



10. Andrew bought several switches and batteries. Twice the number of switches plus 3 times the number of batteries is 50. Five times the number of switches minus twice the number of batteries is 11. How many batteries did he buy?

11. One hundred and twenty tickets were sold to a basketball game. Five times the number of student tickets sold diminished by 160 equals 3 times the number of adult tickets sold. How many student tickets were sold?

12. A garage ordered a total of 150 alternators and generators. Three times the number of alternators added to twice the number of generators is 380. How many alternators were ordered?

13. During a one-week period a mechanic did a total of 35 grease jobs and oil changes. Five times the number of grease jobs increased by 5 equals 4 times the number of oil changes. How many oil changes did he do?

14. A store sold 40 more cases of cola than root beer. If the store had sold 30 more cases of cola and 70 more cases of root beer, a total of 700 cases would have been sold. How many cases of cola were sold?

15. Paula earned a total of 170 marks in history and geography. Twice her history mark decreased by 40 equals her geography mark increased by 33. What was her history mark?

16. If 5 times a smaller number plus 3 times a larger equals 19 and 4 times the smaller plus 7 times the larger is 29, find the numbers.

17. Two angles are complementary. Three times the smaller exceeds twice the larger by 10°. What is the degree measure of each?

REVIEW EXERCISE

A **1.** Solve

(a) $x + 5 = 12$ (b) $2a = 14$ (c) $t - 2 = 8$
(d) $-3m = 12$ (e) $6 + b = 6$ (f) $3d = -3$
(g) $-t = 4$ (h) $3b + 3 = 6$ (i) $3 + x = -6$
(j) $\frac{1}{2}m = 8$ (k) $\frac{y}{3} = 9$ (l) $\frac{m}{-5} = 4$
(m) $a - 2 = -9$ (n) $-3x = 0$ (o) $2t - 2 = 2$

2. Express in the form $y = mx + b$

(a) $3x + y = 9$ (b) $y - 4x = 2$ (c) $-x + y = 11$
(d) $y + 3x + 2 = 0$ (e) $4 + y = -2x$ (f) $x = 3 + y$
(g) $2y + 4x = 6$ (h) $2x + 3y = 7$ (i) $5x - 2y = 11$

3. Solve for x and y

(a) $x - y = 6$, $x + y = 8$ (b) $x + y = 5$, $x - y = 3$ (c) $x + y = 12$, $-x + y = 4$
(d) $3x + y = 12$, $x + y = 4$ (e) $x + 5y = 16$, $x + y = 4$ (f) $x + y = 0$, $2x - y = 9$

B **4.** Solve and check

Test your skill: $(0.2)^3$

(a) $5x - 10 = 15$ (b) $6a = 4a + 8$
(c) $4b = 5b + 3$ (d) $3(x - 1) = 9$
(e) $2(m - 3) - (m + 1) = 8$ (f) $4(2t - 3) - (1 + 3t) = -3$
(g) $2(1 - 3x) - 3(x - 7) = 2(x - 1) - 1$
(h) $3m - (m - 3) + 2(1 - 2m) = 3(1 + m) - 18$
(i) $6 - 3(1 - 2a) - (a - 2) = 5 - 2(1 - 3a)$

5. Solve

(a) $\frac{x - 1}{3} = 6$ (b) $\frac{m}{3} - \frac{1}{2} = \frac{1}{4}$
(c) $\frac{b}{4} - \frac{b}{6} = 3$ (d) $\frac{2b + 1}{3} = 3$
(e) $\frac{2a - 1}{4} = \frac{3a + 2}{6}$ (f) $\frac{x - 7}{6} + 3 = \frac{x + 1}{2}$
(g) $\frac{m + 7}{6} + \frac{1}{2} = \frac{m - 2}{4}$ (h) $\frac{3m + 7}{4} - \frac{m}{3} = -\frac{1}{3}$

6. Solve and check

(a) $(x - 1)(x + 2) = (x + 3)(x + 2)$
(b) $(a - 3)(a + 4) - (a + 2)(a + 1) = -16$
(c) $4x^2 - (2x - 1)(2x + 1) = x$
(d) $(2b - 3)(b - 2) - 2(b + 1)(b + 4) = 6 - 7b$

(e) $(m - 3)(2m + 1) - 2m^2 = 2$
(f) $(t - 3)(t - 2) - (t + 1)(t - 4) = 4$
(g) $2(x - 2)(x + 1) - 3(x + 2)(x + 1) = 1 - x^2$

7. Solve and check

(a) $y = 3x - 1$
$y = 2x + 7$

(b) $y = -2x + 4$
$y = 3x - 6$

(c) $y + 3x = -6$
$y = 2x + 4$

(d) $4a + b = 10$
$3a + 2b = 5$

(e) $3x - 2y = 3$
$4x + 3y = 4$

(f) $5m - 2n = 11$
$2m - 3n = 0$

(g) $4t - 3r = 2$
$2t + 5r = -12$

(h) $5d - 3b = 3$
$6d - 4b = 2$

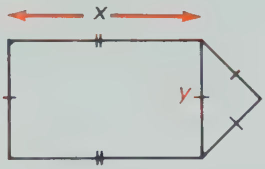
If $3 \times n = 3 + n$,
$n = ?$

8. The sum of 2 numbers is 15. Twice the smaller plus 3 times the larger is 39. Find the numbers.

9. In Melissa's English class there are 8 more girls than boys. If there are 32 students in all, how many girls are there?

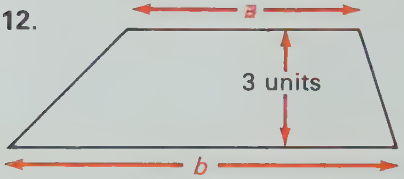
10. The difference in 2 numbers is 5. Twice the larger minus 3 times the smaller is 8. Find the numbers.

11.



The perimeter of the rectangle is 16 units. The perimeter of the triangle is 9 units. What is the perimeter of the whole figure?

12.



The area of the trapezoid is

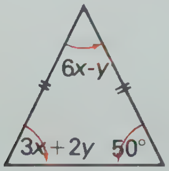
$$3\left(\frac{a + b}{2}\right) = 24 \text{ units}^2$$

If b is 4 units longer than a, find the lengths of a and b.

13. Suppose we make up a sequence starting with terms a and b and add two previous terms to get the next.
$a, b, a + b, a + 2b, 2a + 3b, 3a + 5b, 5a + 8b$
If the last two terms are 21 and 34, what are the first two?

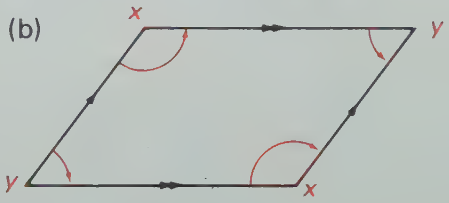
14. Find the value of x and y in each of the following figures.

(a)



$3x + 2y = 50^\circ$
$6x - y = 80^\circ$ (why?)

(b)



$x - y = 40^\circ$
$x + y = 180^\circ$ (why?)

REVIEW AND PREVIEW TO CHAPTER 7

LENGTH, CAPACITY AND MASS

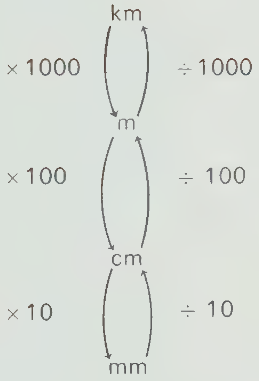
EXERCISE 1

Length

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

1. Complete the table in your notebook.

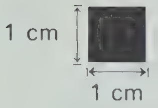
kilometres km	metres m	centimetres cm	millimetres mm
1			
	1		
		1	
			1
	100		
			1000
		1000	



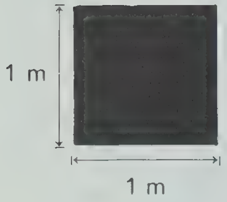
2. Complete the table in your notebook.

km	m	cm	mm
2.5			
0.093			
	576		
	3.2		
			7860
		35.2	
			107.3
		0.79	

3. On a map, a measurement of 1 cm represents 2.5 km. How far apart are two cities if the measurement between them is 5.2 cm?

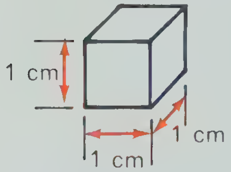


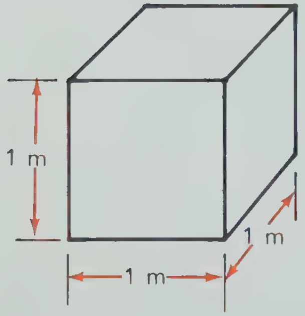
1 square
centimetre
= 1 cm²



1 square
metre
= 1 m²

$$\underset{\text{velocity}}{v} = \frac{\overset{\text{distance}}{d}}{\underset{\text{time}}{t}}$$

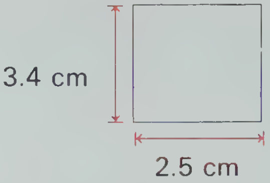




1 cubic centimetre
= 1 cm × 1 cm × 1 cm
= 1 cm³

1 cubic metre
= 1 m³

4. Calculate the area of the rectangle in square centimetres.

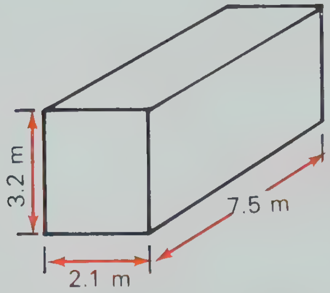


5. A sheet of panelling has dimensions 1.25 m by 2.50 m. What wall area, in square metres, will this sheet cover?

6. Calculate the area of a circle with a radius of 1.5 cm. (Use $\pi = 3.14$.)

7. The distance from Hamilton to Toronto is approximately 64 km. If the trip took $\frac{3}{4}$ h, calculate the average speed for the trip.

8. Calculate the volume of the rectangular solid in m³.



9. The volume of a sphere is given by the formula

$$V = \frac{4}{3}\pi r^3$$

Calculate the volume in cm³ if r, the radius, is 15 cm. (Use $\pi = 3.14$.)

EXERCISE 2

Capacity

1 L = 100 cL
1 cL = 10 mL

Mass

1 kg = 1000 g

1. Complete the table in your notebook.

litre L	centilitre cL	millilitre mL
1		
	1	
		1
		1000
0.01		
	0.1	

2. Complete the table in your notebook.

litre L	centilitre cL	millilitre mL
560		
	3.75	
	0.16	
		1365
0.79		
		0.85

$$\begin{tikzcd} L \arrow[d, bend right, "\times 100"'] \\ cL \arrow[u, bend right, "\div 100"'] \arrow[d, bend right, "\times 10"'] \\ mL \arrow[u, bend right, "\div 10"'] \end{tikzcd}$$

3. Complete the table below.

kilograms kg	grams g
	100
10	
0.01	
1.5	
	5750
	37.5
	0.875

$$\begin{tikzcd} kg \arrow[d, bend right, "\times 1000"'] \\ g \arrow[u, bend right, "\div 1000"'] \end{tikzcd}$$

4. The following are the masses of five men:

72 kg, 81.9 kg, 70.2 kg, 85.9 kg, 73.4 kg.

Calculate the average mass of the men to the nearest tenth of a kilogram.

5. The cost of 908 g of honey is $1.12. Calculate the cost in cents per gram of honey.

6. The density of gold is approximately 19 300 kg/m^3.

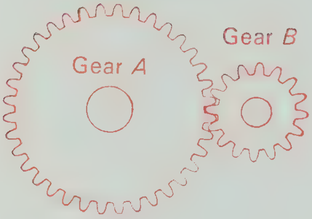
(a) What will be the mass of 0.000 015 m^3 of gold?

(b) What will be the volume of 0.137 03 kg of gold?

7. In Europe, gasoline consumption is usually expressed in L/100 km. If the consumption is 15 L/100 km, how many litres will be required to travel 256 km?

CHAPTER 7

Ratio and Proportion



7.1 RATIO

A **ratio** is a comparison of two or more quantities. In the case of the gears shown at left, the ratio of the number of teeth on gear *A* to the number of teeth on gear *B* is 36:12. Notice that this tells us that there are three times as many teeth on *A* as on *B*. In simplest form, the ratio 36:12 may also be written 3:1.

Ratios which express the same idea are called **equivalent ratios.**

EXAMPLE 1. *At the school dance, there were 25 teachers, 175 girls, and 250 boys. Express the following ratios in simplest form.*

(*a*) *Boys to girls* (*b*) *Girls to boys*
(*c*) *Teachers to students* (*d*) *Teachers to boys*
(*e*) *Boys to girls to teachers.*

Solution

(a) $250:175 = 250 \div 25 : 175 \div 25$
$= 10:7$

(b) $175:250 = 175 \div 25 : 250 \div 25$
$= 7:10$

(c) $25:425 = \frac{25}{425} = \frac{25 \div 25}{425 \div 25} = \frac{1}{17}$

(d) $25:250 = \frac{25}{250} = \frac{25 \div 25}{250 \div 25} = \frac{1}{10}$

$a:b$
is equivalent to
$\frac{a}{b}$

(e) $250:175:25 = 250 \div 25 : 175 \div 25 : 25 \div 25$
$= 10:7:1$

Note that in simplest form, a ratio is expressed in terms of whole numbers.

EXAMPLE 2. *John and Karen share a paper route. During one week John delivered the papers on Monday, Wednesday, Thursday, and Friday, and Karen on Tuesday and Saturday. Their total earnings for the week were $12.*

(*a*) *In what ratio should the total earnings be divided?*
(*b*) *What is each person's share of the total earnings?*

Solution
(a) Since John delivered for four days and Karen two days, the ratio should be 4:2 which simplifies to 2:1.
(b) The ratio $2x:x$ is equivalent to 2:1.

$$2x:x = \frac{2x}{x} = \frac{2}{1} = 2:1$$

Let John's amount in dollars be represented by $2x$ and Karen's amount in dollars be represented by x.

$$\begin{aligned} 2x + x &= 12 \\ 3x &= 12 \\ x &= 4 \\ 2x &= 8 \end{aligned}$$

Therefore Karen's share is \$4 and John's share is \$8.

EXAMPLE 3. *The student council decided that profits from their rock concert should be divided in the ratio 5:3:2 between the social committee, the drama committee, and the school band. If the profit totalled \$350, how much would each group receive?*

Solution Let the amounts in dollars be represented by $5a$, $3a$, and $2a$.

$5a:3a:2a$ is equivalent to 5:3:2

$$\begin{aligned} 5a + 3a + 2a &= 350 \\ 10a &= 350 \\ a &= 35 \end{aligned}$$

$$\begin{aligned} 5a &= 5 \times 35, & 3a &= 3 \times 35, & 2a &= 2 \times 35 \\ &= 175 & &= 105 & &= 70 \end{aligned}$$

Therefore the social committee received \$175, the drama committee \$105, and the school band \$70.

EXERCISE 7-1

A **1.** Express the following ratios in their simplest form.
(a) 4:2 (b) 18:3 (c) 5:35 (d) 12:9
(e) 15:6 (f) 27:18 (g) 48:16 (h) 100:40

2. Express the following ratios in simplest form.
(a) 36:18:9 (b) 8:4:4 (c) 13:26:52 (d) 48:16:24
(e) 42:14:28 (f) 18:45:72 (g) 8:8:20 (h) 76:38:19
(i) $\frac{4}{6}$ (j) $\frac{10}{12}$ (k) $\frac{15}{12}$ (l) $\frac{21}{35}$
(m) $\frac{14}{7}$ (n) $\frac{56}{16}$ (o) $\frac{15}{90}$

3. State the value of a which will make the ratio $4a:6a:10a$ equivalent to:
(a) 4:6:10 (b) 12:18:30 (c) 2:3:5
(d) 400:600:1000 (e) 48:72:120 (f) 1:1.5:2.5

B **4. Quantities being compared in a ratio should be expressed in the same units.** Express, in simplest form, the ratio for each of the following.
(a) 15 cm to 2.5 m (b) 3 l to 400 cm³
(c) 5 m to 23 cm (d) 3 kg to 120 g
(e) 48 cm² to 3000 mm² (f) 90 000 m² to 4 ha

1 m = 100 cm
1 cm² = 100 mm²
1 l = 1000 cm³
1 ha = 10 000 m²

5. **Two-term ratios** in the form $a:b$ may be expressed in the form $\frac{a}{b}$, $b \neq 0$, and then reduced to simplest form. Complete the table below in your notebook using the method illustrated.

Ratio	Simplification	Simplest form
$\frac{1}{2}:\frac{3}{4}$	$\frac{1}{2}/\frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$	2:3
$\frac{1}{2}:3$		
$5:\frac{2}{3}$		
$1\frac{1}{4}:\frac{5}{6}$		
$2:7\frac{1}{2}$		
$1\frac{1}{3}:2\frac{7}{15}$		
$4\frac{2}{5}:3$		

6. Ratios involving fractions and decimals may also be simplified by multiplication. Complete the table below in your notebook using the methods illustrated.

$$\frac{a}{b} \div \frac{c}{d}$$
$$= \frac{a}{b} \times \frac{d}{c}$$

Ratio	Simplification	Simplest form
$\frac{1}{3}:\frac{1}{2}:2$	$\frac{1}{3} \times 6 : \frac{1}{2} \times 6 : 2 \times 6$	2:3:12
0.3:1.7	$0.3 \times 10 : 1.7 \times 10$	3:17
$\frac{1}{5}:\frac{1}{3}$		
0.5:0.3		
2.1:1:0.3		
$\frac{3}{5}:\frac{1}{2}:5$		
2.4:0.8:2		
$1\frac{1}{2}:3:\frac{1}{4}$		

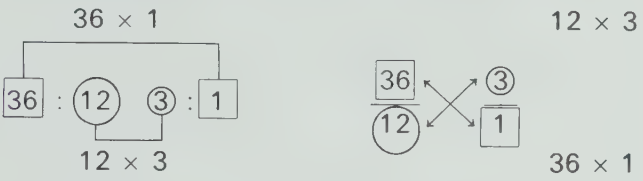
7. Two partners in a business have agreed to share the profits in the ratio of 7:8. If the first year's profits total $8523.75, what will be each man's share?

Test your skill:
what is the L.C.M. of
14 and 49?

8. In the school election, the votes for president were split between Pete, Karen, and Bill in the ratio 4:5:3. If 1152 students voted, how many votes did the winner receive?

9. A mixture of concrete contains 3 parts cement to 4 parts sand to 10 parts crushed gravel. Determine the amounts needed to prepare 1360 m^3 of concrete.

10. A test may be applied to determine **equivalent, two-term ratios.** Notice the pattern for the equivalent ratios 36:12 and 3:1:



Determine which of the following are pairs of **equivalent ratios.**

(a) 3:5, 9:15 (b) $6:1\frac{1}{2}$, 4:16 (c) $2\frac{1}{2}:3$, 18:6

(d) 1.2:3, 2:5 (e) 48:10, 50:240 (f) 5.6:7, 7.2:9

11. The area of a circle is given by the formula

$$A = \pi r^2$$

area

radius

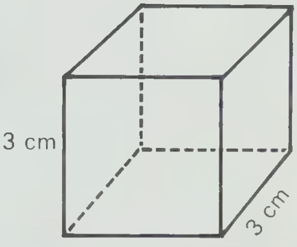
Find the ratio of the areas of two circles with radii 7 cm and 14 cm respectively.

12. The dimensions of a cube are shown in the diagram at right.

(a) Using the formula, $V = lwh$, calculate the volume of the cube, V_1.

(b) If each of the dimensions is doubled, calculate the new volume, V_2.

(c) What is the ratio $V_1 : V_2$ in simplest form?



$$V = lwh$$

Length

Volume

height

width

7.2 PROPORTIONS

Two equivalent ratios can be expressed in a **proportion** such as

$$\frac{42}{12} = \frac{7}{2}$$

or

$$42:12 = 7:2$$

first proportional

second proportional

fourth proportional

third proportional

The proportion is read "42 is to 12 as 7 is to 2".

Several general ideas may be illustrated using the above proportion.

× ÷
before
+ −

Put the numbers 1 to 9 in the spaces to make the statements true:

▨ − ▨ ÷ ▨ = 1
▨ − ▨ × ▨ = 1
▨ + ▨ − ▨ = 1

If 42 : 12 = 7 : 2, then	If $a:b = c:d$, then
1) $\frac{42}{12} = \frac{7}{2}$	$\frac{a}{b} = \frac{c}{d}$
2) $\frac{42}{12} = \frac{7}{2}$ $\therefore\ 42 \times 2 = 12 \times 7$	$\frac{a}{b} = \frac{c}{d}$ $\therefore\ ad = bc$
3) $42:12 = 7:2$ $\therefore\ 42 \times 2 = 12 \times 7$	$a:b = c:d$ $ad = bc$
4) $\frac{42}{7} = \frac{12}{2}$	$\frac{a}{c} = \frac{b}{d}$
5) $42 = 6 \times 7 \quad 12 = 6 \times 2$ constant	$a = kc \quad b = kd$ constant

EXAMPLE 1. *Find the third proportional in the sequence 9, 15, 3.*

Solution Let x represent the third proportional

$$\frac{9}{15} = \frac{x}{3}$$
$$15x = 27$$
$$x = \frac{27}{15}$$
$$x = \frac{9}{5}$$

In the form $\frac{9}{15} = \frac{x}{3}$, the proportion may be easily solved with a slide rule

Therefore the third proportional is $\frac{9}{5}$.

In a proportion such as 3 : 12 = 12 : 48, 12 is known as the **mean proportional** between 3 and 48.

$\frac{a}{b} = \frac{c}{d}$

EXAMPLE 2. *Find the mean proportional between 4 and 9.*

Solution Let x represent the mean proportional

$$\therefore\ 4:x = x:9 \quad \text{or} \quad \frac{4}{x} = \frac{x}{9}$$
$$\therefore\ x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

Therefore the mean proportional is +6 or −6.

Proportions may involve ratios of more than two terms. Problems involving this type of proportion may be solved using the idea of the constant factor. For the proportion 24:18:42 = 4:3:7, notice that

$$24 = 6 \times 4,\ 18 = 6 \times 3,\ 42 = 6 \times 7$$

constant

$$\tfrac{24}{4} = 6 \quad \tfrac{18}{3} = 6 \quad \tfrac{42}{7} = 6$$

For any number of terms,

If $a_1 : a_2 : a_3 = b_1 : b_2 : b_3$, then $a_1 = kb_1$, $a_2 = kb_2$, $a_3 = kb_3$ for some constant value k.

$$\therefore \quad \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

EXAMPLE 3. *If x:15:4 = 7:5:y, determine x and y.*

Solution 1

(i) Since $x:15:4 = 7:5:y$

$$15 = k \times 5$$
$$k = 3$$
$$x = k \times 7 \text{ and } 4 = k \times y$$
$$= 3 \times 7 \qquad 4 = 3 \times y$$
$$= 21 \qquad \tfrac{4}{3} = y$$

Solution 2

(ii) $\dfrac{x}{7} = \dfrac{15}{5} = \dfrac{4}{y}$

$$\frac{x}{7} = \frac{3}{1} \text{ and } \frac{3}{1} = \frac{4}{y}$$
$$x = 21 \text{ and } 3y = 4$$
$$y = \tfrac{4}{3}$$

Put the numbers 1 to 9 in the spaces to make the statements true:

▨ + ▨ ÷ ▨ = 2
▨ − ▨ + ▨ = 2
▨ × ▨ − ▨ = 2

EXAMPLE 4. *The ratio of the area of a triangle to its height is 3:2 when the base is 3* cm. *Calculate the height if the area is to be 3.9* cm².

Solution Let the height in cm be represented by h.

$$3.9 : h = 3:2$$
$$3h = 2 \times 3.9$$
$$h = \tfrac{7.8}{3}$$
$$h = 2.6$$

Therefore the height is 2.6 cm.

EXERCISE 7-2

B

1. Calculate the missing proportional in each case.

(a) $3:x = 4:12$ (b) $\dfrac{a}{12} = \dfrac{14}{4}$ (c) $8:3 = 2:y$

(d) $\dfrac{2}{7} = \dfrac{4}{b}$ (e) $x:8 = 13:4$ (f) $\dfrac{2.5}{x} = \dfrac{5}{6}$

(g) $3.1:a = 12.4:9$ (h) $\dfrac{t}{17.5} = \dfrac{5}{3.5}$ (i) $6:3y = 3y:6$

2. Calculate the missing proportionals in each case.

(a) $14:2:y = 7:x:12$ (b) $\frac{a}{6} = \frac{5}{4} = \frac{b}{3}$

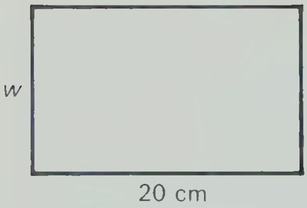
(c) $2.1:x:y = 6.3:5:10$ (d) $\frac{5.2}{2} = \frac{a}{3} = \frac{b}{5}$

(e) $1\frac{1}{2}:x:3 = 2:6:y$ (f) $\frac{a}{2} = \frac{5}{3} = \frac{6}{y}$

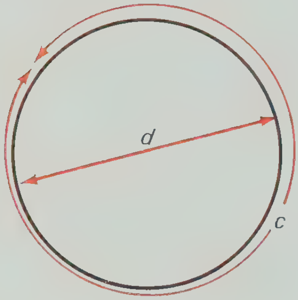
3. Find the fourth proportional in the sequence 8, 3, 6.

4. Find the third proportional in the sequence 4, $2\frac{1}{3}$, 7.

5. Find the mean proportionals between the following pairs of numbers.
(a) 2 and 8 (b) 4 and 15 (c) 4 and 5 (d) 6 and 3

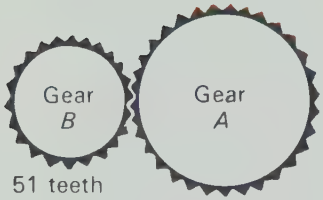


6. The length and width of a rectangle are in the ratio 8:5. If the length is 20 cm, calculate the width.

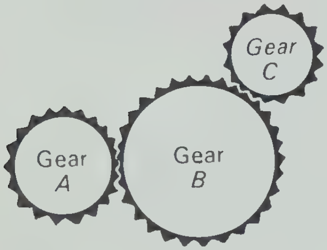


7. The ratio of the circumference of a circle to its diameter is 3.14:1. How far will a wheel with diameter 10 cm roll in one complete revolution?

8. The Grow Green Grass Company recommends that their fertilizer be spread at a rate of 25 kg/500 m^2. How much fertilizer is required for a lawn measuring 15 m by 50 m.



9. The ratio of the number of teeth in gear *A* to the number of teeth in gear *B* is to be 5:3. If gear *B* has 51 teeth, how many teeth are needed on gear *A*?



10. In a set of three gears, the ratio of the number of teeth is given by the proportion

$$A:B:C = 4:5:2$$

If the largest gear has 45 teeth, determine the number of teeth required in each of the other gears.

7.3 INVESTIGATIONS IN VARIATION

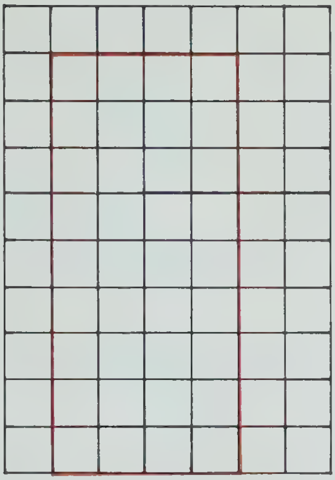
In many problems from science, industry, and business, the idea of variation or change plays an important role. The investigations suggested in the following pages illustrate some of the different forms of change which may take place. The equipment required is easily obtained. What is required from you is imagination, and a certain amount of care in measuring, timing, graphing, and calculating.

INVESTIGATION 7.1 *Rectangles I*

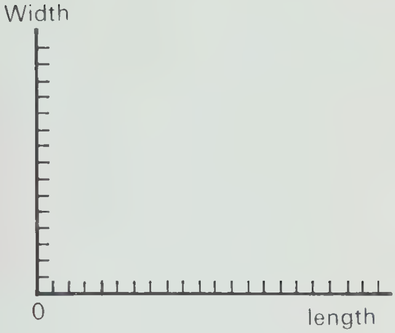
Objective: To graph the relationship between length and width of a rectangle with constant area.

Materials: Squared paper.

Method: On squared paper, outline as many rectangles as you can containing 36 square units. Record the dimensions of each rectangle. Graph your results.



Length	Width
1	
2	
3	
4	
6	
9	
12	
18	
36	



1. Do the points lie in a straight line?

2. Draw a smooth curve through the points.

3. (a) Use your graph to complete the table at right.
(b) Use your results to complete the area column in the table.
(c) What do you notice about the areas?

Length	Width	Area (length × width)
$1\frac{1}{2}$		
	5	
$8\frac{1}{2}$		
	16	

INVESTIGATION 7.2 *Ramps I*

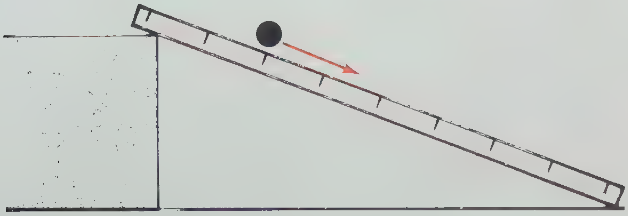
Objective: To graph a relationship between distance and time when slope is constant.

Materials: Stop watch,
something that rolls freely (marble, ball, toy car),
something to roll it on (board, drapery track), 1 to 3 m long.

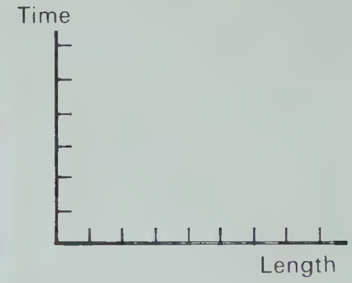
Method: Mark off the rolling surface in suitable equal lengths. Release the object at each of the different levels, and determine the time to reach the bottom. Record and graph the results.

Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ − ▨ ÷ ▨ = 3
▨ + ▨ ÷ ▨ = 3
▨ − ▨ − ▨ = 3



Length l	Time t	Ratio $\frac{l}{t}$



1. Is the graph a straight line?
2. What is kept constant in this experiment?
3. From your graph, how long will the object take to roll $3\frac{1}{2}$ lengths? Check this result experimentally.
4. For each case calculate the ratio $l:t$. Is this ratio a constant?
5. For each case, calculate the product $l \times t$. Is this product a constant?

Put the numbers from 1 to 9 in the spaces to make the statements true:

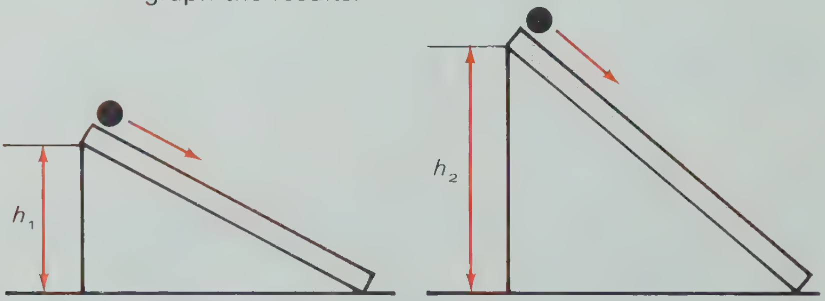
▨ + ▨ ÷ ▨ = 4
▨ × ▨ ÷ ▨ = 4
▨ − ▨ + ▨ = 4

INVESTIGATION 7.3 *Ramps II*

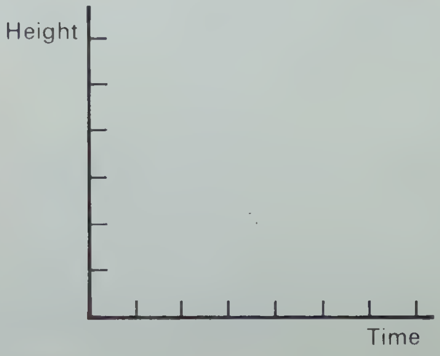
Objective: To graph a relationship between height and time when length is constant.

Materials: Stop watch,
something that rolls freely (marble, ball, toy car),
something to roll it on (board, drapery track), 1 to 3 m long.

Method: Determine the times taken by the object to roll the length of the board as the height of one end of the board is varied. Measure each height. Record and graph the results.



Height	Time

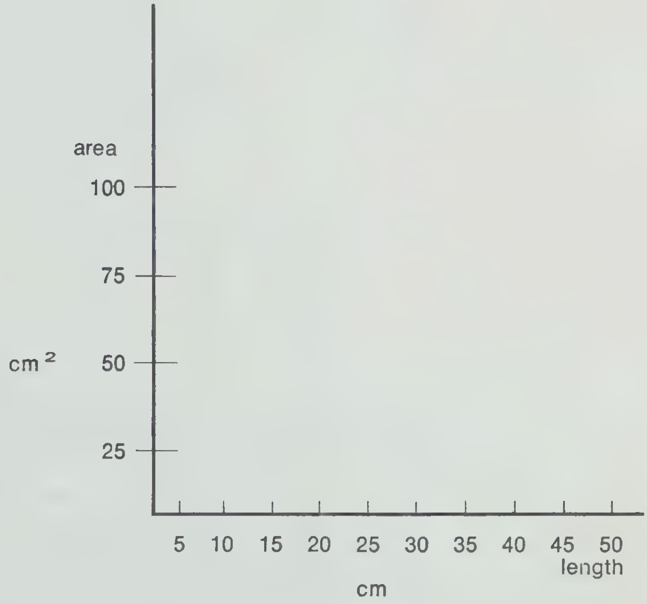


1. Is you graph a straight line?
2. What remains constant in this experiment?
3. From your graph, how long will the object take to roll down the incline if the height is (a) 10 cm (b) 25 cm?
4. Calculate the ratio $\frac{h}{t}$ for each case. Does this ratio remain constant?
5. Calculate the product $h \times t$. Does the product remain constant?

INVESTIGATION 7.4 *Rectangles II*

Objective: To graph the relationship between area and length of side of a rectangle with constant perimeter.
Materials: Loop of string 1 m long.
Method: Construct rectangles as suggested in the table using the loop of string. Complete the table and graph your results.

length	width	area
40 cm		
35		
30		
25		
20		
15		
10		



1. Do the points lie in a straight line?
2. Draw a smooth curve through the points.
3. State the dimensions that give the largest area of the rectangle with perimeter 1 m.

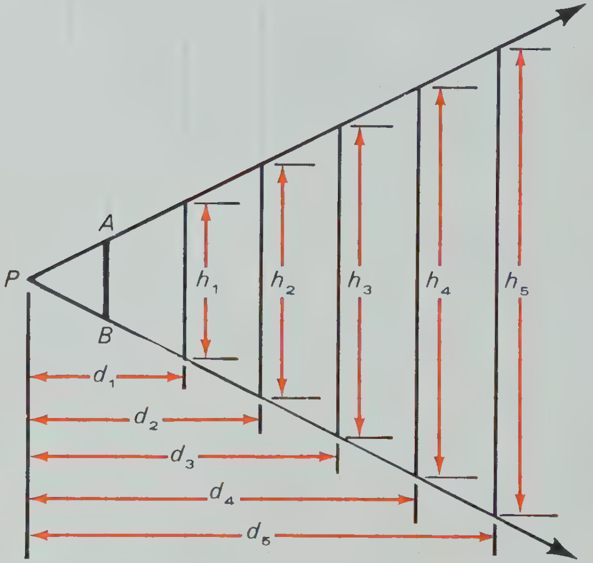
INVESTIGATION 7.5 *Projections I*

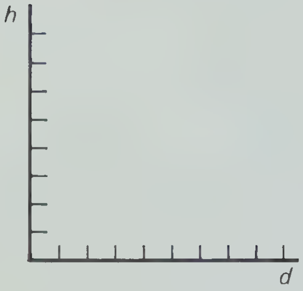
Objective: To investigate the relationship between the height of the shadow of an object and the position of the screen.

Materials: Squared paper,
ruler,
pencil.

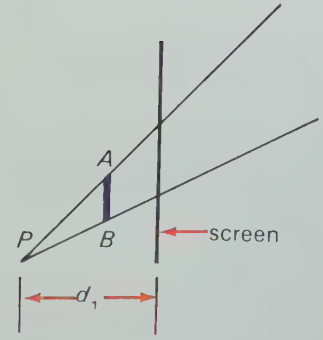
Method: On the grid shown, the point *P* represents a "light source" throwing the "shadow" of the object *AB* onto a "screen."

Distance *d*	Height *h*





1) Measure the distance of the first "screen" from *P*, and the height of the "shadow" h_1. Record the results in a table in your notebook.
2) As the "screen" moves to each new position, the height of the "shadow" changes. Measure the distance *d* and the length *h* for the five positions shown. Record your results in the table.
3) Plot the ordered pairs from the table on a graph such as the one at the right.
4) Draw a smooth curve through the points.
5) Is the graph a straight line?
6) From your graph, what is the height when the distance is 35 mm? 63 mm?
7) At what distance should the "screen" be placed to obtain a "shadow" with height 65 mm?
8) On squared paper, repeat the experiment with
   (a) a larger object,
   (b) the "light source" in another position.

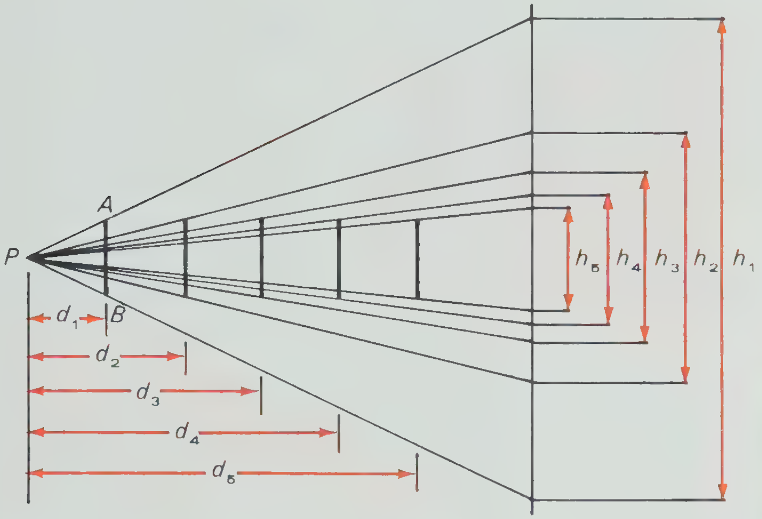


INVESTIGATION 7.6 *Projections II*

Objective: To investigate the relationship between the height of the shadow of an object and the position of the object.

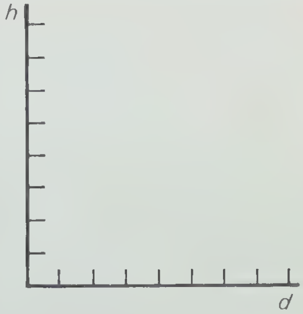
Materials: Squared paper,
ruler,
pencil.

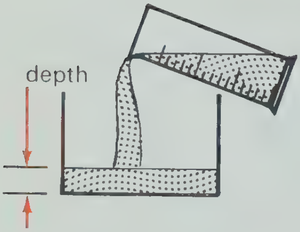
Method: On the grid shown, the point *P* represents a "light source" throwing the "shadow" of the object *AB* on the "screen".



Distance *d*	Height *h*

1) Measure the distance d_1 of the object *AB* from *P*. Measure the height h_1 of the "shadow" on the screen.
Record the results in a table in your notebook.
2) As the object moves further from the "light source," the height changes. Measure the distance *d* and the height *h* for the five positions shown. Record the results in the table.
3) Plot the ordered pairs from the table on a graph.
4) Draw a smooth curve through the points.
5) Is the graph a straight line?
6) From your graph, what is the height when the distance is 3 cm? 7 cm?
7) At what distance should the object be placed to obtain a "shadow" with height 10 cm?





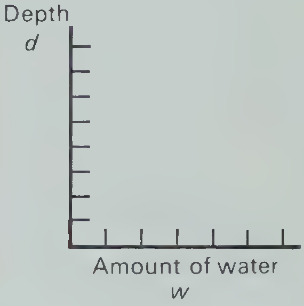
INVESTIGATION 7.7 *Water Levels I*

Objective: To graph the relationship between the amount of water and the depth of water in a given container.

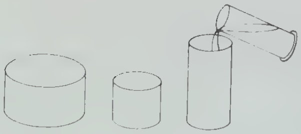
Materials: A tin can (fairly large),
a graduated cylinder or measuring cup.

Method: Measure the depth of water in the can, in centimetres, for various amounts of water added. Record and graph the results. Draw a smooth curve of best fit through the points.

Amount of water	Depth



1. Is the graph a straight line?
2. What remains constant in this experiment?
3. What amount of water will produce a depth of 4 cm? of 10.5 cm?



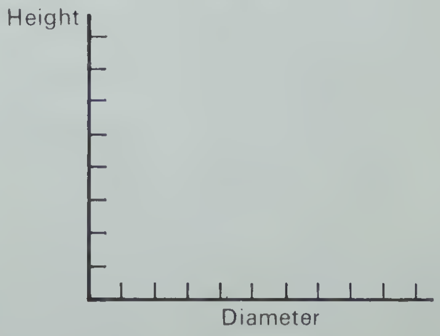
INVESTIGATION 7.8 *Water Levels II*

Objective: To graph the relationship between the depth of water and the diameter of the container for a given amount of water.

Materials: Several cans of different diameter,
a graduated cylinder or measuring cup.

Method: Measure the diameter of each can. Pour the same volume of water into each can. Measure the depth of the water in each can. Record and plot the results. Draw a smooth curve through the points.

Diameter	Depth



1. Is the graph a straight line?
2. What remains constant in this experiment?

7.4 DIRECT VARIATION

In many problems from science, industry, and business the idea of variation or change plays an important part. In this section, **direct variation** will be studied, along with its relationship to the ideas of ratio and proportion. The examples will introduce the language associated with variation.

EXAMPLE 1. *The table of values below shows the distance travelled by a car moving at a constant speed of 20* km/h.

time t (h)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$
distance d (km)	10	20	30	40	50	60	70

(a) Using these results as a set of ordered pairs (t,d), draw a graph showing the relationship between distance and time.
(b) Using the graph, determine the distance travelled in $\frac{3}{4}$ h, in $2\frac{1}{4}$ h.
(c) For three ordered pairs (t,d), determine the value of

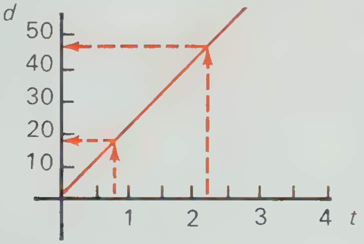
in simplest form. Use your results to determine the equation of the graph.

Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ × ▨ − ▨ = 5
▨ ÷ ▨ + ▨ = 5
▨ + ▨ − ▨ = 5

Solution

(a)



(b) From the graph,
When $t = \frac{3}{4}$, $d = 15$
When $t = 2\frac{1}{4}$, $d = 45$

(c) (i) $(\frac{1}{2},10)$

$$\frac{d}{t} = \frac{10}{\frac{1}{2}} = 10 \times \frac{2}{1} = 20$$

(ii) $(2,40)$

$$\therefore \frac{d}{t} = \frac{40}{2} = 20$$

(iii) $(3\frac{1}{2},70)$

$$\therefore \frac{d}{t} = \frac{70}{3\frac{1}{2}} = 70 \times \frac{2}{7} = 20$$

$\therefore \frac{d}{t} = 20$ or $d = 20t$ is the equation.

Since $\frac{d}{t} = 20$ for any ordered pairs (t_1,d_1) and (t_2, d_2),

$$\therefore \frac{d_1}{t_1} = \frac{d_2}{t_2}$$

If speed is constant, time and distance form an example of a **direct variation between two quantities**—as the time increases, the distance increases.

$$\left.\begin{array}{l} d \text{ varies directly as } t \\ d \text{ varies as } t \\ d \text{ is directly proportional to } t \\ d \text{ is proportional to } t \end{array}\right\} \begin{array}{l}\text{may be written} \\ d \propto t\end{array}$$

In general, if $y \propto x$, then $y = kx$, and $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

k is called the **variation constant.**
y is the **dependent variable** (depends on the value of x).
x is the **independent variable.**

Put the numbers from 1 to 9 in the spaces to make the statements true:
▨ − ▨ + ▨ = 6
▨ ÷ ▨ × ▨ = 6
▨ − ▨ − ▨ = 6

EXAMPLE 2. *If $a \propto b$, and $a = 10$ when $b = 15$*
(*a*) *find the variation constant, k*
(*b*) *determine a when $b = 24$*

Solution

(a) Since $a \propto b$,
$a = kb$
by substitution,
$10 = k(15)$
$k = \frac{10}{15}$
$= \frac{2}{3}$

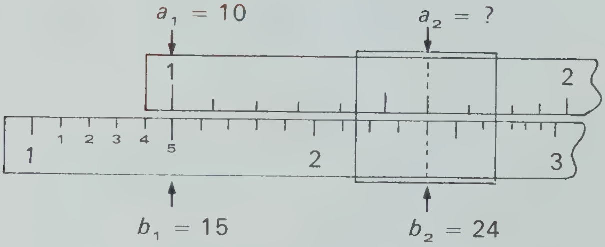
(b) Two solutions are shown
(i) Since $k = \frac{2}{3}$
$a = \frac{2}{3}b$
$= \frac{2}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{8}{\cancel{24}}}{1}$
$= 16$

(ii) Since $a \propto b$

$$\therefore \frac{a_1}{b_1} = \frac{a_2}{b_2}$$

by substitution,

$$\frac{10}{15} = \frac{a_2}{24}$$
$$\therefore 15a_2 = 240$$
$$a_2 = \frac{240}{15}$$
$$= 16$$



EXAMPLE 3. *The perimeter of a square varies as the square root of the area. If the perimeter is 24* cm *when the area is 36* cm², *determine the perimeter when the area is 81* cm².

Solution
Let p represent the perimeter and A represent the area.

$$p \propto \sqrt{A}$$
$$\therefore \quad p = k\sqrt{A}$$

When $p = 24$ and $A = 36$,

$$24 = k\sqrt{36}$$
$$\therefore \quad 24 = k(6)$$
$$\therefore \quad k = 4$$
$$\therefore \quad p = 4\sqrt{A}$$

When $A = 81$,

$$\begin{aligned} p &= 4\sqrt{81} \\ &= 4(9) \\ &= 36 \end{aligned}$$

∴ the perimeter is 36 cm.

EXERCISE 7-4

B **1.** In your notebook, complete the following table:

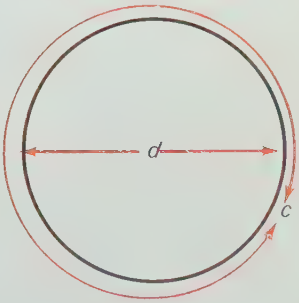
Statement	**Notation**	**Equation**	**Proportion**
a varies directly as b			
	$c \propto d$		
			$\frac{x_1}{y_1} = \frac{x_2}{y_2}$
V is proportional to T			
		$W = kd^2$	
F varies as M			
			$\frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}$

2. For tables (a) and (b) below
(i) Draw a graph illustrating the relationship between the quantities.
(ii) Using the method of Example 1, determine the equation of the graph.
(a) With constant acceleration, the velocity v of a car varies directly as the time t.

t (s)	1	$1\frac{1}{2}$	2	3	4	5
v (m/s)	15	$22\frac{1}{2}$	30	45	60	75

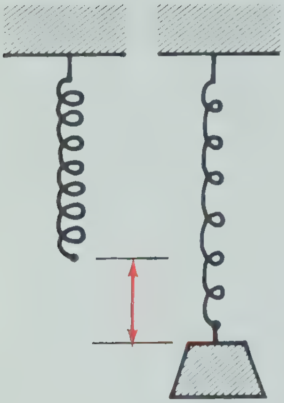
(b) The circumference c of a circle varies as the diameter d.

d (cm)	0	1.0	1.5	2.0	2.6	3.2	4.0	5.0
c (cm)	0	3.1	4.7	6.3	8.2	10.0	12.6	15.7



3. In your notebook, complete the following table as illustrated in part (a).

Statement	Equation	Given Values	Variation Constant k	Equation
(a) x varies directly as y	$x = ky$	$x = 3$ $y = 5$	$3 = k(5)$ $\therefore\ k = \frac{3}{5}$	$x = \frac{3}{5}y$
(b) $V \propto T$		$V = 550$ $T = 300$		
(c) d varies as t		$d = 10.5$ $t = 2.5$		
(d) a varies directly as b^3		$a = 24$ $b = 2$		

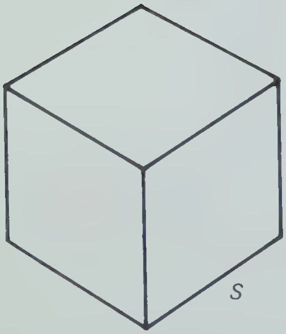


4. p varies directly as q. When $q = 21$, the value of p is 14.
(a) Find the variation constant k.
(b) Determine p when $q = 54$.

5. The area of a circle varies as the square of the diameter. If the area is 154 cm^2 when the diameter is 14 cm, find the area when the diameter is 28 cm.

6. The length that a loaded spring stretches varies directly as the mass of the load. If a mass of 8 kg stretches a spring 2.25 cm, calculate the mass required to stretch the same spring 5.5 cm.

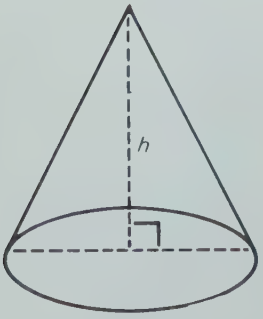
7. The annual interest on a bank loan varies directly as the amount borrowed (called the principal). When the principal is \$200, the interest is \$15. Calculate the interest when the principal is \$350.



8. The mass of a cube varies as the cube of the length S of one edge. When the length is 30 cm the mass is 18 kg.
(a) Calculate the mass when $S = 20$ cm.
(b) Calculate S when the mass is $42\frac{2}{3}$ kg.

9. If the area of the base remains constant, the volume of a cone varies as the height. If the volume is 132 cm^2 when the height is 14 cm, calculate the volume when the height is 21 cm.

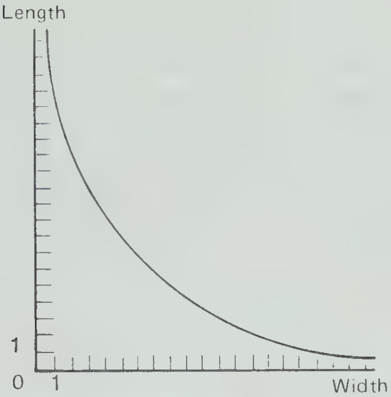
10. The distance travelled by a falling object varies directly as the square of the time.
(a) If an object falls 19.6 m in 2 s, how far will it fall in 3 s?
(b) How far will the object fall in the fourth second?



7.5 INVERSE VARIATION

The relationship between lengths and widths of a rectangle with an area of 36 square units is illustrated in a table and graph such as the one shown below.

Length	72	36	18	12	9	6	4	3	2	1
Width	$\frac{1}{2}$	1	2	3	4	6	9	12	18	36



Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ × ▨ + ▨ = 7
▨ ÷ ▨ + ▨ = 7
▨ + ▨ − ▨ = 7

These results indicate that when the area is kept constant,

(i) the length must increase as the width decreases,
(ii) the width must increase as the length decreases.

This relationship is an example of an **inverse variation,** and can be translated into an equation using the values in the table.

Numerical Values	Notation
$72 \times \frac{1}{2} =$ ▨	$l_1 \times w_1 = 36$
$36 \times 1 =$ ▨	$l_2 \times w_2 =$ ▨
$18 \times 2 =$ ▨	$l_3 \times w_3 =$ ▨
.	.
.	.
.	.

$\therefore$ the equation may be written

$$lw = 36$$

The statement "x varies inversely as y" may be written $x \propto \frac{1}{y}$, and implies

(1) $xy = k$ for some constant k.
(2) $x_1y_1 = x_2y_2$ for corresponding values of x and y.

EXAMPLE 1. *The time required to travel a certain distance varies inversely as the average speed. If a trip requires 8* h *to complete at 55* km/h, *how long will it take at a speed of 48* km/h?

Kilometres per hour = km/h

Solution
Let the time in hours be represented by t and the velocity in km/h be represented by v.

$$\therefore \quad t \propto \frac{1}{v} \qquad \therefore \quad v_1t_1 = v_2t_2$$

$$\therefore \quad 55 \times 8 = 48 \times t_2$$

$$\therefore \quad t_2 = \frac{55 \cdot \cancel{8}^{1}}{\cancel{48}_{6}}$$

$$= 9\tfrac{1}{6}$$

Therefore the trip will take 9 h and 10 min.

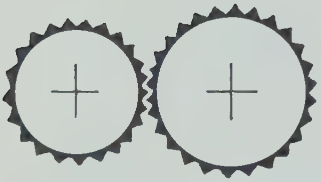
EXERCISE 7-5

B 1. If a varies inversely as b, determine the missing values in the table below.

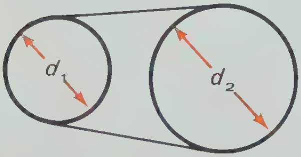
a_1	b_1	a_2	b_2
6	5		10
4		12	5
	2	3	5
5	3	2	
$3\frac{1}{2}$		$1\frac{1}{2}$	2
2.6	5.1		3
4.6	0.6	9.2	

2. The time required to complete a job varies inversely as the number of men used. If 8 men complete a job in 9 d, how long will 24 men take?

3. The volume of a gas varies inversely as the pressure when the temperature remains the same. If the volume of a gas is 60 m³ when the pressure is 5 Pa, what will be the volume if the pressure is increased to 12 Pa?

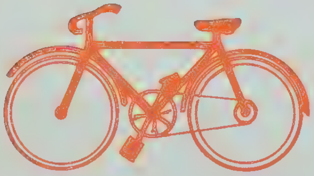


4. When two gears mesh, the speeds vary inversely as the number of teeth. If a gear with 48 teeth rotates at 150 r/min and drives a gear with 18 teeth, what will be the speed of the smaller gear?



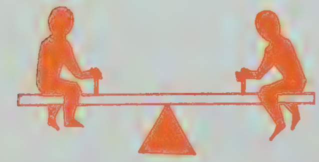
5. If x varies inversely as y^2, and $x = 7$ when $y = 3$, determine x when $y = 6$.

6. At constant tension, the frequency f, of the note produced by a guitar string varies inversely as the length, l. If a frequency of 512 vibrations per second occurs with a length of 75 cm, find the frequency when the length is 60 cm.



7. When two pulleys are connected by a belt, the speed, in revolutions per minute, varies inversely as the diameter. If the diameter of a pulley rotating at 120 r/min is 9 cm, what will be the diameter of a pulley rotating at 180 r/min?

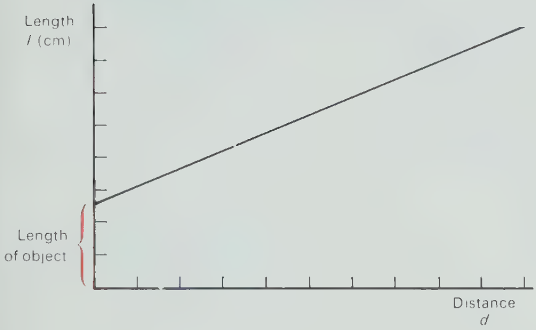
8. On a bicycle, the number of teeth on the pedal-driven sprocket is 40 and on the rear wheel sprocket is 18. If John is pedalling at 72 r/min, at what r/min is the rear wheel turning?

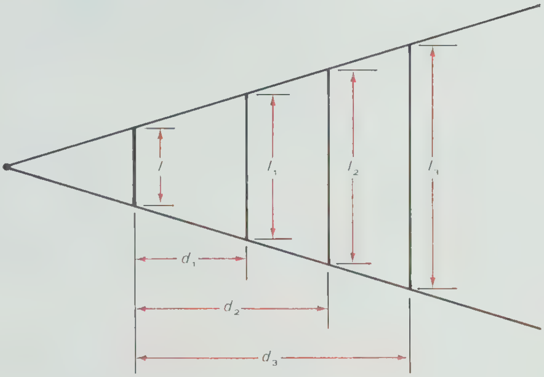


9. To balance on a seesaw a person has to sit at a distance from the support which varies inversely with his mass. If Mary has a mass of 44 kg and sits 1.5 m from the support, how far away should Dean, with a mass of 55 kg, sit?

7.6 PARTIAL VARIATION

From the results of experiments, similar to Investigations 7.5 and 7.6, it was found that the relationship between an object and its shadow depends on the distance of the shadow from the object. However, if the light source remains in the same place, the final size of the shadow depends also on the size of the original object (a constant value).





Distance	1.15	1.83	2.3	3.45
Length	2.25	3.0	3.5	4.25

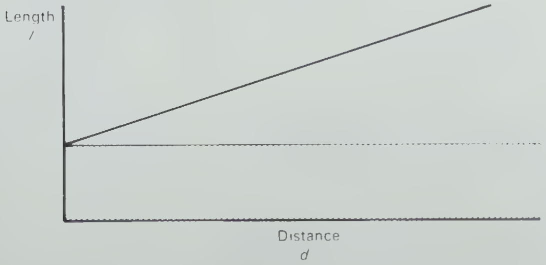
Some typical results for this investigation, shown above, illustrate that this is a form of variation. However, it does not fit into the pattern of either direct or inverse variation.

$\frac{l}{d}$	$l \times d$
$\frac{2.25}{1.15} = 1.95$	$2.25 \times 1.15 = 2.59$
$\frac{3.00}{1.83} = 1.64$	$3.0 \times 1.83 = 5.5$
$\frac{3.50}{2.30} =$ ▨	$3.5 \times 2.3 =$ ▨

Direct variation $\frac{y}{x} = k$

Inverse variation $xy = k$

If the constant length of the object is ignored, this becomes an example of direct variation—as the distance increases, the length increases.



Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ × ▨ + ▨ = 8
▨ ÷ ▨ + ▨ = 8
▨ − ▨ + ▨ = 8

A variation, such as the one in this example, is known as a **partial variation,** and involves two parts,

(i) a constant term
(ii) a direct or inverse variation

EXAMPLE 1. *The PQRS TV repair company charges a $4.50 service charge for house calls plus an hourly rate of $2.75 for labour.*

(a) Calculate the total charge for a house call if the repairman worked for 1 h *and 20* min.

(b) Express the total charge as a formula.

Solution

(a) Total charge = service charge + cost of labour

$$= 4.50 + (1\tfrac{1}{3} \times 2.75)$$
$$= 4.50 + 3.67$$
$$= 8.17$$

The total bill came to $8.17.

(b)

$$\underset{\text{total charge}}{T} = \underset{\text{service charge}}{4.50} + 2.75\underset{\text{number of hours}}{h}$$

EXERCISE 7-6

B **1.** (a) The Nu Car Rental agency charges a basic rate of $5 plus $0.09/km. Find the total cost for renting a car and driving 240 km.

(b) Express the total charge as a formula.

2. The Best Way Car rental agency has two plans for the rental of a car on a weekly basis.

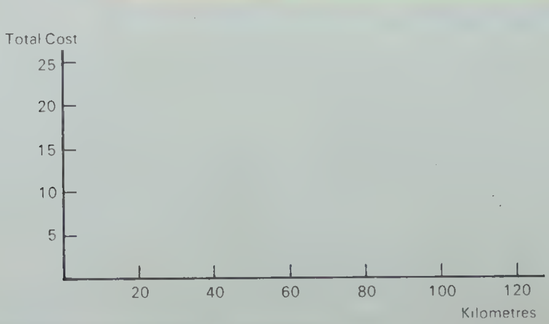
Plan A	$15 plus $0.05/km
Plan B	$5 plus $0.15/km

(a) Which is the best plan if you estimate you will be travelling 150 km? How much will you save?

(b) To decide the distance at which one plan becomes favourable, complete the tables below, and graph the results on the same axes.

Kilometres	20	30	50	60	90	120
Cost Plan A						

Kilometres	20	30	50	60	90	120
Cost Plan B						



Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ × ▨ + ▨ = 9
▨ × ▨ − ▨ = 9
▨ ÷ ▨ + ▨ = 9

From your graph determine the approximate kilometres travelled at which one plan becomes better than the other.

3. The relationship between number of guests and total cost is given by the equation

$$C = \frac{5}{2}n + 150$$

cost in dollars number of guests

(a) What is the cost for 100 guests? 200? 400?
(b) How many guests can be served for \$400? \$500? \$600?

4. The velocity of a car is given by the formula

$$v = at$$

velocity acceleration time

m = metres
s = seconds

2.5 m/s² means that in every second, the velocity increases 2.5 m/s.

(a) For an acceleration $a = 2.5\ \text{m/s}^2$ complete the table below in your notebook.

t	0	1	2	3	4	5
v						

(b) Using the table in (a) graph the relation $v = 2.5t$ in your notebook.
(c) What type of variation is illustrated by your graph?
(d) If the car is already moving when acceleration begins, the velocity is given by the formula

$$v = \underset{\substack{\uparrow \\ \text{initial velocity}}}{u} + at$$

If $u = 5.5\ \text{m/s}$, $a = 2.5\ \text{m/s}^2$, complete the table below and graph the results on the same axes as in part (b)

t	0	1	2	3	4	5
v						

(e) What type of variation is illustrated by the graph of

$$v = 5.5 + 2.5t?$$

Put the numbers from 1 to 9 in the spaces to make the statements true:

▨ + ▨ − ▨ = 10
▨ ÷ ▨ + ▨ = 10
▨ − ▨ + ▨ = 10

REVIEW EXERCISE

A

1. Express the following ratios in simplest form:

(a) 5:10	(b) 3:9	(c) 8:2	(d) 18:8
(e) 40:24	(f) 36:16	(g) 5:10:20	(h) 16:18:6
(i) 15:36:3	(j) 4:14:16	(k) 27:36:9	

2. From the list below, pick out 4 pairs of equivalent ratios:

7:28, 14:2, 9:6, 4:16, 8:10,
56:8, 27:18, 12:6, 7:1, 32:40

$a:b:c = x:y:z$ is equivalent to $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

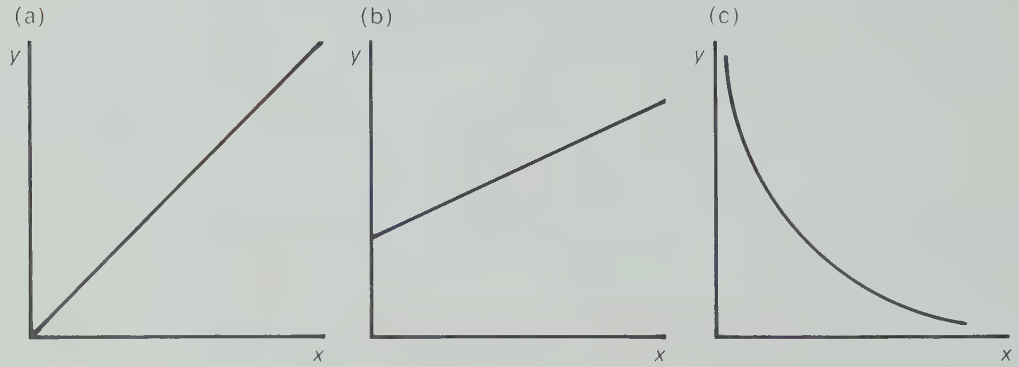
3. In the proportion $3:2 = 6:4$ what do we call the 3? the 2? the 6? the 4?

4. In the proportion $3:x = x:12$ what do we call the x?

5. In a mathematics class there are 18 girls and 24 boys. State the following ratios (in simplest form).

(a) Boys to girls, (b) Girls to boys,
(c) Girls to entire class, (d) Boys to entire class.

6. What type of variation is illustrated by each of the graphs below?



$a:b = c:d$ means $a \times d = b \times c$

7. Express the following ratios in simplest form.

(a) 9 cm to 1 m (b) 3 kg to 8 g
(c) 0.3 m to 36 cm (d) 1 h 15 min to 25 min
(e) 18 months to 2 years (f) 40 cents to 2 dollars
(g) 5.8 cm to 38 mm (h) 3.25 m to 12.5 cm

8. Calculate the value of the variable in each proportion.

(a) $x:12 = 3:4$ (b) $a:27 = 2:3$
(c) $1.5:x = 3:2$ (d) $5:6 = b:42$
(e) $3.5:6 = 10.5:y$ (f) $5:2 = a:1.2$
(g) $3.6:x = 1.2:1.5$ (h) $0.4:3 = 12:d$

$\frac{a}{b} = \frac{c}{d}$ means $ad = bc$

9. Calculate the value for each variable.

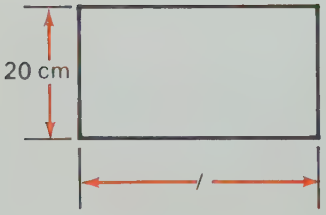
(a) $a:3:5 = 2:b:10$ (b) $x:y:2 = 3:15:4$
(c) $1.5:1:6 = y:3:z$ (d) $6:a:5 = b:3:1.25$

10. Determine the value of the variables in each proportion.

(a) $\frac{2}{y} = \frac{1}{12}$ (b) $\frac{3}{13} = \frac{15}{x}$ (c) $\frac{18}{96} = \frac{x}{64}$

(d) $\frac{3.5}{a} = \frac{2.1}{3}$ (e) $\frac{81}{5.4} = \frac{0.9}{b}$ (f) $\frac{x}{3} = \frac{y}{5} = \frac{10}{4}$

(g) $\frac{30}{a} = \frac{5}{6} = \frac{2}{b}$ (h) $\frac{w}{4} = \frac{l}{5} = \frac{12}{15}$ (i) $\frac{1.2}{x} = \frac{y}{3.0} = \frac{1.6}{2.4}$



11. The length and width of a picture frame are to be in the ratio 1.6:1. If the width is 20 cm what will be the length?

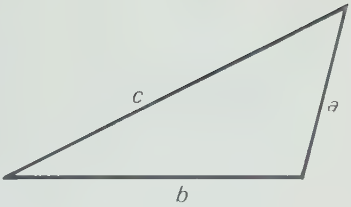
12. Lori saves \$24 in 9 weeks. At this rate, how many weeks will be required to save \$56?

13. A recipe requires 0.5 L of flour for 4 dozen cookies. How much flour will be required for 10 dozen cookies?

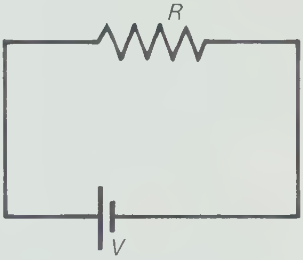
14. The ratio of the sides of the triangle at right is given by the proportion

$$a:b:c = 1:1.7:2$$

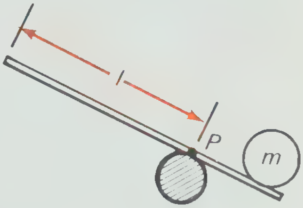
Calculate the lengths a and c to the nearest tenth of a centimetre when $b = 6.9$ cm.



15. The voltage, V, in an electrical circuit varies directly as the resistance, R, when the current is constant. If the voltage is 9 V when the resistance is 100 Ω, calculate the voltage when the resistance is 500 Ω.



16. The mass of rock which may be moved varies directly as the length of the lever extending from the point P at right. A 240 kg rock may be moved by a lever extending 3 m past P.
(a) How much mass may be moved if the lever extends 2 m?
(b) What length is necessary to move a 300 kg rock?

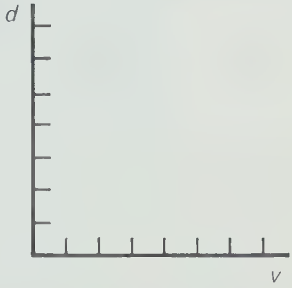


17. The stopping distance required for a car varies directly as the square of the velocity. For a car travelling 10 m/s the stopping distance is 15 m.
(a) Calculate the variation constant, k.
(b) Complete the table in your notebook.

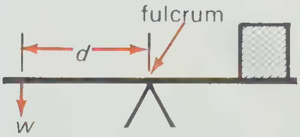
velocity (m/s)	0	5	10	15	20	25
distance (m)						

(c) Plot the results from the table on a graph such as the one at right. Draw a smooth curve through the points.
(d) Using your graph, complete the table below in your notebook.

velocity	7.5	14				
distance			18	35		55



18. If the propelling force is constant, the acceleration of a rocket varies inversely as the mass of the rocket. If the acceleration of a rocket is 7.7 m/s² when the mass is 52 t, what will the acceleration be if the mass is 28 t?



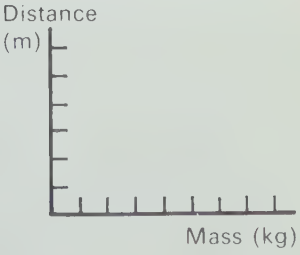
1 tonne (t) = 1000 kg

19. The mass needed to balance a lever varies inversely as the distance of the mass from the fulcrum. To balance a particular lever, a mass of 32 kg must be placed 3.5 m from the fulcrum.
(a) Calculate the variation constant.
(b) Complete the table below in your notebook.

Mass (kg)	4	8	12	16	24	32
Distance (m)						

(c) Plot the results from the table on a graph as illustrated at right. Draw a smooth curve through the points.
(d) From your graph, complete the table below in your notebook.

D	5	14.5		
M			4.5	7



REVIEW AND PREVIEW TO CHAPTER 8

$$\begin{aligned} 6\tfrac{3}{4}\% &= 6.75\% \\ &= 6.75 \times \tfrac{1}{100} \\ &= 0.0675 \end{aligned}$$

1. When working with a calculator, percentages are handled best as decimal fractions. Convert the following percentages to decimals.

(a) 7% (b) 5% (c) 45% (d) $6\frac{1}{2}\%$
(e) 12.5% (f) 100% (g) $7\frac{3}{4}\%$ (h) 75%
(i) 8.5% (j) $3\frac{1}{4}\%$ (k) $9\frac{1}{8}\%$ (l) $5\frac{1}{3}\%$
(m) $\frac{3}{4}\%$ (n) $2\frac{1}{8}\%$ (o) $12\frac{3}{4}\%$ (p) $\frac{1}{2}\%$
(q) $1\frac{1}{2}\%$ (r) 4.75% (s) $\frac{3}{8}\%$ (t) $10\frac{1}{4}\%$

$$\begin{aligned} 0.045 &= 0.045 \times 100\% \\ &= 4.5\% \\ &= 4\tfrac{1}{2}\% \end{aligned}$$

2. Convert the following fractions to percentages.

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) 0.06
(e) 0.075 (f) 0.14 (g) 0.055 (h) 0.55
(i) 0.0475 (j) 0.061 25 (k) 0.455 (l) 0.003 75
(m) 0.0125 (n) 0.0375 (o) 0.105 (p) 0.0875

$$\begin{aligned} &5\tfrac{1}{2}\% \text{ of } \$200 \\ &= 0.055 \times \$200 \\ &= \$11.00 \end{aligned}$$

3. Find the following amounts to the nearest cent.

(a) 6% of \$470 (b) 3% of \$1500 (c) 2% of \$750
(d) $4\frac{1}{2}\%$ of \$1200 (e) $6\frac{3}{4}\%$ of \$500 (f) $10\frac{3}{4}\%$ of \$1000
(g) 12% of \$74.50 (h) $4\frac{1}{2}\%$ of \$15.50 (i) 8.5% of \$400
(j) 5.25% × \$25 (k) 7.5% × \$142 (l) 110% × \$700
(m) $106\frac{1}{2}\%$ × \$250 (n) 103% × \$650 (o) 112.75% × \$1000

$$\begin{aligned} 3\tfrac{3}{4}\% \text{ of } x &= \$22.50 \\ 0.0375x &= \$22.50 \\ x &= \frac{\$22.50}{0.0375} \times \frac{10\ 000}{10\ 000} \\ &= \frac{\$225\ 000}{375} \\ &= \$600 \end{aligned}$$

4. Solve the following equations.

(a) 6% of x = \$2.50 (b) $3\frac{1}{2}\%$ of x = \$1.25
(c) 4% of x = \$12.25 (d) $2\frac{3}{4}\%$ of x = \$9.30
(e) 12% of x = \$275 (f) 15% of x = \$35.50
(g) 103% of x = \$412 (h) 108% of x = \$162
(i) $104\frac{1}{2}\%$ of x = \$1567.50 (j) $103\frac{3}{4}\%$ of x = \$539.50
(k) 112% of x = \$5880 (l) $108\frac{1}{2}\%$ of x = \$271.50

5. Find the total of the following amounts and calculate 7% tax to the nearest cent.

(a) \$6.49, \$12.84, \$7.35, \$47.92
(b) \$137.50, \$256.14, \$583.12, \$142.19
(c) \$2324.18, \$7215.93, \$1597.64, \$1482.95
(d) \$748.30, \$126.15, \$384.17, \$68.92
(e) \$83.92, \$27.53, \$38.15, \$75.06
(f) \$489.30, \$725.89, \$302.74, \$396.70

6. Find the average of the following amounts.

(a) \$475.50, \$84.20 (b) \$532.50, \$49.60
(c) \$5842.00, \$394.70 (d) \$84.96, \$7.50
(e) \$37.60, \$8.20 (f) \$1530.80, \$214.20
(g) \$158.32, \$19.24 (h) \$759.82, \$137.40
(i) \$158.24, \$37.50

Personal Finance

8.1 MANAGING SMALL FUNDS

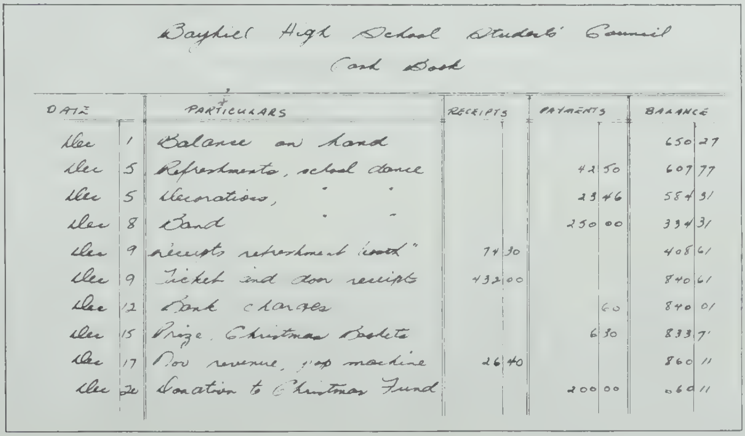
Most organizations and clubs become involved with money, either incidentally to meet minor expenses, or as a primary function such as in charity drives. One of the officals the club will require is a treasurer. The duties of the treasurer should be set out in the constitution of the club and will follow the outline below.

Duties of a treasurer

A Keep a record of all financial dealings of the club in a cash book;
B Keep the club funds in a separate bank account;
C Obtain a receipt or cancelled cheque for all expenditures of club funds and give a receipt for all dues or money received;
D Present a treasurer's report at regular meetings;
E Submit the financial records to be audited as required by the constitution.

Let us suppose that you have been elected treasurer of the Bayhill High School Students' Council effective January 1.

A The first thing you should do is look over the cash book to familiarize yourself with the system being used. The form shown here is suitable where there is no need to keep cash on hand and the transactions are relatively few.



Bayhill High School Students' Council
Cash Book

DATE		PARTICULARS	RECEIPTS		PAYMENTS		BALANCE	
Dec	1	Balance on hand					650	27
Dec	5	Refreshments, school dance			42	50	607	77
Dec	5	Decorations, ″ ″			23	46	584	31
Dec	8	Band ″ ″			250	00	334	31
Dec	9	Receipts refreshment booth″	74	30			408	61
Dec	9	Ticket and door receipts	432	00			840	61
Dec	12	Bank charges				60	840	01
Dec	15	Prize, Christmas Baskets			6	30	833	71
Dec	17	Nov revenue, pop machine	26	40			860	11
Dec	20	Donation to Christmas Fund			200	00	660	11

EXERCISE 8-1

Answer the following questions in your notebook:

1. What is the first entry on the monthly sheet?

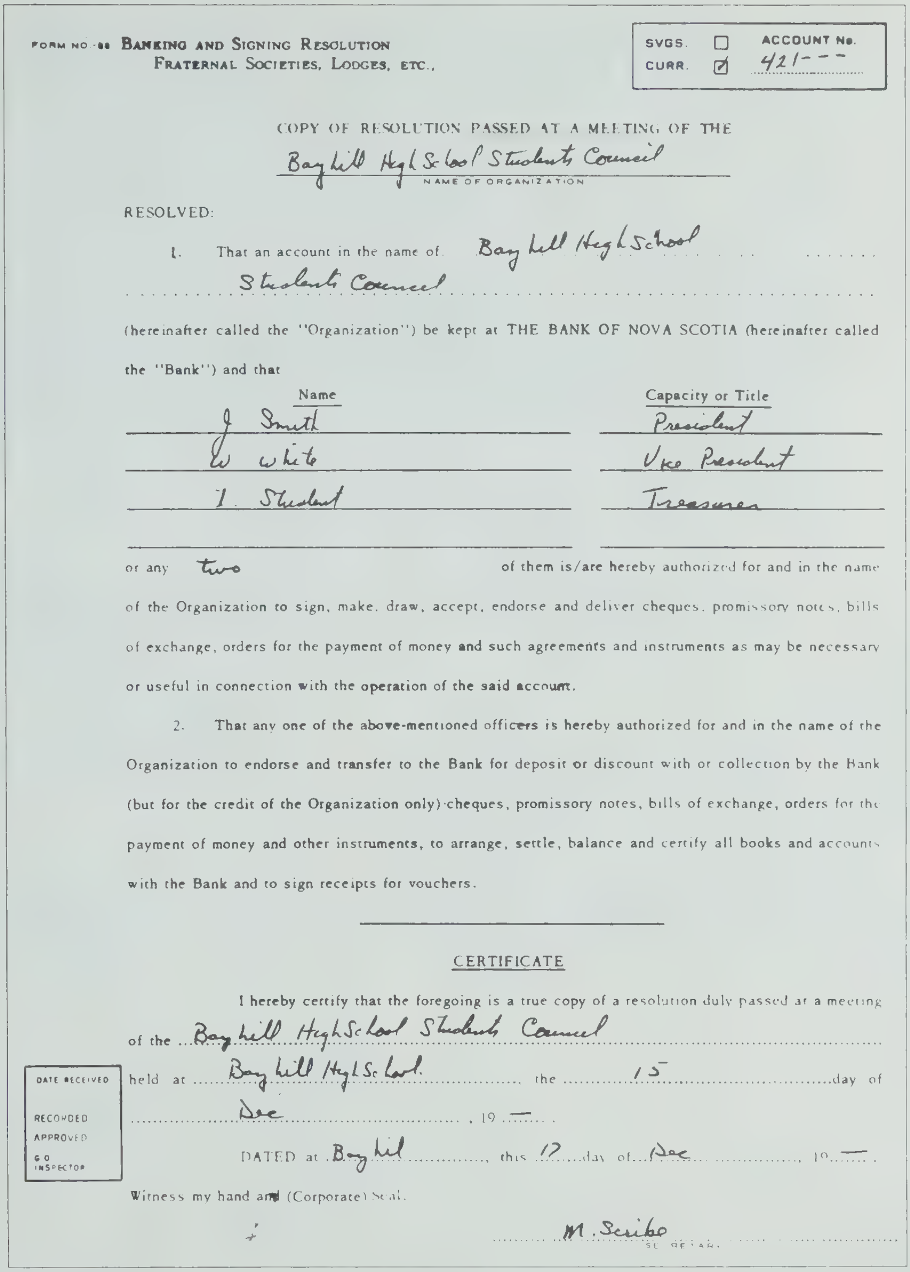
2. What are two differences between items for receipts and items for payments?

3. In your first month as treasurer the following transactions take place. Make up the January sheet for the cash book bringing the balance forward from December.

ITEM

(i)	Jan. 4	paid $10.25 to Acme Printers for tickets to the school play.
(ii)	Jan. 6	paid $25 to Tripp Bus Lines for a spectator bus.
(iii)	Jan. 7	received $16.50 in bus fares for the spectator bus.
(iv)	Jan. 10	received $5 Downtown Restaurant $\frac{1}{4}$ page advertisement on play program.
(v)	Jan. 12	paid $27.50 to Theatrical Supplies Ltd., makeup.
(vi)	Jan. 15	received $5 Showplace Theatre, $\frac{1}{4}$ page advertising on play program.
(vii)	Jan. 17	received $10 Athletic Sports Equipment Ltd., $\frac{1}{2}$ page advertising on play program.
(viii)	Jan. 20	paid $7.25 McGraw-Hill Ryerson, Christmas Book Prize.
(ix)	Jan. 22	paid $30 Tripp Bus Lines, Basketball Team exhibition game.
(x)	Jan. 24	received $2.30 December revenues from cold drink machine.
(xi)	Jan. 28	paid $15.30 Stationers Ltd., cardstock for play program.
(xii)	Jan. 28	December bank charges, 40¢.

B It will be necessary to inform the bank of the change in officers of the organization and have the signing authority on the bank account changed. This will be done on a Banking and Signing Resolution Form, and your signature will be kept on file at the bank on a signature card. Note that the Resolution Form allows the club to specify how many members of the executive committee will be required to sign cheques and withdrawal slips.



FORM NO. 88 **BANKING AND SIGNING RESOLUTION**
FRATERNAL SOCIETIES, LODGES, ETC.,

SVGS. ☐ CURR. ☑ ACCOUNT No. 421---

COPY OF RESOLUTION PASSED AT A MEETING OF THE

Bay hill High School Students Council
NAME OF ORGANIZATION

RESOLVED:

1. That an account in the name of Bay hill High School Students Council (hereinafter called the "Organization") be kept at THE BANK OF NOVA SCOTIA (hereinafter called the "Bank") and that

Name	Capacity or Title
J Smith	President
W white	Vice President
T. Student	Treasurer

or any two of them is/are hereby authorized for and in the name of the Organization to sign, make, draw, accept, endorse and deliver cheques, promissory notes, bills of exchange, orders for the payment of money and such agreements and instruments as may be necessary or useful in connection with the operation of the said account.

2. That any one of the above-mentioned officers is hereby authorized for and in the name of the Organization to endorse and transfer to the Bank for deposit or discount with or collection by the Bank (but for the credit of the Organization only) cheques, promissory notes, bills of exchange, orders for the payment of money and other instruments, to arrange, settle, balance and certify all books and accounts with the Bank and to sign receipts for vouchers.

CERTIFICATE

I hereby certify that the foregoing is a true copy of a resolution duly passed at a meeting of the Bay hill High School Students Council held at Bay hill High School, the 15 day of Dec, 19—.

DATED at Bay hil, this 17 day of Dec, 19—.

Witness my hand and (Corporate) Seal.

M. Scribe
SECRETARY

DATE RECEIVED
RECORDED
APPROVED
G O INSPECTOR

The most satisfactory kind of account for this type of organization is a current account. The purpose of the account is to hold the organization's funds so that bills may be paid by cheque, with a record of all transactions being supplied on a monthly basis. No interest is paid on the funds in the account and there will be a charge for each deposit or cheque written on the account. These charges vary from time to time but for this example we will assume them to be 20¢ per item with one free item for each $50 minimum monthly balance.

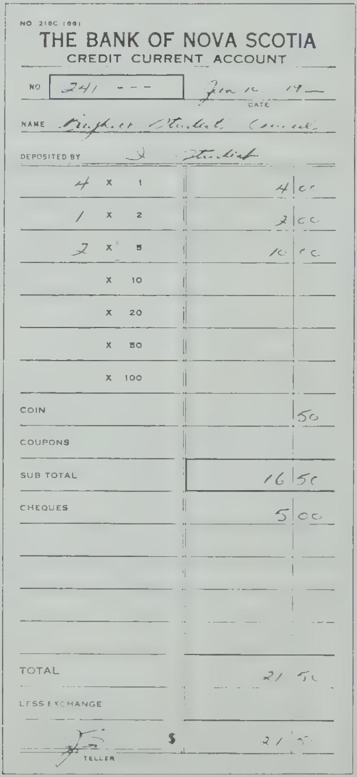
EXAMPLE 1. *From the December entries it is noted that there were five payments and three receipts of funds with a minimum monthly balance of $334.31. Assuming that all transactions were handled by the bank and no cash was kept on hand, what were the bank charges for December?*

Solution	Total number of items (5 + 3)	8
	Number of free items (334 ÷ 50)	6
	Charged items (8 − 6)	2
	Charges (2 @ 20 ¢)	40¢

From the bank you will receive a deposit book, a cheque book and a monthly statement.

Making a deposit

The deposit slips come in a pad with alternate blank pages and a sheet of carbon paper. The deposit slip should be filled out before going to the bank, left in the pad and given to the teller with the deposit. The teller will check the deposit against what you have written on the slip; if it is correct he will remove the deposit slip and stamp and initial your copy, then return the pad to you. This is your receipt for the deposit.



NO 218C (66)

THE BANK OF NOVA SCOTIA

CREDIT CURRENT ACCOUNT

NO 241 --- Jan 10 19—
DATE

NAME Brighton Student Council

DEPOSITED BY J. Student

4 x 1	4	00
1 x 2	2	00
2 x 5	10	00
x 10		
x 20		
x 50		
x 100		
COIN		50
COUPONS		
SUB TOTAL	16	50
CHEQUES	5	00
TOTAL	21	50
LESS EXCHANGE		
TELLER $	21	50

On January 10, you make a deposit of the $16.50 received for bus fares (item 3) and the $5 advertising revenue (item 4). The $16.50 was cash collected from the passengers and the $5 was a cheque.

Page 168 shows the deposit slip. Note each entry on the slip, including the breakdown of the cash deposit into the number of bills in each denomination. Before the cheque can be deposited, it must be endorsed. To safeguard against loss or theft, the cheque should be given the restrictive endorsement shown in the margin.

For deposit only to the account of Bayhill Students Council

J. Student

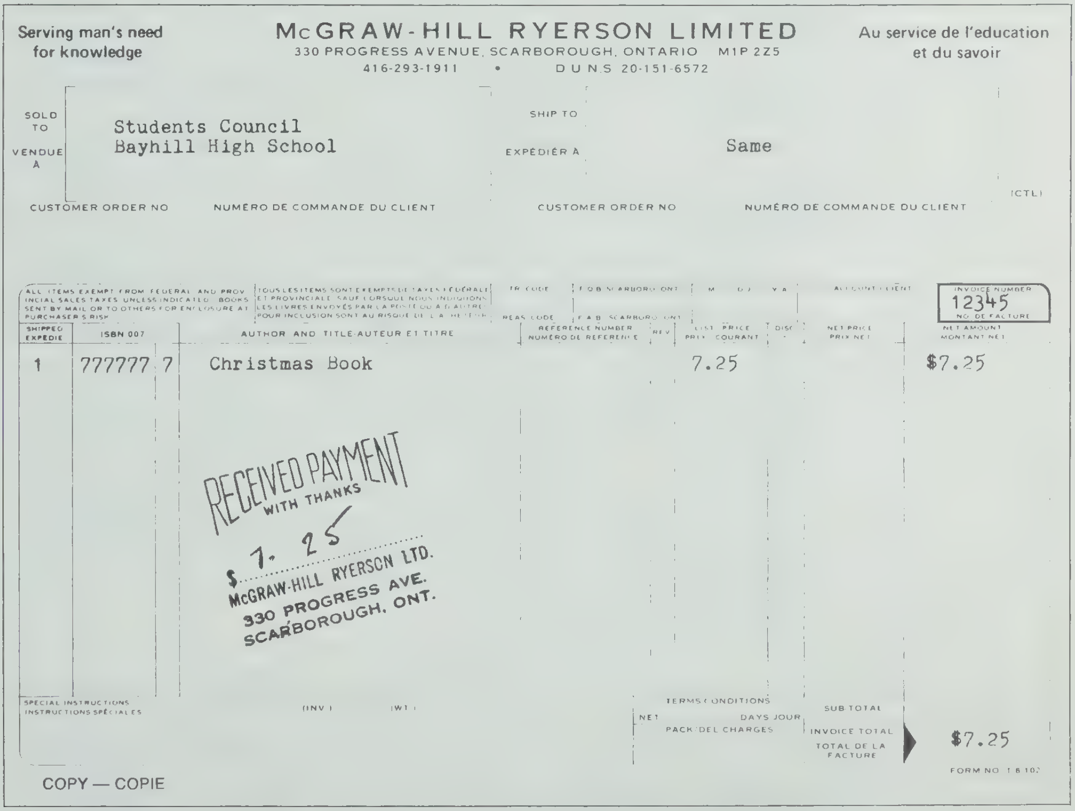
EXERCISE 8-1 *(continued)*

4. In your notes make up a deposit slip for January 24 for a deposit consisting of a $5 cheque (item 6), a $10 cheque (item 7) and $21.30 cash made up of one $10 bill, two $5 bills, one $1 bill and 30c change.

5. Show the endorsement used on the cheques.

6. Calculate the bank charges for January on 12 items with a minimum balance of $560.00.

C For all expenditures you must obtain a receipt or cancelled cheque. A receipted copy of an invoice is acceptable for your records.



Serving man's need for knowledge

McGRAW-HILL RYERSON LIMITED
330 PROGRESS AVENUE, SCARBOROUGH, ONTARIO M1P 2Z5
416-293-1911 • D U N S 20-151-6572

Au service de l'education et du savoir

SOLD TO / VENDUE A
Students Council
Bayhill High School

SHIP TO / EXPÉDIÉR A
Same

(CTL)

CUSTOMER ORDER NO NUMÉRO DE COMMANDE DU CLIENT

CUSTOMER ORDER NO NUMÉRO DE COMMANDE DU CLIENT

ALL ITEMS EXEMPT FROM FEDERAL AND PROVINCIAL SALES TAXES UNLESS INDICATED. BOOKS SENT BY MAIL OR TO OTHERS FOR ENCLOSURE AT PURCHASER'S RISK

TOUS LES ITEMS SONT EXEMPTS DE TAXES FÉDÉRALE ET PROVINCIALE SAUF LORSQUE NOUS INDIQUONS. LES LIVRES ENVOYÉS PAR LA POSTE OU A D'AUTRES POUR INCLUSION SONT AU RISQUE DE L'ACHETEUR

TR CODE | F.O.B. SCARBORO, ONT | M D/J Y A | ACCOUNT CLIENT

REAS CODE | F.A.B. SCARBORO, ONT

INVOICE NUMBER 12345 NO. DE FACTURE

SHIPPED EXPÉDIÉ	ISBN 007	AUTHOR AND TITLE-AUTEUR ET TITRE	REFERENCE NUMBER NUMÉRO DE REFERENCE	REV	LIST PRICE PRIX COURANT	DISC	NET PRICE PRIX NET	NET AMOUNT MONTANT NET
1	777777 7	Christmas Book			7.25			$7.25

RECEIVED PAYMENT WITH THANKS
$ 7.25
McGRAW-HILL RYERSON LTD.
330 PROGRESS AVE.
SCARBOROUGH, ONT.

SPECIAL INSTRUCTIONS INSTRUCTIONS SPÉCIALES (INV) (WT)

TERMS CONDITIONS NET DAYS JOUR PACK/DEL CHARGES

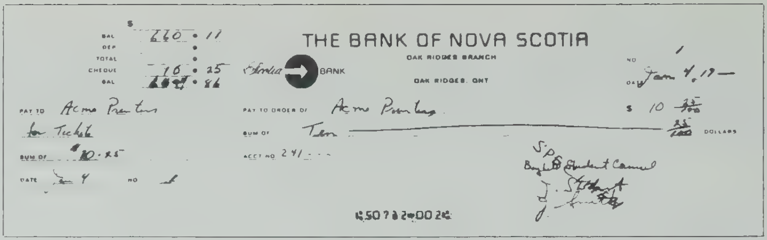
SUB TOTAL

INVOICE TOTAL TOTAL DE LA FACTURE $7.25

FORM NO. 18 102

COPY — COPIE

The bank supplied cheques with stubs for your use. The stubs give room to record the transaction and act as your copy until the cancelled cheque is returned to you from the bank. The stub also gives a current balance for the account.



BAL $ 660 . 11
DEP
TOTAL
CHEQUE 10 . 25
BAL 649 . 86

PAY TO Acme Printers
for Tickets
SUM OF $10.25
DATE Jan 4 NO 1

THE BANK OF NOVA SCOTIA
OAK RIDGES BRANCH
OAK RIDGES, ONT

NO 1
DATE Jan 4, 19—

PAY TO ORDER OF Acme Printers $ 10 25/100
SUM OF Ten 25/100 DOLLARS
ACCT NO 241-

S.P.S. Student Council
J. Stewart
J. Smith

50782-002

EXAMPLE 2. *Acme Printers Ltd. is paid on January 4, $10.25 by cheque (item 1). The balance before the payment is $660.11. Write out the cheque and stubs.*

Solution The stub is made out first. It contains a space for any deposit that has been made so that the balance in the account can be brought up to date. It should contain sufficient information for you to account fully for the expenditure. It also ensures that there is enough money in the account to cover the cheque. Note carefully each of the entries on the stub and cheque, particularly the form of the signature showing the organization name, and write up the following:

EXERCISE 8-1 *(continued)*

7. Make out a cheque to Tripp Bus Lines to pay for the spectators' bus (item 2).

8. Make out a cheque to Theatrical Supply Ltd. for $27.50 (item 5). Recall that a deposit of $21.50 was made on January 10 and should have been entered on the stub.

D At the end of each month the books should be balanced and a treasurer's report should be prepared for the next meeting. In balancing the books, it is necessary to show that the sum of the cash on hand plus the money in the bank equals the balance shown in the cash book. It is also the responsibility of the treasurer to see that all outstanding bills have been paid.

To prepare you for January, we shall examine what happened in December.

First, look at the December bank statement. Does it really give us the balance in the bank at the end of December? On comparing the entries on the statement with the cheque stubs the treasurer found that the cheque for $6.30 had not been processed:

Debits		Credits	Date M D	Balance
			11–23	690.36
40.09			11–25	650.27
42.50	23.46		12–09	584.31
	0.60 SC		12–10	583.71
250.00			12–11	333.71
		506.30	12–12	840.01
		26.40	12–18	866.41
200.00			12–21	666.41

In such a case the treasurer should make up a Bank Reconciliation Statement to show the true bank balance.

Bayhill Students' Council
Bank Reconciliation Statement
for December 31, 19__

Balance as shown on statement	$666.41
Less outstanding cheque #425	6.30
True Balance	$660.11

Finally a treasurer's report was drawn up showing all receipts and payments. The sum of the opening balance plus receipts must equal the payments plus closing balance. Notice the double line setting off these two sums.

Bayhill Students' Council
Treasurer's Report
December 31, 19__

Balance, Dec. 1		$650.27
Receipts:		
Refreshment booth	$74.30	
Dance Admissions	432.00	
Pop machine	26.40	
		532.70
		$1 182.97
Payments:		
Refreshments	42.50	
Decorations	23.46	
Band	250.00	
Bank charges	0.60	
Christmas Basket Prize	6.30	
Donation, Christmas Fund	200.00	522.86
Cash on hand	0.00	
Bank Balance	660.11	660.11
		$1 182.97

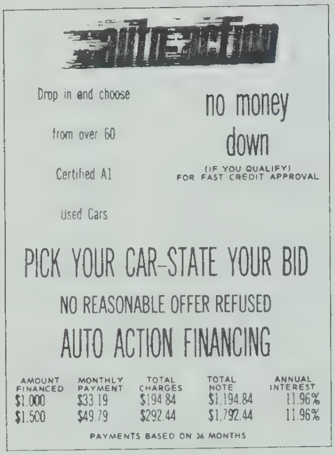
Signature ______________
Treasurer

EXERCISE 8-1 *(continued)*

9. When you receive the January bank statement, the cheque for $15.30 (item 11) has not been processed. The balance indicated on the statement is $617.51.

(a) Draw up a bank reconciliation statement.
(b) Draw up a treasurer's report for January.

E The job of the auditor is to verify the treasurer's work. He will require bank statements, cheques and receipts, and copies of your reconciliation statements and reports. In order to make the job as easy as possible all records should be kept neat and legible and each month's cheques and receipts should be bundled, dated and stored.



auto action

Drop in and choose

from over 60

Certified A1

Used Cars

no money down

(IF YOU QUALIFY) FOR FAST CREDIT APPROVAL

PICK YOUR CAR–STATE YOUR BID

NO REASONABLE OFFER REFUSED

AUTO ACTION FINANCING

AMOUNT FINANCED	MONTHLY PAYMENT	TOTAL CHARGES	TOTAL NOTE	ANNUAL INTEREST
\$1,000	\$33.19	\$194.84	\$1,194.84	11.96%
\$1,500	\$49.79	\$292.44	\$1,792.44	11.96%

PAYMENTS BASED ON 36 MONTHS

8.2 FINANCING A MAJOR PURCHASE

Sooner or later the time will come when you wish to make a major purchase, and the decision will have to be made to buy now and pay later or to save now and buy later. There are several options to "Qualified" purchasers, the qualifications being usually age, credit rating, and a record of steady employment. These options will be:

A Pay cash.
B Pay a down payment with the balance in time payments.
C No down payment, the total price in time payments.

Many businesses operate on a strictly cash basis. In this way they avoid expensive record keeping, billing operations and losses through bad debts. People wishing to buy from such a store on a credit basis must use third party financing. They may borrow the money from a bank, finance company, or credit union.

EXERCISE 8-2

1. (a) List 10 articles which you would buy for cash.
(b) List 10 articles which you would probably use credit to purchase.
(c) List the characteristics of a purchase which put it in the credit category.

(a) amount of purchase
(b) non-consumable goods
(c) . . . .

2. For one week keep a list of advertisements you see on television, in the newspapers or magazines advertising credit. Divide them into two groups—those advertising direct financing, where the seller offers the credit, and those advertising third party financing such as banks and finance companies.

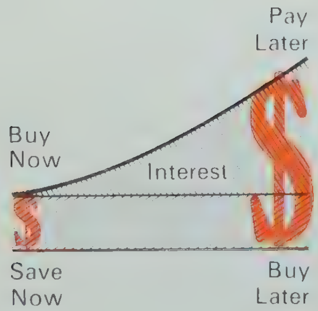
8.3 SIMPLE INTEREST

Interest is the rent paid for the use of someone else's money. The rate at which this interest is paid is expressed as a percentage of the amount of money loaned per unit of time. This amount of money loaned is called the principal.

INTEREST = PRINCIPAL × RATE × TIME

The total sum of money repaid, principal plus interest, is called the amount.

AMOUNT = PRINCIPAL + INTEREST



Where is your "now" on the payment scale?

EXAMPLE 1. *The Go Gas Co. charges 1.5% per month interest on overdue accounts. Ms. Lafleur allows a bill of $94 to go one month overdue. How much interest will be charged? What amount must be repaid?*

Solution

$P =$ \$94, $r = 1.5\%$ per month, $t = 1$ month.

$$\begin{aligned} I &= Prt \\ &= 94 \times 1.5\% \times 1 \\ &= 94 \times 0.015 \times 1 \\ &= 1.41 \end{aligned} \qquad \begin{aligned} A &= P + I \\ &= 94 + 1.41 \\ &= 95.41 \end{aligned}$$

Ms. Lafleur is charged \$1.41 interest and repays \$95.41.

Interest = Service charge
A = Total amount repaid.
I = Rent paid for the use of someone else's money.
P = The amount of money borrowed.
r = The rate at which the interest is charged expressed as a percentage of the principal per time unit.
t = Number of time units.

EXAMPLE 2. *\$6.75 interest is paid on a loan of \$450 borrowed at 6%/a. Find the time period of the loan.*

Solution

$I =$ \$6.75, $P =$ \$450, $r = 6\%/\text{a}$

$$\begin{aligned} I &= Prt \\ \frac{I}{Pr} &= \frac{Prt}{Pr} \\ t &= \frac{I}{Pr} \end{aligned} \qquad \begin{aligned} t &= \frac{6.75}{450 \times 6\%} \\ &= \frac{6.25}{450 \times 0.06} \\ &= 0.25 \end{aligned}$$

The money was borrowed for $\frac{1}{4}$ a. What tells us that the unit time is a year?

$I = Prt$

$t = \frac{I}{Pr}$

$P =$ ▨

$r =$ ▨

Per Annum

EXAMPLE 3. *\$1200 is borrowed for three months. The interest charged is \$63.*
(a) What monthly interest rate is charged?
(b) What is the annual interest rate charged?

Solution

(a) $I =$ \$63, $P =$ \$1200, $t = 3$ months

$$\begin{aligned} I &= Prt \\ \frac{I}{Pt} &= \frac{Prt}{Pt} \\ r &= \frac{I}{Pt} \end{aligned} \qquad \begin{aligned} r &= \frac{63}{1200 \times 3} \\ &= 0.0175 \\ &= 1.75\% \end{aligned}$$

$r = \frac{I}{Pt}$

The interest rate is $1\frac{3}{4}\%/\text{month}$. What tells us the time unit is months?

(b) $(1\frac{3}{4}\%/\text{month}) \times (12\ \text{months}/\text{a}) = 21\%/\text{a}$.

EXAMPLE 4. *\$21.90 interest is charged on a loan taken for six months at 12%/a. What principal was borrowed?*

Solution

$I =$ \$21.90, $r = 12\%/\text{a}$, $t = 6$ months $= \frac{1}{2}$ a

$P = \frac{I}{rt}$

$$I = Prt$$
$$\frac{I}{rt} = \frac{Prt}{rt}$$
$$P = \frac{I}{rt}$$

$$P = \frac{21.90}{12\% \times \frac{1}{2}}$$
$$= \frac{21.90}{0.12 \times 0.5}$$
$$= 365$$

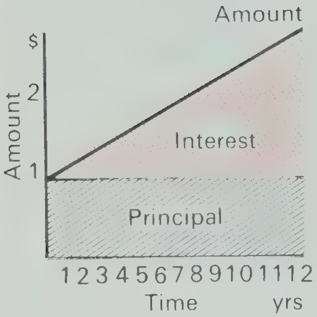
The principal is $365.

Why was the time changed from 6 months to $\frac{1}{2}$ a?

EXERCISE 8-3

B 1. Complete the following table in your notebook.

	Principal	Rate	Time	Interest
(a)	$500	10%/a	6 months	
(b)	$6050	$8\frac{1}{2}$%/a	$\frac{3}{4}$ a	
(c)	$245		$1\frac{1}{2}$ a	$36.75
(d)	$750		9 months	$67.40
(e)		$1\frac{1}{4}$%/ month	10 months	$24.52
(f)		6%/a	$1\frac{3}{4}$ a	$10.50
(g)	$825	1%/ month		$57.75
(h)	$1500	$7\frac{1}{2}$%/a		$22.50



The amount repaid after *n* years when $1 is borrowed at 12%/a simple interest

2. A boat and motor cost $2500. If Mr. Wilson pays $1700 cash and the balance at the end of 4 months with a service charge of 2%/month simple interest, what amount must he pay at the end of the period? What is the total cost of the boat?

3. Lo-Price Department Store advertises a service charge of 1½%/month. If your account has an unpaid balance of $273.50, what will the service charge be at the end of the month?

4. Ms. Sue Lee has a $15 000 mortgage on her house on which she pays interest at ¾%/month, a $2000 automobile debt on which she pays 1%/month and a $300 colour TV debt which is costing her 1½% per month. How much interest will she pay this month?

5. A debt of $2400 is paid off at the rate of $200/month plus 1% interest on the unpaid balance. Make up a table of payments under the headings:

Month	Unpaid Balance	Interest	Payment on Principal	Total Payment
1	$2400	$24.00	$200	$224.00
2	2200	/////	200	//////

Calculate the total amount repaid.

8.4 THE EFFECTIVE INTEREST RATE

Arrange four 9's to make 100.

Most articles bought on a time payment plan are paid for by a number of equal payments. The law now requires that the true rate of interest be stated in the advertisement and agreement; however, it is still worthwhile to be able to check this figure for yourself.

EXAMPLE 1. *A motorcycle is priced at \$695 cash or 24 payments of \$34.50. Find the true rate of interest.*

Solution The amount of money borrowed is \$695. Each payment pays the interest for the previous month and part of the principal. The principal is decreasing through the term of the loan until at the time of the last payment it is approximately \$34.50. In calculating an approximation to the true interest rate we shall use the **average principal.**

What would happen if the payment was less than the month's interest?

$$\frac{\$695.00 + \$34.50}{2} = \$364.75$$

Average Principal

$$= \frac{\text{Principal} + \text{Last Payment}}{2}$$

The **interest** charged is the difference between the cash price and the total amount repaid.

$$\begin{aligned} I &= (24 \times 34.50) - 695 \\ &= 828 - 695 \\ &= 133 \end{aligned}$$

The interest is \$133. The time of the loan is 2 years.

$$\begin{aligned} r &= \frac{I}{Pt} \\ &= \frac{133}{364.75 \times 2} \\ &= \frac{133}{729.50} \\ &= 0.18 \\ &= 18\% \end{aligned}$$

The true interest rate is approximately 18%

Note the steps:

1. Calculate the average principal.

   $P_{av} = \frac{1}{2}(\text{cash price} - \text{down payment} + \text{last payment})$

2. Calculate the interest charged.

   $I = (\text{total amount paid}) - (\text{cash price})$

3. Time of the loan is the total time in years over which the payments last.
4. $r = \dfrac{I}{P_{av} \times t}$

EXAMPLE 2. *Find the true interest rate charged on a leather coat priced at \$125 cash or \$25 down and 12 monthly payments of \$9.25.*

Solution

1. Since a \$25 down payment was made at the time of purchase the principal borrowed was \$100 (\$125 − \$25).

   $P_{av} = \frac{1}{2}(100.00 + 9.25)$
   $= 54.63$ The average principal is \$54.63.

2. $$\begin{aligned} I &= (12 \times 9.25) - 100 \\ &= 111 - 100 \\ &= 11 \quad \text{The interest is \$11.} \end{aligned}$$

3. $t = 1$ The time of the loan is 1 a.

4. $$\begin{aligned} r &= \frac{11}{54.63 \times 1} \\ &\doteq 20\% \end{aligned}$$

Interest is being charged at approximately 20% per annum.

EXERCISE 8-4

1. Calculate the true interest rate being charged for each of the following repayment schemes.

	Principal	Monthly Repayment	Interest or Service Charge Included
(a)	\$56.50	12 of \$5.00	\$3.50
(b)	\$150.00	24 of \$7.30	\$25.20
(c)	\$850.00	18 of \$51.00	\$68.00
(d)	\$1500.00	36 of \$46.50	\$174.00
(e)	\$3000.00	36 of \$95.00	\$420.00

2. A camera and projector for home movies cost \$240 cash or \$12 per month for 24 months. Find the true rate of interest.

3. A 20″ colour TV set has a cash price of \$500 or \$17 per month for 36 months. Find the true rate of interest.

4. An electric guitar and amplifier cost \$240. Bill Smith pays \$40 cash and finances the remaining \$200 at \$12 per month for 18 months. What is the true rate of interest?

5. A ten-speed bicycle is listed at \$84.98 cash or \$15 down and 12 months to pay at \$6.50 per month. Find the true interest rate.

6. True interest rate may also be approximated by the formula:

$$r = \frac{2NI}{P(n + 1)}$$

where N = the number of payments per year. (For monthly payments $N = 12$.)
I = interest charged
P = principal
n = the total number of payments (not including the down payment if any).

Apply this formula to the data in questions 2 to 5 and compare the results.

8.5 SOURCES OF CREDIT

When a purchaser decides to buy goods before he can afford to pay the full price he must find a source of credit.

Monthly Payment Chart

Balance	Monthly Payment
Up to 40.00	5.00
40.01 to 50.00	6.00
50.01 to 70.00	7.00
70.01 to 100.00	8.00
100.01 to 120.00	9.00
120.01 to 170.00	10.00
170.01 to 200.00	11.00
200.01 to 220.00	12.00
220.01 to 240.00	13.00
240.01 to 260.00	14.00
260.01 to 280.00	15.00
280.01 to 320.00	16.00
320.01 to 340.00	17.00
340.01 to 360.00	18.00
360.01 to 380.00	19.00
380.01 to 400.00	20.00

Special Accounts— up to 36 months to pay

For Appliances . . .

Balance	Monthly Payment
80.01 to 100.00	5.00
100.01 to 120.00	5.50
120.01 to 140.00	6.00
140.01 to 160.00	6.50
160.01 to 180.00	7.00
180.01 to 210.00	7.50
210.01 to 220.00	8.00
220.01 to 230.00	8.50
230.01 to 250.00	9.00
250.01 to 260.00	9.50
260.01 to 280.00	10.00
280.01 to 290.00	10.50
290.01 to 300.00	11.00
300.01 to 320.00	11.50
320.01 to 330.00	12.00
330.01 to 350.00	12.50
350.01 to 360.00	13.00
360.01 to 370.00	13.50
370.01 to 390.00	14.00
390.01 to 400.00	14.50

For Larger Purchases totaling over \$400 . . .

of durable goods for your home . . . furniture, major appliances and fixtures

Balance	Monthly Payment
400.01 to 420.00	15.00
440.01 to 460.00	16.50
500.01 to 510.00	18.00
550.01 to 560.00	20.00
600.01 to 610.00	21.50
650.01 to 670.00	23.50
700.01 to 710.00	25.00
750.01 to 770.00	27.00
790.01 to 800.00	28.00

The most common sources are:

- **A** The retailer who sells the goods (direct financing)
- **B** Consumer finance companies
- **C** Banks —(third party financing)
- **D** Credit Unions

A: Direct Financing

One of the most popular types of credit arrangements used by many customers of large department stores and oil companies is the 30-Day Revolving Charge Account. Purchases are billed at the end of the month and if paid for within 30 d of the billing date no service charge is added.

If the bill is not completely paid a minimum monthly payment is determined by the amount of the purchase, and to this is added a service charge determined by the previous month's unpaid balance.

A sample scale of payments and charges is shown in the margin. The ownership of all goods remains with the vendor (seller) until they are completely paid for.

Previous Unpaid Balance	Service Charge
$ – $ 10	$ .08
10.01 – 20	.24
20.01 – 30	.40
30.01 – 40	.56
40.01 – 50	.72
50.01 – 60	.88
60.01 – 70	1.04
70.01 – 80	1.20
80.01 – 90	1.36
90.01 – 100	1.52
100.01 – 110	1.68
110.01 – 120	1.84
120.01 – 130	2.00
130.01 – 140	2.16
140.01 – 150	2.32
150.01 – 160	2.48
160.01 – 170	2.64
170.01 – 180	2.80
180.01 – 190	2.96
$190.01 – $200	$3.12

What do you notice about the way the service charges increase?

What rate is being charged?

EXAMPLE 1. *Susie McMaster purchased a portable stereo record player for $89.98, including provincial sales tax. Find the total of payments and charges according to the schedule shown.*

Solution From the monthly payment chart the minimum monthly payment on a purchase of from $70 to $100 is $8 per month.

Payment #	Balance	Service Charge	Total	Minimum Payment	Balance after Payment
1	$89.98	$0.00	$89.98	$ 8.00	$81.98
2	81.98	1.36	83.34	8.00	75.34
3	75.34	1.20	76.54	8.00	68.54
4	68.54	1.04	69.58	8.00	61.58
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
12	10.14	0.24	10.38	8.00	2.38
13	$ 2.38	$0.08	$ 2.46	$ 2.46	$ 0.00
Total service charges		$8.48		$98.46	
Total amount repaid		$98.46			

B and C: Financial Institutions

Rather than finance your purchase at the store you buy from you may prefer to borrow the money from a lending agency and purchase for cash. This may give you the option of purchasing at a "cash only" outlet, perhaps for a lower price.

The main sources of cash open to the general public are banks, trust companies and consumer finance companies. For

Addition:

hocus
pocus
presto

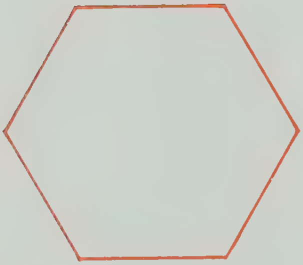
small consumer loans not secured by collateral, procedures and interest rates at banks, trust companies, or finance companies will be very similar. In general the loan companies will be willing to take greater risks in return for larger interest charges.

If the borrower has bonds or other securities to use as collateral for the loan, banks offer other types of loans at lower rates.

D: Credit Unions

A credit union is an organization of people who usually have a common interest in that they work for the same company or live in the same community. The members deposit savings with the branch in the form of purchased shares in the credit union. The funds are then available to be loaned out to the members at a rate of interest which is usually about 1% per month. Profits made by the credit union are distributed among the members on a per share basis and are called dividends. Loans from a credit union are life insured at no extra cost to the borrower.

If the interest rates of the commercial lending institutions drop, the credit union may declare a rebate to the borrower as part of the year-end dividend. In this way they stay competitive.



How many diagonals has a hexagon?

EXAMPLE 2. *Mr. W. Byres belongs to a credit union at the factory where he works. He has had a loan approved for $2100 to purchase a used car on 1980 01 07. He will repay the loan at the rate of $100 per month on the principal plus 1% per month interest on the unpaid balance. Show a payment schedule for the first three and last two payments.*

Solution

Date	Debt to Date	Principal Payment	Interest	Amount Due	Balance
1980 02 17	$2100	$100	$21.00	$121.00	$2000
1980 03 17	2000	100	20.00	120.00	1900
1980 04 17	1900	100	19.00	119.00	1800
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1982 09 17	200	100	2.00	102.00	100
1982 10 17	100	100	1.00	101.00	0

EXERCISE 8-5

1. Copy the table in example 2 and complete the missing portion.

2. Complete a repayment table for the purchase of a pool table with a cash price of $179.98 plus 7% provincial sales tax. Use the amounts shown in the tables on page 176, 177.

3. (a) Using the average monthly balance for each service charge given in the table in the margin on page 176 calculate the monthly service charge rate.

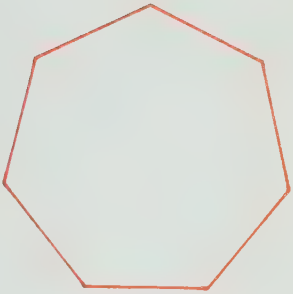
Insert brackets to make the statement true:

$3 - 5 + 2 \times 3 + 8 = -10$

(b) From the rates in (a) calculate the approximate annual rate. (Twelve times the monthly rate.)

(c) Check the credit arrangements given in a catalogue for any large retail store, gas company, or building supply distributor and compare the data with the results of (a) and (b).

4. A ten-speed Road Master bicycle sells for \$98.85 plus sales tax. To reduce the service charge the purchaser decides to make a down payment of \$28.85 plus tax and pay the remainder at \$10 per month plus service charge. Make up a schedule of payments using the service charge table given.



How many diagonals has a septagon?

5. Mrs. Ann Jones borrows \$3000 from her credit union for a new car. She will repay the loan at a rate of \$100/month plus interest of ¾% on the unpaid balance. Draw up a schedule of payments showing the first two and last three payments.

6. Jack Wilson has had money deducted from his pay each month to buy Canada Savings Bonds. At the present time he owns \$5000 in bonds. Now Jack has decided to purchase a summer cottage and needs a down payment of \$1500. Rather than sell any bonds he has gone to the bank manager and arranged a demand payment loan of \$1500, using the bonds as collateral. Because the bank manager holds a sufficient value of Jack's bonds to cover the loan should Jack default, there is no risk involved and the loan is made at 7½%. If the loan is repaid at \$75/month, draw up a schedule of payments for the first 6 months showing:

$$I = \frac{31}{365} \times 0.075 \times 1500$$
$$= 9.55$$

1	2	3	4	5	6
Date	**Principal**	**Payment**	**Interest**	**Payment on Principal** 3–4	**Balance** 2—5
Jan. 5	\$1500.00				
Feb. 5	1500.00	\$75.00	\$9.55	\$65.45	\$1434.55
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

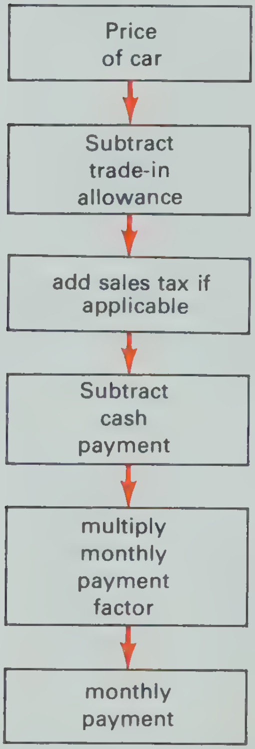
7. A demand payment loan for \$2500 earns interest at $7\frac{3}{4}\%$/a calculated on the monthly balance. Payments of \$100 monthly pay the accrued interest and reduce the principal. Draw up a schedule of payments, as for question 6, for the first 6 payments.

8.6 PURCHASING AN AUTOMOBILE

When someone shops for an automobile, several important decisions must be made. Many of these decisions are based on the person's ability to pay for the original purchase plus upkeep. In this section, we shall do some of the mathematics that helps us make these decisions.

Since the purchase of an automobile is a great expense, many people must borrow a portion of the cost. There are many places to borrow the money — the dealer, a bank, trust company, credit union, or finance company.

The following gives an estimate of the monthly payment.


Price of car
Subtract trade-in allowance
add sales tax if applicable
Subtract cash payment
multiply monthly payment factor
monthly payment

Monthly Payment Factor	
15%	
Number of Payments	*Factor*
24	0.048 486 6
30	0.040 178 5
36	0.034 665 3
18%	
Number of Payments	*Factor*
24	0.049 924 1
30	0.041 639 2
36	0.036 152 3

The monthly payment factor multiplies the amount to be financed to give the monthly payment. To select the correct monthly payment factor, you must know the annual rate of interest and the number of monthly payments.

$$\text{Monthly Payment} = \begin{matrix}\text{Amount to}\\ \text{be financed}\end{matrix} \times \begin{matrix}\text{Monthly}\\ \text{payment factor}\end{matrix}$$

EXAMPLE 1. *Mary Parker has just purchased a new Mountaineer van for $11 840. She was allowed $3280 on her Bonsai trade-in and financed the rest at 18% over 30 months. Applicable sales tax is 7%. Estimate the monthly payment.*

Solution

Price of car to be purchased	*$ 11 840*
Trade-in allowance (if any)	*$ 3 280*
Difference	*$ 8 560*
Sales Tax (where applicable)	*$ 599.20*
Total (may be found from $8560 × 1.07)	*$ 9 159.20*
Cash payment (if any)	*0*
Amount to be financed	*$ 9 159.20*
Monthly Payment Factor (18%, 30 mon.)	*0.0416392*
Approximate Payment	*$ 381.38*

∴Mary will make 30 payments of $381.38 each.

Monthly Payment Factor	
15%	
Number of Payments	*Factor*
24	0.048 486 6
30	0.040 178 5
36	0.034 665 3
18%	
Number of Payments	*Factor*
24	0.049 924 1
30	0.041 639 2
36	0.036 152 3

EXERCISE 8-6

1. Use the monthly payment factors to find an estimate of the monthly payment for the following automobile loans at 18%.
(a) $4580 for 24 months
(b) $7418 for 36 months
(c) $8810 for 30 months

2. Find an estimate of the monthly payment for the following automobile loans at 15%.
(a) $3125 for 30 months
(b) $2835 for 24 months
(c) $5230 for 36 months

3. Find an estimate of the monthly payment for the following automobile loans.
(a) $8426 at 18% for 36 months
(b) $2155 at 15% for 24 months
(c) $5580 at 15% for 30 months
(d) $4625 at 18% for 30 months
(e) $9280 at 15% for 36 months

4. Find an estimate of the monthly payment for each of the following car purchases:
(a) A new Z85 for $9500 with a trade-in allowance of $3550; sales tax is 7%; the car is financed for 36 months at 18%.
(b) A used Vulcan costing $5280 with no trade-in; there is a down payment of $2000, and the rest is financed at 15% for 24 months; 7% sales tax is charged.
(c) A new Jetstar van costing $11 850 with a Carsun trade-in worth $3215; the balance is financed at 18% for 36 months; 5% sales tax is charged.

5. Eric Jensen buys a new Towncoup for $12 500. He trades in a Hawk for $1850 and pays $2000 cash, then finances the rest for 36 months at 18%. 8% sales tax is charged. Estimate the monthly payment.

6. Rene Rivard buys a new Skodar for $18 500 and has a down payment of $6500. He finances the balance at 15% over 30 months. Estimate the monthly payment if there is a sales tax of 7%.

7. Ruth Jarvis buys a new CS2 for $14 850 and has a trade-in worth $3600. She finances the balance with the dealer at 18% for 36 months. No sales tax is paid.
(a) Find the monthly payment.
(b) What is the total amount repaid?
(c) How much interest is paid in 36 months?

8. Tom Dow buys a new SS15 for $19 250 and trades in a motorbike for $2500. 7% sales tax is paid. The balance is financed at 18% for 36 months.
(a) Find the monthly payment.
(b) What is the total amount repaid?
(c) How much interest is paid in 36 months?

REVIEW EXERCISE

1. The following financial transactions were entered into by the Bayhill Students' Council for April.

Apr. 1 Balance on hand $743.20

	Expenditures		Receipts
Apr 4	$ 36.20	Apr 12	$ 12.36
Apr 17	$ 17.42	Apr 15	$ 2.50
	$ 9.30	Apr 23	$247.50
Apr 23	$157.90		
Apr 30	$ 16.20		

Make up a suitable item description for each amount and complete the sheet for the cash book.

2. Make up a bank deposit slip for Bayhill Students' Council account, April 25.

10—$1 bills
4—$2 bills
6—$5 bills
1—$10 bill
$4.65 in coins
Cheques for $12.36, $2.50

3. What is the difference between direct financing and third party financing?

4. Fill in the missing quantities in the following simple interest table. Interest rate is shown per annum.

	I	P	r	t
(a)	▨	$500	6%	30 d
(b)	▨	$1250	12%	2 mon
(c)	$36.50	▨	8%	6 mon
(d)	$5.20	▨	10%	30 d
(e)	$4.50	$1200	▨	3 mon
(f)	$12.00	$2400	▨	60 d
(g)	$3.55	$3600	12%	▨ d
(h)	$4.25	$970	10%	▨ d

5. A 24″ colour TV costs $456.95 cash or 30 monthly payments of $17.20. What is the effective annual interest rate being charged?

6. A mini trail-bike costs $372.50 cash or 24 monthly payments of $18.30. What is the effective annual interest rate being charged?

7. (a) A loan of $2500 is paid at a rate of $85/month, principal and interest. The loan bears interest at ¾%/month. Draw up a schedule of payments for the first 5 months. (As for Ex. 8.5 Qu. 6.)
(b) A loan of $2500 is paid at a rate of $85/mon on the principal, plus the interest of ¾%/mon on the unpaid balance. Draw up a schedule of payments for the first five months. (As for Section 8.5, Example 2.)
(c) Compare the balances at the end of 5 months under each repayment scheme.

8. A new car is purchased for $7580 with a trade in allowance of $2400. 7% sales tax is paid on the price difference. If a $1500 cash payment is made and the remainder is financed at 15% for 30 months, find the monthly payment and the total interest charged.

REVIEW AND PREVIEW TO CHAPTER 9

1. (a) Find the total for each of the following accounts:

(i)	(ii)	(iii)	(iv)
$ 125.32	$ 25.60	$ 635.00	$ 36.40
14.56	45.22	52.18	27.12
749.15	13.68	394.70	82.17
26.40	179.43	25.42	36.59
$	$	$	$

(b) Find the total of the four accounts.

2. (a) Find the total for each of the following accounts. A quantity marked Cr. is a credit and should be subtracted from the account.

(i)	(ii)	(iii)	(iv)
$ 14.32	$ 275.84	$ 294.73	$ 413.49
36.24	97.36	749.90	785.13 Cr.
52.43	142.12 Cr.	476.42	9.74
17.29 Cr.	743.15	194.31	183.62
36.21	74.20	684.17 Cr.	942.12 Cr.
9.85	349.70 Cr.	481.46 Cr.	71.00
$	$	$	$

(b) Add 7% sales tax to each account.

3. (a) In the following accounts only items marked T are taxable at 7%. Find the amount due in each case.

	(i)	(ii)	(iii)	(iv)
Items	$ 46.43 T	$ 140.30	$ 463.19	$ 836.41
	134.19	273.92 T	984.30	592.18
	28.32 T	42.18 T	598.40 T	742.38 T
	479.06 T	390.00	923.95 T	136.59
	91.24	47.62	726.39	84.16 T
	6.20	183.40 T	283.12 T	257.30 T
	18.47	97.51	38.40 T	79.16

(b) Find the total amount due for the four accounts and the total tax to be remitted.

CHAPTER 9

Matrices

9.1 INTRODUCTION TO MATRICES

Collections or stacks of numbers are used in many different ways. At the end of a half inning of a baseball game, the radio announcer gives a list such as:

4 runs
6 hits
1 error
2 left on base

Bowling scores can be written as follows:

	Game 1	Game 2	Game 3	Total
R. Brown	211	206	214	631

The standings for a hockey league can be shown as

	G	W	L	T	P
Vancouver	33	19	10	4	42
Edmonton	31	16	9	6	38
Montreal	32	16	12	4	36
Toronto	32	13	17	2	28

These collections of numbers can be written as

$$\begin{pmatrix} 4 \\ 6 \\ 1 \\ 2 \end{pmatrix}, \quad (211\ 206\ 214\ 631), \quad \begin{pmatrix} 33 & 19 & 10 & 4 & 42 \\ 31 & 16 & 9 & 6 & 38 \\ 32 & 16 & 12 & 4 & 36 \\ 32 & 13 & 17 & 2 & 28 \end{pmatrix}$$

A matrix is a rectangular array of numbers enclosed by parentheses.

The individual numbers in a matrix are called the *elements* or *entries* of the matrix.

The dimensions of a matrix are determined by the number of rows (horizontal) and the number of columns (vertical).

C
ROWS
L
U
M
N
S

Other examples of matrices are:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 7 & 8 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 9 \\ 4 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 & 2 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix}$$

Matrix A shown above has dimensions 3 × 2 (read "three by two"). Matrix B is a 2 × 2 matrix (*square matrix*). Matrix C has only one row and is called a *row matrix*. Matrix D is called a *column matrix*.

EXERCISE 9-1

A 1. State the dimensions of each matrix.

(a) $\begin{pmatrix} 2 & 4 & 7 \\ 3 & 1 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 3 & 4 \\ 1 & 8 & 0 \\ 4 & 7 & 6 \end{pmatrix}$ (c) $\begin{pmatrix} 5 & 2 \\ 3 & 7 \\ 4 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 6 & 1 & 5 & 2 \end{pmatrix}$ (e) $\begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & 4 & 5 & 8 \\ 3 & 7 & 9 & 2 \end{pmatrix}$

(g) $\begin{pmatrix} 6 \end{pmatrix}$ (h) $\begin{pmatrix} 7 & 9 \end{pmatrix}$ (i) $\begin{pmatrix} 4 & 9 \\ 2 & 2 \end{pmatrix}$

2. If $D = \begin{pmatrix} 4 & 3 & 6 & 2 \\ 5 & 9 & 2 & 5 \\ 7 & 1 & 8 & 9 \end{pmatrix}$, state

(a) the dimensions of D
(b) the entries in the second column
(c) the entries in the third row
(d) the entries in the first column
(e) the entry in the second row and third column
(f) the entry in the third row and fourth column
(g) the entry in the first row and second column
(h) the entry in the first row and second column

3. The following table is used at a ticket office to determine the cost of hockey tickets.

Number of Tickets	Gold	Red	Blue	Yellow	Grey
1	8.50	7.30	5.10	4.25	3.15
2	17.00	14.60	10.20	8.50	6.30
3	25.50	21.90	15.30	12.75	9.45
4	34.00	29.20	20.40	17.00	12.60
5	42.50	36.50	25.50	21.25	15.75

(a) What do 3 Golds cost?
(b) What do 4 Blues cost?
(c) What do 2 Yellows cost?
(d) What is the cost of 3 Reds and 4 Greys?
(e) What is the cost of 2 Golds, 3 Blues and 4 Yellows?
(f) What is the cost of 1 Blue, 5 Golds and 2 Greys?

4. The following table gives the highway distances (in kilometres) between several cities.

	Calgary	Halifax	Montreal	Ottawa	St. John's	Toronto	Vancouver	Winnipeg
Calgary	0	4917	3651	3523	4579	3427	1054	1331
Halifax	4917	0	1273	1467	422	1821	6003	3606
Montreal	3651	1273	0	201	933	549	4730	2333
Ottawa	3523	1467	201	0	1130	397	4576	2192
St. John's	4579	422	933	1130	0	1482	5664	3266
Toronto	3427	1821	549	397	1482	0	4505	2108
Vancouver	1054	6003	4730	4576	5664	4505	0	2397
Winnipeg	1331	3606	2333	2192	3266	2108	2397	0

(a) How far is it from St. John's to Vancouver?
(b) How far is it from Halifax to Calgary?
(c) How far is it from Ottawa to Montreal?
(d) Write the row matrix that gives the distances from Winnipeg to the other cities.
(e) Write the column matrix that gives the distances from Toronto to the other cities.
(f) Write the row matrix that gives the round trip distances from Montreal to the other cities.
(g) Write the column matrix that gives the round trip distances from Vancouver to the other cities.
(h) Road maps use distance tables one half the size of the one shown. Why is this possible?

5. Frank recorded this month's pop sales (in cases) to various stores in a table as follows.

April	**Cola**	**Ginger ale**	**Orange**	**Soda**
Big M	13	19	10	6
Astro	15	21	4	4
Cosmic	21	16	13	5
Sim's	8	5	15	2
Pete's	15	12	3	7

(a) How many cases of Cola did Cosmic buy?
(b) How many cases of Soda did Sim's buy?
(c) How many cases of Orange did Astro buy?
(d) Write Pete's purchases as a row matrix. What was the total purchase?
(e) Write the ginger ale sales as a column matrix. What was the total ginger ale sales?
(f) Find the total sales to each store.
(g) What were Frank's total sales for April?

9.2 ADDITION OF MATRICES

The following example will illustrate that we can add matrices only if they have the same dimensions.

EXAMPLE 1. *The manager of a warehouse that distributes appliances records the orders received in tables. The following shows the orders received in November and December from two stores.*

November	*Stoves*	*Fridges*	*T.V.'s*	*Radios*
Store A	*10*	*5*	*6*	*13*
Store B	*9*	*8*	*5*	*17*

December	*Stoves*	*Fridges*	*T.V.'s*	*Radios*
Store A	*7*	*3*	*15*	*22*
Store B	*6*	*2*	*13*	*27*

Express the orders for each month as a matrix and find the total order for each store.

Solution The orders for each month can be expressed as matrices as follows.

$$N = \begin{pmatrix} 10 & 5 & 6 & 13 \\ 9 & 8 & 5 & 17 \end{pmatrix}, \qquad D = \begin{pmatrix} 7 & 3 & 15 & 22 \\ 6 & 2 & 13 & 27 \end{pmatrix}$$

The total order is

$$\begin{aligned} N + D &= \begin{pmatrix} 10 & 5 & 6 & 13 \\ 9 & 8 & 5 & 17 \end{pmatrix} + \begin{pmatrix} 7 & 3 & 15 & 22 \\ 6 & 2 & 13 & 27 \end{pmatrix} \\ &= \begin{pmatrix} 10+7 & 5+3 & 6+15 & 13+22 \\ 9+6 & 8+2 & 5+13 & 17+27 \end{pmatrix} \\ &= \begin{pmatrix} 17 & 8 & 21 & 35 \\ 15 & 10 & 18 & 44 \end{pmatrix} \end{aligned}$$

Store A ordered 17 stoves, 8 fridges, 21 T.V.'s and 35 radios. Store B ordered 15 stoves, 10 fridges, 18 T.V.'s and 44 radios.

Matrices can be added if they have the same dimensions.

To add matrices we add corresponding entries.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

The matrices $\begin{pmatrix} 4 & 6 \\ 1 & 9 \end{pmatrix}$ and $\begin{pmatrix} 3 & 1 & 4 \\ 8 & 7 & 6 \end{pmatrix}$ cannot be added because they do not have the same dimensions.

EXERCISE 9-2

1. Perform the following additions, where possible.

(a) $\begin{pmatrix} 5 & 6 \\ 9 & 3 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 7 & 9 \\ 3 & 8 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 2 & 9 \\ 7 & 13 & 21 \end{pmatrix}$

(c) $\begin{pmatrix} 6 & 9 \\ 11 & 12 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & -9 & 8 \\ 5 & 2 & -11 \\ 16 & -14 & 12 \end{pmatrix} + \begin{pmatrix} -9 & 8 & -4 \\ 13 & 7 & 15 \\ -10 & 11 & 21 \end{pmatrix}$

(e) $\begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 & 0 & 4 \end{pmatrix}$

(f) $\begin{pmatrix} -32 & 15 \\ 19 & -24 \\ 63 & -81 \end{pmatrix} + \begin{pmatrix} 29 & 47 \\ -35 & -92 \\ 46 & -83 \end{pmatrix}$

(g) $\begin{pmatrix} 461 & 537 \\ 492 & 876 \end{pmatrix} + \begin{pmatrix} 687 & 395 \\ 766 & 848 \end{pmatrix}$

(h) $\begin{pmatrix} 425 & 387 \\ 576 & 418 \\ 822 & 798 \end{pmatrix} + \begin{pmatrix} 607 & 420 \\ 539 & 841 \\ 762 & 156 \end{pmatrix}$

2. Perform the following additions.

(a) $\begin{pmatrix} 5 & 4 \\ 6 & 10 \end{pmatrix} + \begin{pmatrix} 11 & 3 \\ -6 & -7 \end{pmatrix} + \begin{pmatrix} 15 & 0 \\ 4 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 6 \\ 8 \\ 9 \end{pmatrix} + \begin{pmatrix} 11 \\ 15 \\ -6 \end{pmatrix} + \begin{pmatrix} -4 \\ -7 \\ 9 \end{pmatrix}$

(c) $\begin{pmatrix} 10 & 9 \\ 11 & 16 \\ -12 & -8 \end{pmatrix} + \begin{pmatrix} 21 & 13 \\ 24 & -10 \\ -6 & 13 \end{pmatrix} + \begin{pmatrix} 25 & -9 \\ -16 & -20 \\ 14 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 6 & 9 & -3 \\ 4 & 8 & 7 \\ -3 & -2 & 5 \end{pmatrix} + \begin{pmatrix} 11 & 0 & 2 \\ -5 & 0 & 7 \\ -4 & 11 & 2 \end{pmatrix} + \begin{pmatrix} 5 & 1 & 3 \\ -6 & 4 & 8 \\ -9 & 2 & 13 \end{pmatrix}$

(e) $\begin{pmatrix} 156 & 315 & 482 \end{pmatrix} + \begin{pmatrix} 918 & 113 & 325 \end{pmatrix}$

(f) $\begin{pmatrix} 7314 & 4832 \\ 2968 & 5175 \end{pmatrix} + \begin{pmatrix} 6823 & 313 \\ 407 & 529 \end{pmatrix}$

3. The results for November, December and January for a hockey league are as follows

Nov.	Won	Lost	Tied
Aces	7	4	1
Blues	6	4	2
Kings	5	6	1
Reds	4	8	0

Dec.	Won	Lost	Tied
Aces	3	9	2
Blues	5	8	1
Kings	8	6	0
Reds	10	3	1

Jan.	Won	Lost	Tied
Aces	5	5	0
Blues	7	2	1
Kings	3	6	1
Reds	4	6	0

(a) Express the results for each month as a matrix.
(b) Find the sum of the matrices.
(c) If a win is worth 2 points and a tie 1 point, what team was in first place at the end of the 3 months?
(d) What team was in last place?

9.3 SUBTRACTION OF MATRICES

The negative of 4 is -4. Similarly, the negative of matrix A is $-A$. *For example*

$$\text{If } A = \begin{pmatrix} 3 & -1 \\ 2 & -5 \end{pmatrix}, \text{ then } -A = \begin{pmatrix} -3 & 1 \\ -2 & 5 \end{pmatrix}$$

Each entry in $-A$ is the negative of the corresponding entry in A. To subtract real numbers, we can *add* the negative. For example

$$\begin{aligned} 7 - 3 &= 7 + (-3) \\ &= 4 \end{aligned}$$

We also define the subtraction of matrices in terms of addition.

If $A = \begin{pmatrix} 3 & 6 \\ 5 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & -2 \\ 4 & 5 \end{pmatrix}$

$$\begin{aligned} \text{then } A - B &= A + (-B) \\ &= \begin{pmatrix} 3 & 6 \\ 5 & -4 \end{pmatrix} + \begin{pmatrix} -6 & 2 \\ -4 & -5 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 8 \\ 1 & -9 \end{pmatrix} \end{aligned}$$

EXERCISE 9-3

1. Perform the following subtractions, where possible.

(a) $\begin{pmatrix} 3 & 12 \\ 6 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 9 \\ 4 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 12 & 15 & 11 \\ 21 & 6 & 13 \end{pmatrix} - \begin{pmatrix} 4 & 8 & 0 \\ 13 & 4 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 5 & 6 \\ -8 & 4 \end{pmatrix} - \begin{pmatrix} 13 & -4 \\ -1 & 6 \end{pmatrix}$ (d) $\begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 7 & 3 \\ -8 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & -3 \\ -7 & 4 \\ -2 & 6 \end{pmatrix} - \begin{pmatrix} 5 & -6 \\ 6 & -8 \\ -5 & 0 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & -1 & 5 \\ -3 & 4 & 2 \end{pmatrix} - \begin{pmatrix} -4 & -6 & 8 \\ 5 & 6 & 7 \end{pmatrix}$

(g) $\begin{pmatrix} 5 & 9 & -6 \\ 4 & -3 & -8 \\ 0 & 6 & -4 \end{pmatrix} - \begin{pmatrix} 9 & -2 & -3 \\ 0 & -1 & -6 \\ 4 & 2 & 7 \end{pmatrix}$ (h) $\begin{pmatrix} 4 & -6 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 0 & -6 & 1 \\ 2 & -5 & 4 \end{pmatrix}$

2. Simplify.

(a) $\begin{pmatrix} 5 & 4 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ 5 & -8 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 4 & 7 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -3 \\ -4 & 6 \\ 5 & 2 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ -9 & -10 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 10 \\ 4 & -3 \\ -1 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} 14 \\ -6 \\ 19 \end{pmatrix} + \begin{pmatrix} 21 \\ -8 \\ -22 \end{pmatrix} - \begin{pmatrix} 6 \\ -8 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 12 \\ 13 & -14 \end{pmatrix} - \begin{pmatrix} -6 & 8 \\ 9 & -2 \end{pmatrix} - \begin{pmatrix} 7 & -11 \\ 12 & 15 \end{pmatrix}$

(e) $\begin{pmatrix} 10 & 11 & 12 \\ -4 & 9 & 5 \\ -6 & 3 & 7 \end{pmatrix} + \begin{pmatrix} 12 & 21 & 13 \\ -6 & -18 & 10 \\ 4 & -11 & 16 \end{pmatrix} - \begin{pmatrix} 3 & -21 & 10 \\ 15 & -6 & 0 \\ 1 & 4 & 2 \end{pmatrix}$

(f) $\begin{pmatrix} 46 & 83 \\ 25 & -18 \end{pmatrix} - \begin{pmatrix} 31 & -22 \\ -16 & 55 \end{pmatrix} + \begin{pmatrix} 73 & -24 \\ -37 & 54 \end{pmatrix}$

9.4 SCALAR MULTIPLICATION

When working with matrices, a real number is often called a scalar. Multiplying a matrix by a real number such as

$$\underset{\text{scalar}}{2} \times \underbrace{\begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix}}_{\text{matrix}}$$

is called scalar multiplication.

Recall that if x is a real number, then

$$2x = x + x$$

Similarly, if A is a matrix, then

$$2A = A + A$$

For example, if

$$A = \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix}, \text{ then}$$

$$\begin{aligned}2A &= A + A\\ &= \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix} + \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix}\\ &= \begin{pmatrix}10 & 2\\ 6 & 8\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}2 \times 5 & 2 \times 1\\ 2 \times 3 & 2 \times 4\end{pmatrix}\end{aligned}$$

Similarly,

$$\begin{aligned}3A &= A + A + A\\ &= \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix} + \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix} + \begin{pmatrix}5 & 1\\ 3 & 4\end{pmatrix}\\ &= \begin{pmatrix}15 & 3\\ 9 & 12\end{pmatrix}\\ &= \begin{pmatrix}3 \times 5 & 3 \times 1\\ 3 \times 3 & 3 \times 4\end{pmatrix}\end{aligned}$$

In general,

$$\boxed{\begin{array}{l}\text{If } k \text{ is a real number, then}\\ k\begin{pmatrix}a & b\\ c & d\end{pmatrix} = \begin{pmatrix}ka & kb\\ kc & kd\end{pmatrix}\end{array}}$$

EXAMPLE 1.

Simplify $2\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix} + 3\begin{pmatrix}-4 & 5\\ 2 & 6\end{pmatrix}$

Solution

$$\begin{aligned}2\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix} + 3\begin{pmatrix}-4 & 5\\ 2 & 6\end{pmatrix} &= \begin{pmatrix}4 & -2\\ 6 & 8\end{pmatrix} + \begin{pmatrix}-12 & 15\\ 6 & 18\end{pmatrix}\\ &= \begin{pmatrix}-8 & 13\\ 12 & 26\end{pmatrix}\end{aligned}$$

EXERCISE 9-4

A 1. Perform each scalar multiplication.

(a) $2\begin{pmatrix}1 & 5\\ 6 & 3\end{pmatrix}$ (b) $4\begin{pmatrix}3\\ 1\\ -6\end{pmatrix}$ (c) $\frac{1}{2}\begin{pmatrix}8 & 10\\ 4 & 6\end{pmatrix}$

(d) $-2\begin{pmatrix}3 & 0\\ -1 & 4\end{pmatrix}$ (e) $-3(1 \quad 0 \quad -6)$ (f) $10\begin{pmatrix}2 & 3 & -6\\ 4 & -1 & -7\end{pmatrix}$

B 2. Simplify.

(a) $2\begin{pmatrix}1 & 3\\4 & 7\end{pmatrix}+5\begin{pmatrix}2 & 3\\0 & 2\end{pmatrix}$ (b) $3\begin{pmatrix}2 & 4\\1 & 6\\8 & 2\end{pmatrix}+4\begin{pmatrix}1 & -1\\2 & 0\\1 & -1\end{pmatrix}$

(c) $4\begin{pmatrix}2 & -1\\3 & -6\end{pmatrix}-\begin{pmatrix}5 & -6\\0 & -7\end{pmatrix}$

(d) $\begin{pmatrix}1 & 4 & -6\\3 & 8 & 1\\2 & 0 & -2\end{pmatrix}-\begin{pmatrix}5 & -2 & 4\\-3 & 1 & -2\\6 & 0 & -7\end{pmatrix}+2\begin{pmatrix}5 & 6 & -1\\3 & -2 & 1\\4 & 3 & 8\end{pmatrix}$

(e) $5\begin{pmatrix}1\\4\\-6\end{pmatrix}-3\begin{pmatrix}2\\-4\\2\end{pmatrix}+4\begin{pmatrix}1\\-3\\2\end{pmatrix}$

(f) $2\begin{pmatrix}11 & 5\\-3 & 4\\-1 & 0\end{pmatrix}-3\begin{pmatrix}-1 & -3\\4 & -6\\5 & 0\end{pmatrix}-\begin{pmatrix}5 & 9\\-1 & 3\\4 & -2\end{pmatrix}$

3. If $A=\begin{pmatrix}2 & 1\\3 & 4\end{pmatrix}$, $B=\begin{pmatrix}-1 & 3\\-4 & 0\end{pmatrix}$ and $C=\begin{pmatrix}5 & -6\\2 & 4\end{pmatrix}$,
Simplify each of the following.

(a) $A+B$ (b) $A+C$ (c) $A-B$
(d) $B-C$ (e) $2A-B$ (f) $3A+2B-C$
(g) $B-2C+3A$ (h) $B-A-C$

4. A bus company uses the following table to determine the highway distances (in kilometres) between several cities.

	Albany	Baltimore	Boston	Buffalo	Cleveland	Washington
Albany	0	513	272	453	746	587
Baltimore	513	0	640	586	557	62
Boston	272	640	0	733	1024	704
Buffalo	453	586	733	0	298	595
Cleveland	746	557	1024	298	0	579
Washington	587	62	704	595	579	0

(a) Express this table as a matrix.
(b) Use scalar multiplication to determine the round trip distances between the cities.

9.5 MULTIPLICATION OF MATRICES

It would seem that we should multiply matrices in a manner similar to addition, corresponding entry by corresponding entry. However, if we do multiply this way, the applications are extremely limited.

We will illustrate matrix multiplication by determining the revenue of a theatre for a three day period.

Tickets	**Adult**	**Student**
Thursday	151	47
Friday	173	50
Saturday	170	103

Price (dollars)	
Adult	4
Student	3

Before using matrix multiplication, we will use a familiar method to determine the theatre's revenue for each day.

Thursday: $151(4) + 47(3) = 745$
Friday: $173(4) + 50(3) = 842$
Saturday: $170(4) + 103(3) = 989$

The ticket sales can be written as $\begin{pmatrix} 151 & 47 \\ 173 & 50 \\ 170 & 103 \end{pmatrix}$.

The price of a ticket can be written as $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

(Ticket Sales) × (Price per Ticket) = Revenue

or $\begin{pmatrix} 151 & 47 \\ 173 & 50 \\ 170 & 103 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 745 \\ 842 \\ 989 \end{pmatrix}$

The familiar method of calculating the revenue suggests a pattern for matrix multiplication. The entry 745 in the product is obtained by multiplying the entries in row 1 of $\begin{pmatrix} 151 & 47 \\ 173 & 50 \\ 170 & 103 \end{pmatrix}$ by the entries in $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and adding. That is, $151 \times 4 + 47 \times 3 = 745$.

Similarly, the 842 in the product matrix is determined by multiplying the entries in row 2 of $\begin{pmatrix} 151 & 47 \\ 173 & 50 \\ 170 & 103 \end{pmatrix}$ by the entries in $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and adding. That is, $173 \times 4 + 50 \times 3 = 842$.

Similarly, $170 \times 4 + 103 \times 3 = 989$.

$$\begin{aligned}\begin{pmatrix} 151 & 47 \\ 173 & 50 \\ 170 & 103 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} &= \begin{pmatrix} 151 \times 4 + 47 \times 3 \\ 173 \times 4 + 50 \times 3 \\ 170 \times 4 + 103 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 604 + 141 \\ 692 + 150 \\ 680 + 309 \end{pmatrix} \\ &= \begin{pmatrix} 745 \\ 842 \\ 989 \end{pmatrix}\end{aligned}$$

We obtain the entries of the product matrix by adding the products of the elements of a row of the first matrix with the corresponding elements of a column in the second matrix.

To multiply matrices, it is not necessary that they have the same dimensions. What is necessary is that the number of columns of the first matrix be the same as the number of rows of the second matrix.

EXAMPLE 1. *Find the product of*

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}$$

Solution

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&&\\&&\end{pmatrix}$$

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&14&\\&&\end{pmatrix}$$

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&14&10\\&&\end{pmatrix}$$

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&14&10\\13&&\end{pmatrix}$$

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&14&10\\13&12&\end{pmatrix}$$

$$\begin{pmatrix}2&3\\1&4\end{pmatrix}\begin{pmatrix}1&4&5\\3&2&0\end{pmatrix}=\begin{pmatrix}11&14&10\\13&12&5\end{pmatrix}$$

EXERCISE 9-5

1. Perform the following multiplications, where possible.

(a) $\begin{pmatrix}2&3\\4&1\end{pmatrix}\begin{pmatrix}5&1\\2&4\end{pmatrix}$

(b) $\begin{pmatrix}6&7\\9&3\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix}$

(c) $\begin{pmatrix}6&4\\2&3\\1&7\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix}$

(d) $\begin{pmatrix}2&4\\1&6\end{pmatrix}\begin{pmatrix}5&2\\3&1\end{pmatrix}$

(e) $\begin{pmatrix}5&7\\3&4\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}$

(f) $\begin{pmatrix}2&4&3\\1&7&6\\5&8&2\end{pmatrix}\begin{pmatrix}4&0&3\\7&2&1\\8&3&4\end{pmatrix}$

2. The following table shows the number of vehicles parked in a downtown parking lot over a three day period.

	Cars	Buses
Thursday	72	6
Friday	81	2
Saturday	94	3

The charge for cars is \$3 and for buses \$15.

(a) Write the number of vehicles parked as a matrix.
(b) Write the charge for each as a column matrix.
(c) Use matrix multiplication to determine revenue for Thursday, Friday and Saturday.
(d) What was the total revenue for the three days.

3. The table gives the number of games won, lost and tied by four hockey teams.

Team	Won	Lost	Tied
Aces	29	31	6
Blades	29	29	8
Blues	32	28	6
Flyers	28	30	8

A win is worth 2 points, a loss 0 points and a tie 1 point.
(a) Write the games won, lost and tied as a matrix.
(b) Write the points awarded as a column matrix.
(c) Use matrix multiplication to determine the points earned by each team.

4. The following table indicates the tickets sold for three baseball games.

	Boxes	Reserved	Bleachers
Friday	2100	8134	2824
Saturday	3090	9213	1101
Sunday	2520	8475	1316

Box seats cost \$9 each, reserved \$7 each and bleachers \$4 each.
(a) Write the ticket sales for the three days as a matrix.
(b) Write the cost for the tickets as a column matrix.
(c) Use matrix multiplication to determine the ticket revenue for each day.
(d) What is the total revenue for the three day period?
(e) What is the total revenue from the box seats for the three day period?

REVIEW EXERCISE

1. Add, if possible.

(a) $\begin{pmatrix} 3 & 2 \\ 5 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 7 \\ 11 & 13 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 1 & 8 \\ -2 & 4 & 5 \\ -3 & 1 & 7 \end{pmatrix} + \begin{pmatrix} -5 & 2 & 4 \\ -8 & 3 & -3 \\ 4 & 1 & 6 \end{pmatrix}$

(c) $\begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix} + \begin{pmatrix} 9 & 1 \\ 13 & -3 \\ 4 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & -3 & 4 \\ 5 & 16 & -9 \end{pmatrix} + \begin{pmatrix} 12 & 13 & -2 \\ 8 & -11 & -16 \end{pmatrix}$

(e) $\begin{pmatrix} 6 \\ 1 \\ 5 \\ 4 \end{pmatrix} + \begin{pmatrix} -8 \\ -3 \\ 7 \\ 2 \end{pmatrix}$

(f) $\begin{pmatrix} 8 & 1 \\ 6 & 5 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 6 & 5 \\ -2 & 1 & 4 \\ 7 & 6 & 8 \end{pmatrix}$

2. Simplify.

(a) $\begin{pmatrix} 5 & 6 \\ 3 & 9 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 13 & 15 & 22 \\ 14 & 9 & 16 \\ 11 & 24 & 18 \end{pmatrix} - \begin{pmatrix} 9 & 6 & 11 \\ 13 & 5 & 0 \\ 4 & 19 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & -2 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} 4 & -3 \\ 0 & -2 \end{pmatrix} - \begin{pmatrix} 5 & -1 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 6 \\ -1 & -1 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} 0 & -5 \\ 5 & 3 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 4 & 6 \\ 10 & 9 \\ 3 & -1 \end{pmatrix}$

(e) $3\begin{pmatrix} 4 & 2 \\ 3 & 7 \end{pmatrix}$ (f) $-2\begin{pmatrix} 8 & 1 & 5 \\ 4 & -2 & 3 \\ 8 & -1 & 0 \end{pmatrix}$ (g) $\frac{1}{2}\begin{pmatrix} 6 \\ 8 \\ -4 \end{pmatrix}$

(h) $2\begin{pmatrix} 4 & 0 \\ -1 & 3 \end{pmatrix} + 5\begin{pmatrix} -2 & 1 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$

(i) $5\begin{pmatrix} 1 & 0 \\ -2 & 3 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 3 & -4 \\ 2 & 1 \\ 4 & -6 \end{pmatrix} - 2\begin{pmatrix} 3 & 4 \\ -1 & 2 \\ -2 & -3 \end{pmatrix}$

3. Multiply.

(a) $\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 4 & 1 \\ 2 & 5 & 7 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 0 \\ 4 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 5 & 6 \\ 8 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 4 & 2 \\ 0 & 3 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

(e) $\begin{pmatrix} 5 & 6 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & 4 & 1 \\ 3 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 0 \end{pmatrix}$

4. The following table gives the number of volleyballs, soccer balls and footballs ordered from Pro Sports by three high schools.

	Volleyballs	Soccer balls	Footballs
South H.S.	35	12	24
Westside H.S.	26	11	21
Eastern H.S.	31	13	22

Volleyballs cost \$9 each, soccer balls \$21 each and footballs \$35 each.

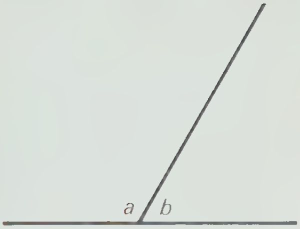
(a) Write the total order as a matrix.
(b) Write the cost of each as a column matrix.
(c) Use matrix multiplication to find the amount owing to Pro Sports by each school.
(d) What is Pro Sports' income from footballs?
(e) What is the total amount owing to Pro Sports by the three schools?

REVIEW AND PREVIEW TO CHAPTER 10

1. Supplementary Angles

State the supplement of

(a) 100° (b) 50° (c) 30° (d) 75° (e) 120°

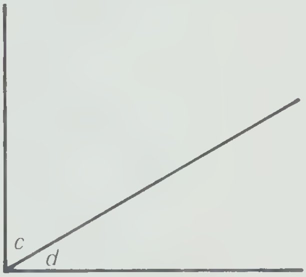


$a + b = 180$

2. Complementary Angles

State the complement of

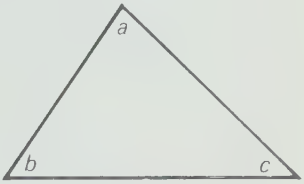
(a) 40° (b) 80° (c) 25° (d) 45° (e) 75°



$c + d = 90$

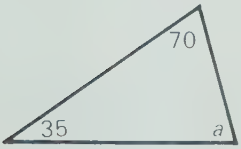
3. Triangle Angle Sum

Calculate the size of the indicated angles

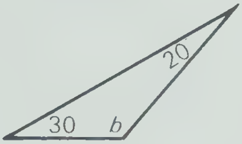


$a + b + c = 180$

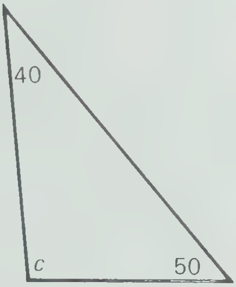
(a)



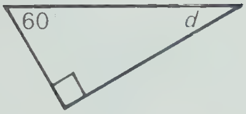
(b)



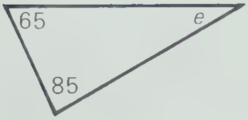
(c)



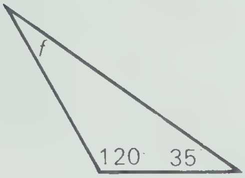
(d)



(e)

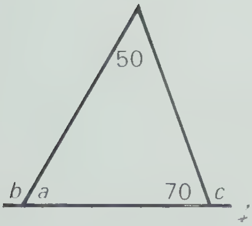


(f)

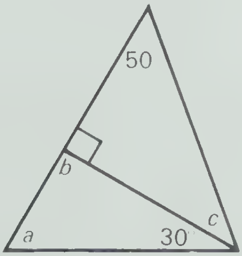


4. Calculate the size of the indicated angles

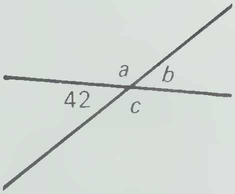
(a)



(b)



(c)



5. Simplify the following

(a) 5.2^2 (b) 3.75^2 (c) $(4\frac{1}{2})^2$ (d) 17^2 (e) 2.5^2

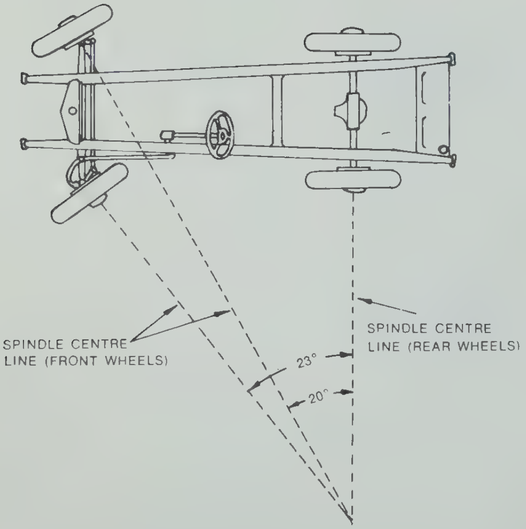
(f) $\sqrt{65.5}$ (g) $\sqrt{348}$ (h) $\sqrt{526}$ (i) $\sqrt{841}$ (j) $\sqrt{28}$

CHAPTER 10

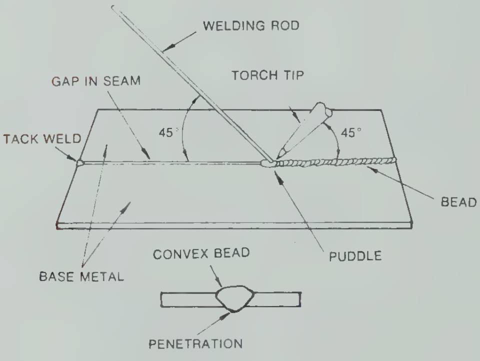
Numerical Applications of Geometry

A project or communication that involves a diagram requires a knowledge of **geometry**. Geometry, from Greek words meaning "land measurement", is used whenever you read a blue print, alter a pattern, make a measurement, or even give a friend directions to go from your school to your home. Interior decorators use geometry as they work from plans and sketches so that their finished product is organized and appealing. Geometry is the language of science; it is difficult to understand many scientific theories unless you can visualize geometrically.

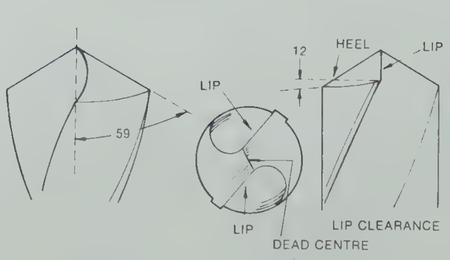
Angles at Work


SPINDLE CENTRE LINE (FRONT WHEELS)
SPINDLE CENTRE LINE (REAR WHEELS)
23°
20°

Toe-Out on Turns



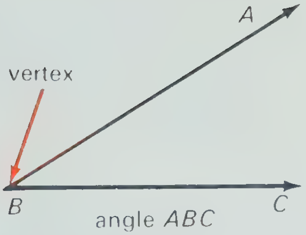
Welding a Butt Joint in the Flat Position



Drill Point

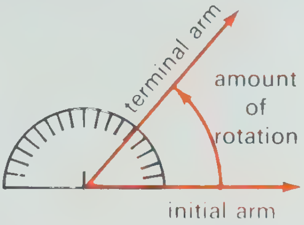
10.1 ANGLES

The **measure of an angle** is the amount of rotation of a line about the vertex from the initial arm to the terminal arm. Angles can be measured in revolutions, degrees, minutes and seconds.



A **protractor** measures angles to the nearest degree.

One revolution = 360° (degrees)
1° = 60′ (minutes)
1′ = 60″ (seconds)



EXAMPLE 1. *Calculate the number of degrees through which (a) the minute hand, (b) the hour hand of a clock travels in 22 min.*

Solution

(a) In 1 minute the minute hand rotates $\frac{1}{60} \times 360° = 6°$
In 22 minutes the minute hand rotates $22 \times 6° = 132°$

(b) In 60 minutes the hour hand rotates $\frac{1}{12} \times 360° = 30°$
In 22 minutes the hour hand rotates $\frac{22}{60} \times 30° = 11°$

If it is 21:00 now, what time will it be 1625 h from now?

EXERCISE 10-1

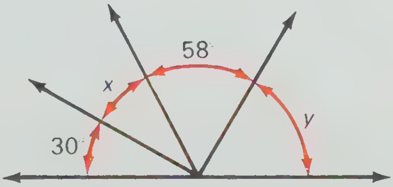
B **1.** If one revolution (1 r) is equal to 360°, find the number of degrees in:

(a) $\frac{1}{2}$ r (b) $\frac{2}{3}$ r (c) $\frac{1}{8}$ r (d) 5.7 r
(e) 3.5 r (f) 4.625 r (g) 6.28 r (h) 0.027 r

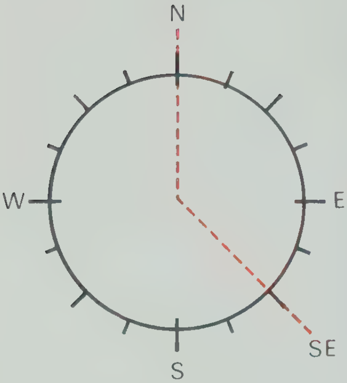
2. Calculate the number of degrees through which
(i) the minute hand, (ii) the hour hand of a clock travels in
(a) 16 min (b) 44 min (c) 30 min (d) 3 h

3. From the given diagram, find the measure of the angle marked x if the angle y is equal to:

(a) 35° (b) 65° (c) 27°
(d) 55° (e) 16°



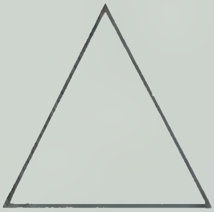
4. A shaft is rotating at 30°/s. Find
(a) the number of r/s
(b) the number of r/min

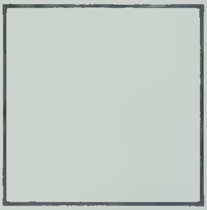


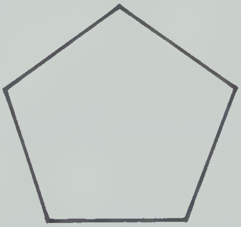
5. A **compass rose** is used on navigation charts, with directions measured clockwise from the north line which is taken as 0°. Give the number of degrees between SE and the direction
(a) S (b) W (c) N (d) SE (e) SSE (f) NNE (g) NE (h) ESE

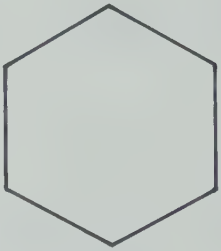
10.2 ANGLES OF POLYGONS

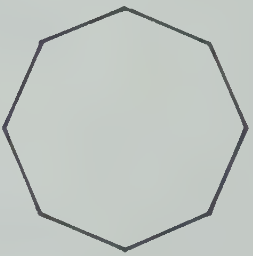
A polygon is a closed plane figure consisting of three or more straight sides. The polygon is **regular** if its sides are equal and its angles are equal. The equilateral triangle, square, regular pentagon, regular hexagon, and regular octagon are the more common regular polygons.


Equilateral triangle


Square

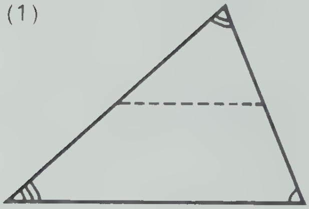

Regular Pentagon


Regular Hexagon

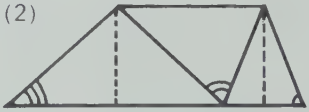

Regular Octagon

The **sum of the angles** of a triangle can be found by direct measurement or by paper folding. Take a cut-out triangle and draw a line joining the midpoints of two of the sides (1). Fold along this line (2), then fold again along the dotted lines to have figure (3). This demonstrates that the sum of the angles of a triangle is a straight angle.

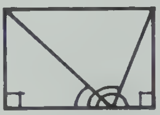
(1)



(2)



(3)

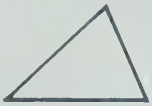
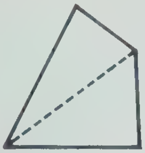
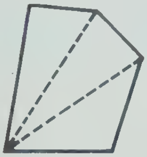
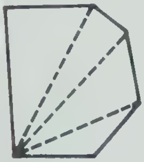
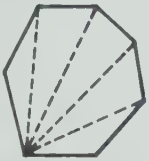


The sum of the angles of a triangle is 180°

This relationship can be used to find the sum of the angles of any polygon as in the exercise which follows.

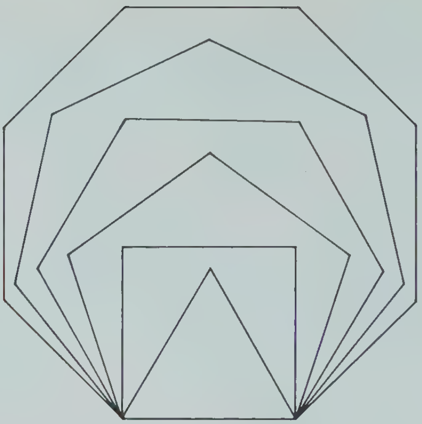
EXERCISE 10-2

1. Complete the following table in your notebook.

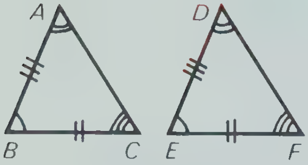
Polygon	Number of sides	Number of triangles	Angle sum of the polygon	Number of angles in the polygon	Angle of the regular polygon
 triangle	3	1	180°	3	60°
 quadrilateral	4	2	360°	4	90°
 pentagon	5				
 hexagon	6				
 septagon	7				
octagon	8				
decagon	10				
dodecagon	20				
n-gon	*n*				

Time Zones
What time is it in Vancouver when it is 10:00 in Halifax?

2.



Using a convenient scale, draw the above figure using ruler, compasses and protractor.



Congruent Triangles

$\triangle ABC \cong \triangle DEF$
$\cong$ means "is congruent to"

10.3 CONGRUENCE

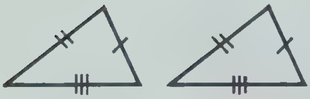
Two figures are **congruent** if they have the same shape and the same size. This means that the corresponding angles are equal, and the corresponding sides have the same length.

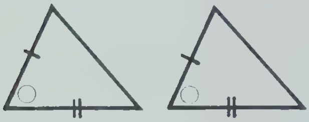
Two triangles are congruent if:

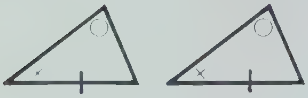
SSS the three sides of one triangle are respectively equal to the three sides of another triangle.

SAS two sides and the contained angle of one triangle are respectively equal to two sides and the contained angle of another triangle.

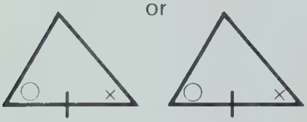
ASA two angles and one side of one triangle are respectively equal to two angles and one side of another triangle.







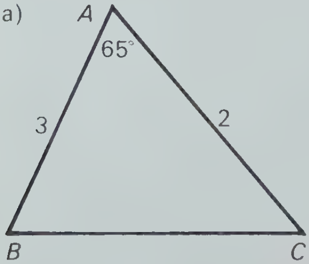
or

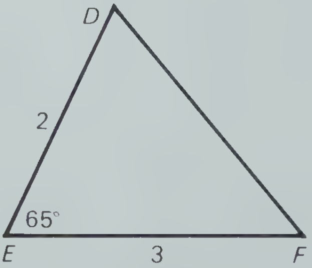


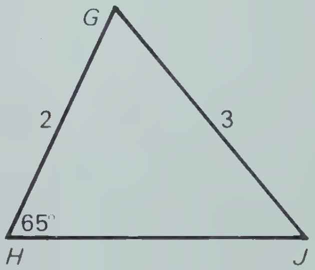
EXERCISE 10-3

B 1. Which of the following triangles is not congruent to the other two?

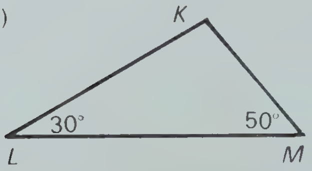
(a)

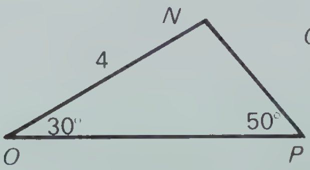


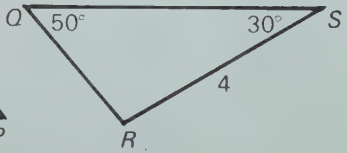




(b)

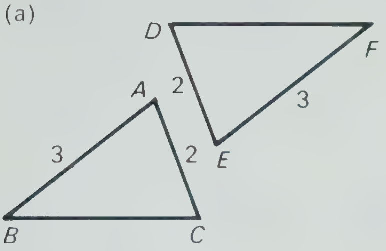




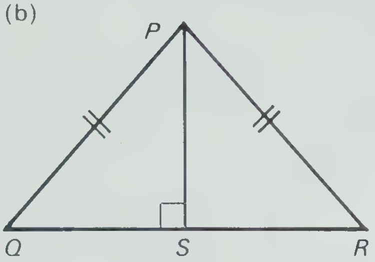


2. State what additional information is required so that the following pairs of triangles are congruent.

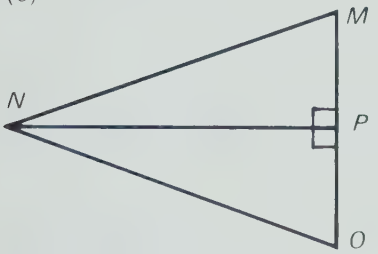
(a)



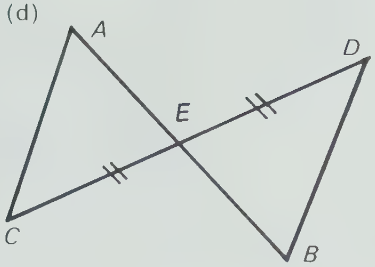
(b)



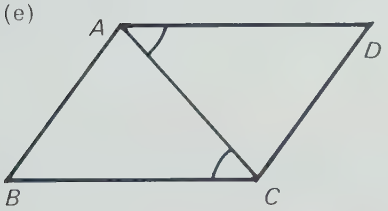
(c)



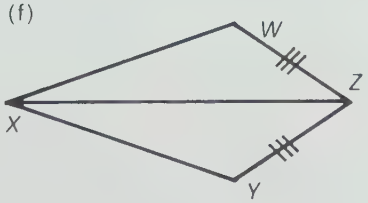
(d)



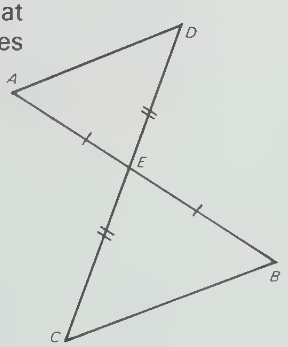
(e)



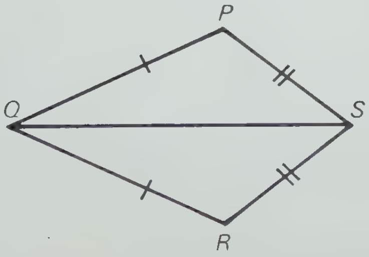
(f)



3. Line segments AB and CD intersect at E so that $AE = EB$ and $ED = EC$. Why are the two triangles congruent?

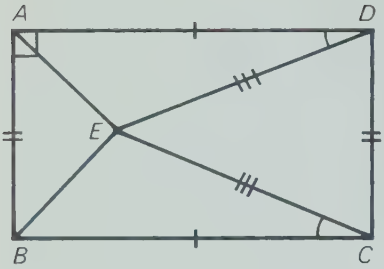


4. In quadrilateral $PQRS$, $SP = SR$ and $QP = QR$. Why are the two triangles to be congruent?

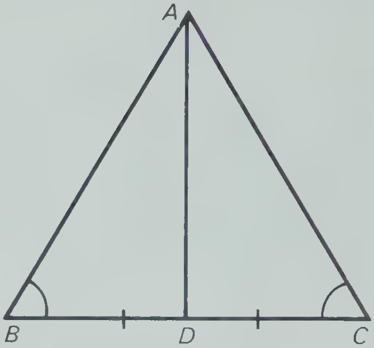


5. State the pairs of congruent triangles:

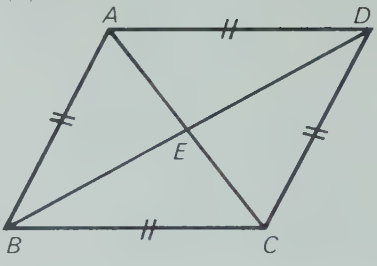
(a)



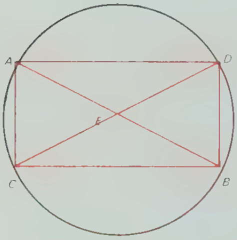
(b)



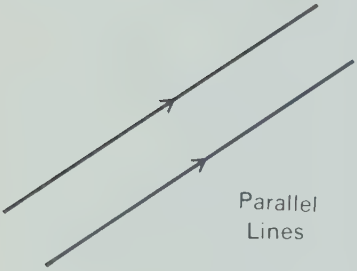
(c)



6. AB and CD are two diameters of a circle whose centre is E. If $\angle EDB = 65°$, calculate the measure of $\angle DBE$, $\angle DEB$, $\angle EAC$, and $\angle ACE$. Why are the triangles AEC and DEB congruent?



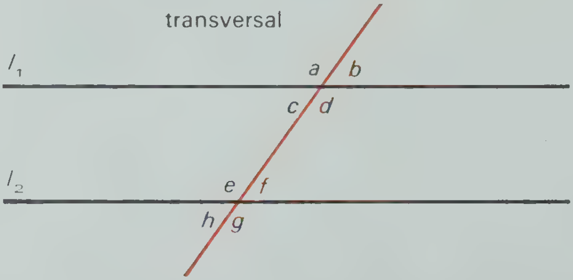
10.4 PARALLEL LINES

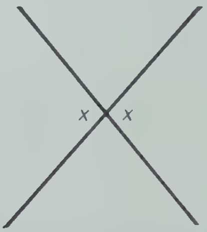


Parallel Lines

Two straight lines are parallel if they lie in the same plane and do not intersect. The edges of a straight sidewalk, railway tracks, and the edges of a ruler are examples of parallelism. List ten examples of parallelism in the classroom.

When a transversal intersects two parallel lines, four pairs of opposite angles are formed.





opposite angles

In Figure 1, the transversal intersects the parallel lines l_1 and l_2 to form four pairs of opposite angles. How are the angles forming each pair related?

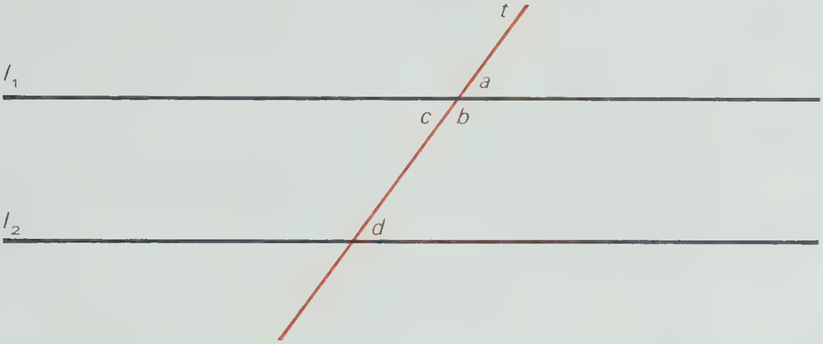
The angles marked b and f are **corresponding angles**; the angles marked d and f are called **alternate angles**. Angles c and f are **interior angles on the same side of the transversal**.

Name (a) three other pairs of corresponding angles;
(b) another pair of alternate angles.

INVESTIGATION 10-4

To find the relationship between the angles formed when a transversal intersects parallel lines:

1. (a) Draw a pair of parallel lines l_1 and l_2 using the edges of a ruler. Draw any transversal, *t*, to the parallel lines l_1 and l_2. Label your diagram as in Figure 2.



(b) Measure and record the size of each angle. Repeat this procedure using four more transversals inclined at any angle. Complete the following table in your workbook.

Case	$\angle a$	$\angle b$	$\angle c$	$\angle d$	$\angle b + \angle d$
1					
2					
3					
4					
5					

2. Use your observations recorded in the table of question 1 to answer the following:

(a) Find a pair of corresponding angles and tell how they are related.

(b) Find a pair of alternate angles and tell how they are related.

(c) Name a pair of interior angles and tell how they are related.

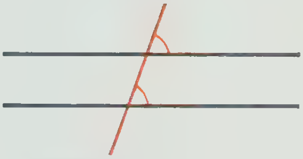
Parallel Lines

If a transversal meets two parallel lines, then

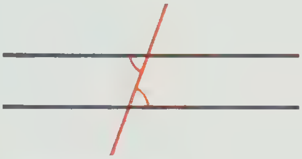
(a) the alternate angles are equal;

(b) the corresponding angles are equal;

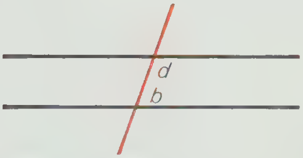
(c) the interior angles on the same side of the transversal are supplementary.



corresponding angles



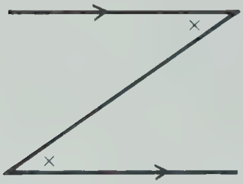
alternate angles



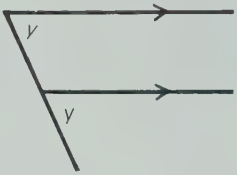
interior angles on the same side of a transversal

The parallel lines properties can be remembered by the *Z*, *F*, and ⊏, patterns formed.

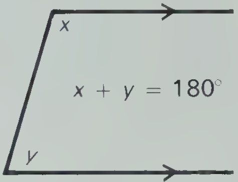
Alternate Angles



Corresponding Angles



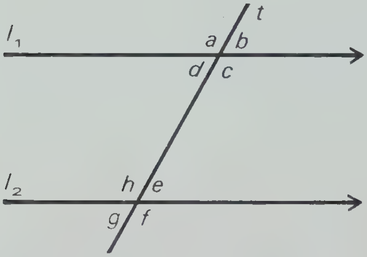
Interior Angles



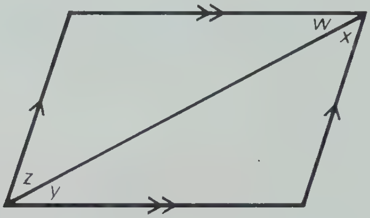
EXERCISE 10-4

1. From each of the following diagrams, make a list of:
   - (i) alternate angles
   - (ii) corresponding angles
   - (iii) interior angles on the same side of the transversal.

(a)

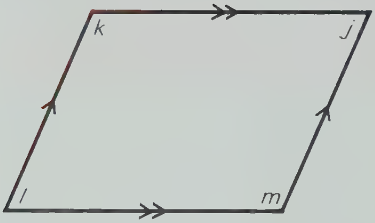


(b)

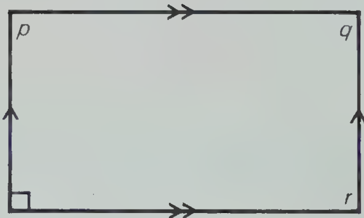


Time Zones
What time is it in Rome when it is 14:00 in Wininipeg?

(c)

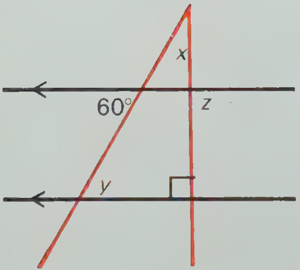


(d)

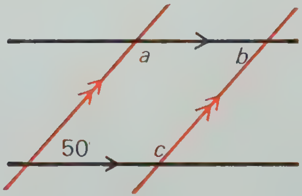


2. Calculate the measures of the indicated angles.

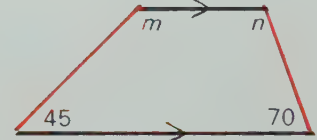
(a)



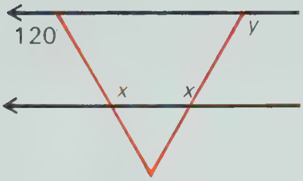
(b)



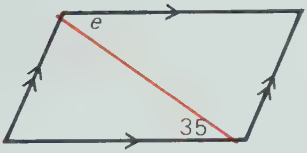
(c)



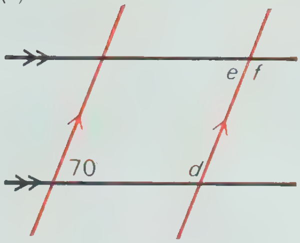
(d)



(e)

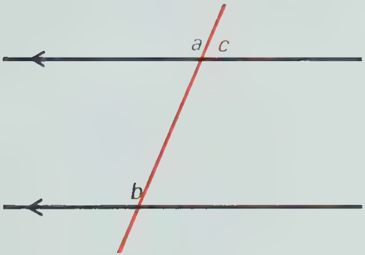


(f)

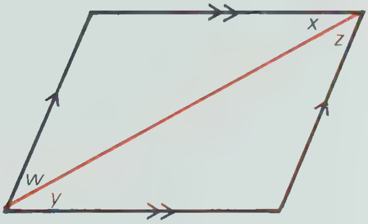


3. State pairs of equal angles.

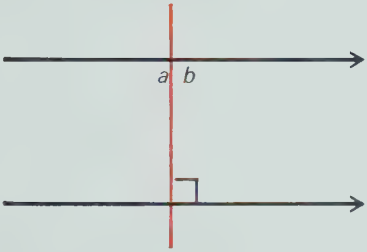
(a)



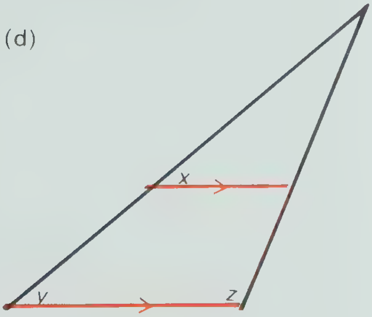
(b)



(c)



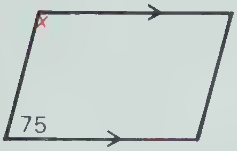
(d)



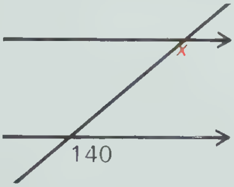
Insert signs to make
3 ▨ 2 ▨ 5 ▨ 7 = 4
▨ true statement.

4. Calculate the value of *X* in the following:

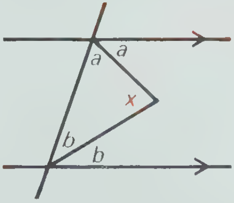
(a)



(b)

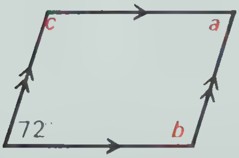


(c)

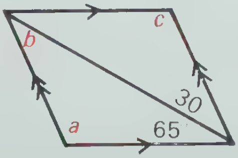


5. In each parallelogram, calculate the values of *a*, *b* and *c*.

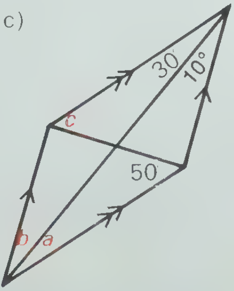
(a)



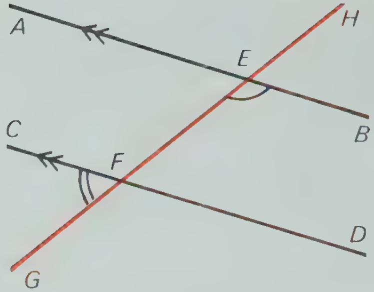
(b)



(c)

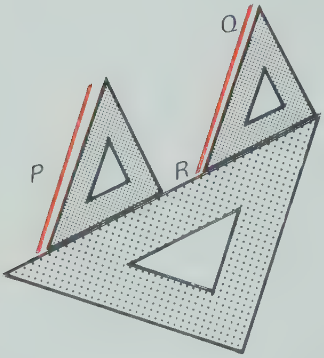


6.



In the diagram at left, $AB \parallel CD$ and the transversal HG intersects AB and CD at E and F respectively.

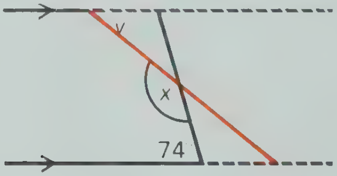
Copy the diagram and calculate the required value of $\angle CFG$, if $\angle BEF$ is equal to:
(a) 125° (b) 126° (c) 127° (d) 128° (e) 129° (f) 130° (g) $X°$



7. Triangles can be used to draw parallel lines. In Figure 3, RQ is a given line and a line through P parallel to RQ is required. First align a triangle with the given line RQ using another triangle for a base as shown. Move the first triangle to position through the given point P and draw the required line.

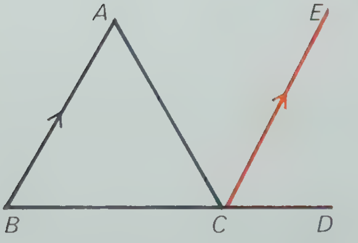
(a) Explain why this method is justified.
(b) Draw your own transversal, and measure the corresponding angles formed to determine whether the lines are indeed parallel.
(c) Using the triangles, draw another set of parallel lines and measure the corresponding angles formed to determine whether the lines are parallel in this case as well.

8.



In the diagram at left, calculate the measure of the angle marked x when y is equal to: (a) 34° (b) 35° (c) 36° (d) 38° (e) 40°

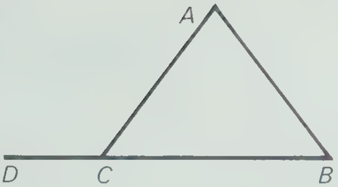
9. Exterior Angle of a Triangle



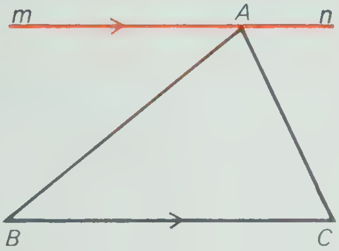
(a) $\angle ACD$ is an **exterior angle** of $\triangle ABC$ and $CE \parallel BA$. State the alternate angles that are equal and the corresponding angles that are equal. Show that $\angle ABC + \angle BAC = \angle ACD$. **The exterior angle of a triangle is equal to the sum of the two interior opposite angles.**

(b) Repeat part (a) of this question for the case where $\angle ABC$ is obtuse.

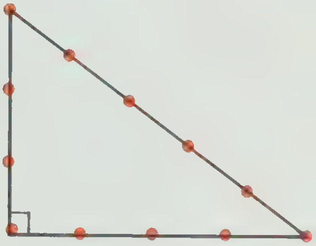
10. The figure at right represents part of an A-frame structure. Calculate the angle at A if $\angle ABC = \angle ACB$ and $\angle DCA$ is equal to
(a) 120° (b) 100° (c) 140° (d) 150°



11. $\triangle ABC$ is any triangle, and the line through A is parallel to BC.
(a) Name alternate angles that are equal.
(b) Use your answer to show that the sum of the angles of a triangle is 180°.



10.5 THE RIGHT TRIANGLE AND PYTHAGORAS

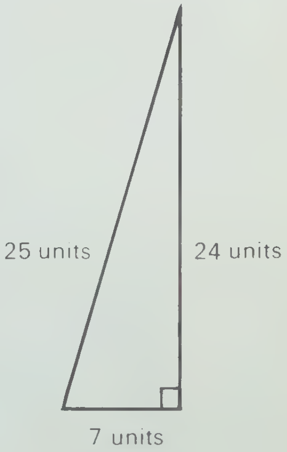


A triangle containing a right angle, 90°, is called a right triangle. The side opposite the right angle is called the hypotenuse. In order to measure and organize their lands, the Egyptians needed to use and to construct a right angle. To do this, men known as "rope stretchers" took a rope and tied 12 equally spaced knots on the rope. Then they staked the rope on the ground, giving a right triangle with sides in the ratio 3:4:5. Although they knew that a rope with 12 knots worked, they did not search for other ways of knotting a rope to produce a right triangle. The Egyptians were satisfied with this method and it never occurred to them to search for other methods or to ask why the relationship of sides in the ratio 3:4:5 gave a right triangle. It was not until more than 2000 years later at about the time of the Golden Age of Greece that the answer to the question "Why?" was found. Credit for finding the answer is given to a mathematician named Pythagoras.

INVESTIGATION 10-5

1. (a) Using ruler and compasses only, construct a triangle with sides 3 cm, 4 cm and 5 cm.
(b) Measure and record the size of each angle to the nearest degree.
(c) Is the triangle a right triangle?

2. The triangle in question 1(a) is sometimes referred to as a "3:4:5 triangle." Construct a 6:8:10 triangle using 1 cm as a unit.
(a) Is the 6:8:10 triangle also a right triangle?
(b) Suggest possible dimensions for three other right triangles.



7:24:25

3. (a) Using suitable units and scale, construct the following triangles and determine which are right triangles
(i) 9:12:15 (ii) 2:3:4 (iii) 5:12:13 (iv) 8:15:17
(b) Which of the following triangles have sides which are multiples of the sides of the right triangles shown above?
(i) 10:24:26 (ii) 16:30:34 (iii) 12:16:20 (iv) 15:20:25
(v) 8:14:15

Addition:
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ned
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dane

4. (a) Complete the following table by constructing right triangles and recording the lengths of the sides:

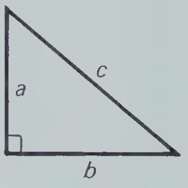
Squares of the Sides of a Right Triangle		
Shortest Side Squared	Second Side Squared	Hypotenuse Squared
$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$8^2 = 64$	$10^2 = 100$
9^2		
12^2		
15^2		
5^2		
8^2		
7^2		

3:4:5
5:12:13
7:24:25
9:40:▨
11:▨:▨
13:▨:▨
. . . . .

(b) In the first line of the table in (a), $3^2 = 9$, $4^2 = 16$, and $5^2 = 25$. How are the numbers 9, 16, and 25 related? Test this relationship for all the entries in the table.

(c) From your observations above, tell how the squares of the sides of a right triangle are related.

(d) If a, b, and c represent the lengths of the sides of a right triangle where c is the hypotenuse, state the relationship between a, b, and c.



In any right triangle

$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse and a and b are the lengths of the other two sides.

EXERCISE 10-5

A **1.** Use the formula $a^2 + b^2 = c^2$ to determine whether the following triples represent the lengths of the sides of a right triangle.

(a) (30,40,50)
(b) $(1,2,\sqrt{5})$
(c) $(2,4,2\sqrt{5})$
(d) (1,1,2)
(e) $(3,4,\sqrt{7})$
(f) $(1,\sqrt{3},2)$
(g) $(1,\frac{3}{4},\frac{5}{4})$
(h) $(1,1,\sqrt{2})$

2. If a, b, and c are the lengths of the sides of a right triangle, where c is the hypotenuse, then complete the following tables.

(a)

a	b	c
4	3	
4	4	
12	5	
6	8	

(b)

a	b	c
7		25
	8	17
1		2
	1	$\sqrt{2}$

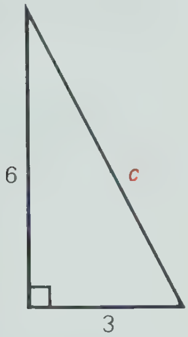
Why is it not necessary to state the units used for the measures in questions 1 and 2?

3. In each case indicate which two whole numbers are nearest the given number.

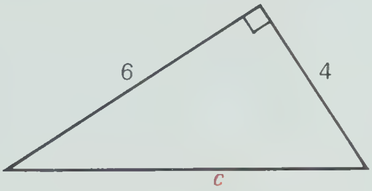
(a) If c^2 is 18, then c is between ________ and ________.
(b) If c^2 is 61, then c is between ________ and ________.
(c) If a^2 is 91, then a is between ________ and ________.
(d) If b^2 is 157, then b is between ________ and ________.
(e) If c^2 is 600, then c is between ________ and ________.
(f) If a^2 is 175, then a is between ________ and ________.

B **4.** Use the Pythagorean Theorem to find c^2 for each of the following right triangles. Then use the slide rule (or calculator) to find c.

(a)



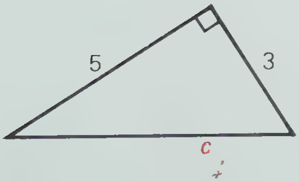
(b)



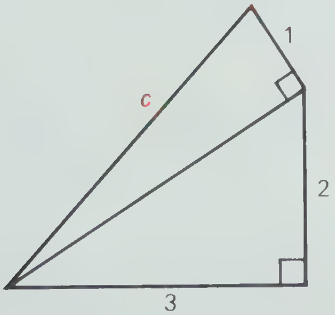
Starting with "can," and changing one letter at a time, can you reach the word "pit" in three changes?

can
1.
2.
3. pit

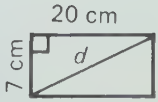
(c)

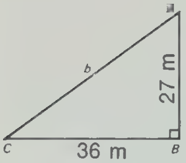


(d)

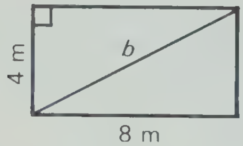


5. A rectangular piece of steel plate is 20 cm long and 7 cm wide. Calculate the length of the diagonal, to 3 significant figures.

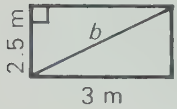




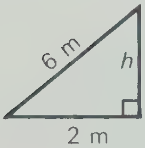
6. How many metres of cable will be required to reach from a point *A*, 27 m up on a tower, to a point *C* on the ground 36 m from the base *B* of the pole?



7. How long a pipe is needed to make a diagonal brace for a gate 4 m high and 8 m long?

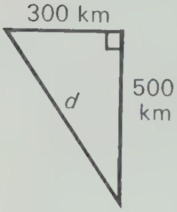


8. How long a board is needed to make a diagonal brace for a garage door 3 m by 2.5 m?



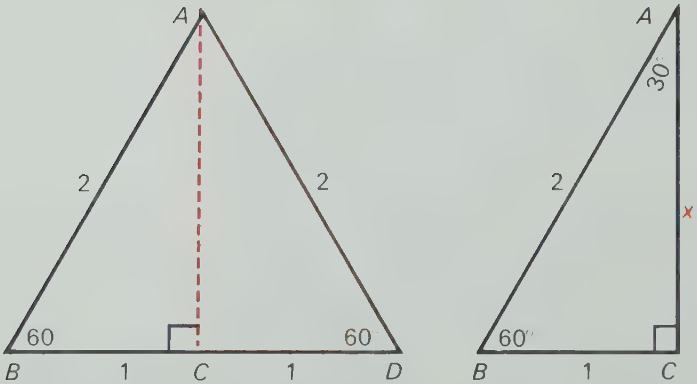
9. (a) How high up on a building will a 6 m ladder reach if its lower end must be 2 m from the building?
(b) How far down the wall will the top of the ladder slide if the lower end moves another metre away from the wall?

Make a diagram and express both answers to two decimal places.

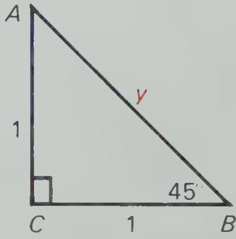


10. Leaving Halifax, the aircraft carrier *Birdsnest* sails 300 km east, and then sails 500 km south. How far is the ship from Halifax? Make a diagram and express your answer to the nearest kilometre.

11. Two special triangles:



30°–60°–90° Triangle



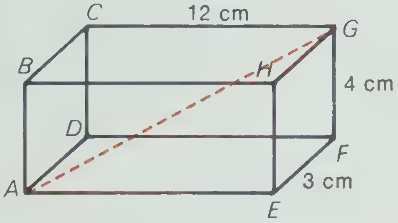
45°–45°–90° Triangle

(a) The 30°–60°–90° triangle is found by taking an equilateral triangle, *ABD*, with each side 2 units, and drawing the perpendicular

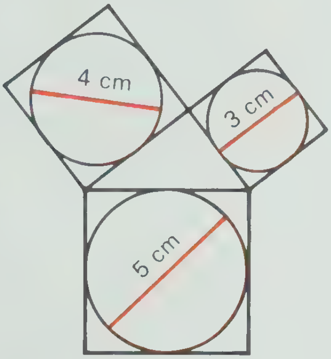
AC to form the required triangle, ABC. In $\triangle ABC$, $AB = 2$ and $BC = 1$. Make a diagram and calculate the length of AC, and express your answer as a Pythagorean triple, $(1,x,2)$. Repeat this process beginning with the equilateral triangle, ABD, having sides equal to 10 units.

(b) The 45°–45°–90° triangle is formed by taking an isosceles triangle, ABC, with $\angle C = 90°$ and $AC = BC = 1$ unit. Make a diagram, calculate the length of AB, and express your answer as a Pythagorean triple, $(1,1,y)$. Repeat this process beginning with the isosceles triangle, ABC, having the equal sides AC and BC equal to 5 units.

12. Find the length of the diagonal, AG, of the rectangular prism pictured at the right. Make diagrams to show the position of the diagonal in the face $ABCD$ and in the face $BCGH$.



13. The given diagram consists of three pipes with diameters 3, 4, and 5 cm.

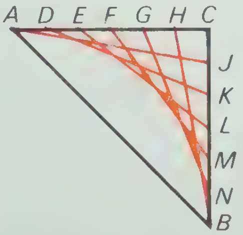


(a) Prove that the same amount of water will flow through the 3 and 4 cm pipes as will flow through the 5 cm pipe provided the pressure is the same in all the pipes.

(b) Find the size of pipe that will carry the same amount of water as a 12 cm pipe and a 5 cm pipe.

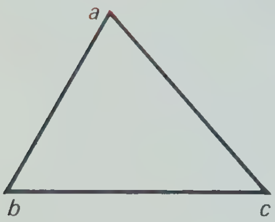
14. What is the longest rod, to the nearest 0.1 cm that can be placed inside a trunk 30 cm wide, 30 cm deep, and 50 cm long?

15. Draw $\triangle ABC$, with $\angle ACB = 90°$ and $AC = BC = 6$ cm. Divide AC in 1 cm intervals at D, E, F, G, and H. Then divide CB in 1 cm intervals at J, K, L, M, and N. Join and calculate the lengths of AJ, DK, EL, FM, GN, and HB. Give your answer to three significant figures by using your slide rule or the calculator. Check by measurement.

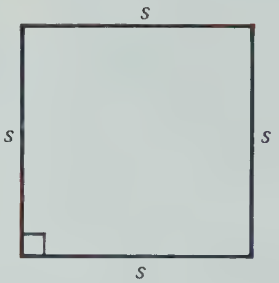


10.6 PERIMETER, AREA, AND VOLUME FORMULAS

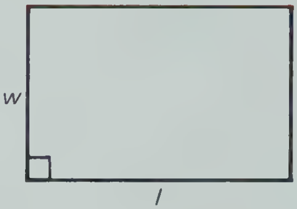
Perimeter



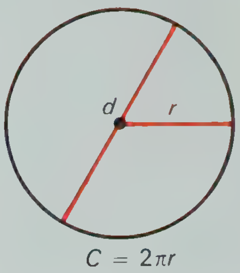
$P = a + b + c$



$P = 4s$



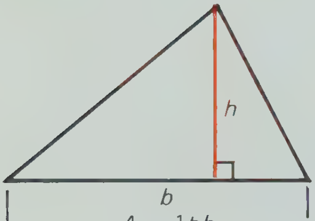
$P = 2(l + w)$



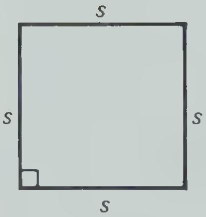
$C = 2\pi r$

$C = \pi d$

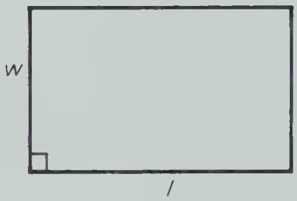
Area



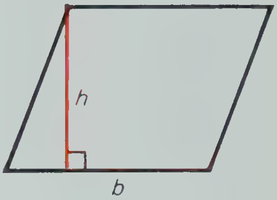
$A = \frac{1}{2}bh$



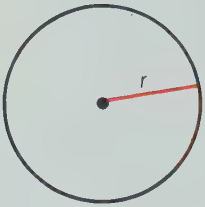
$A = s^2$



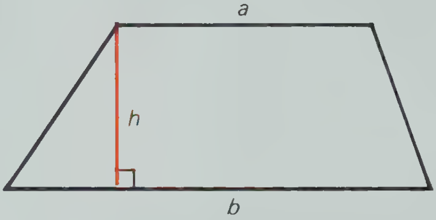
$A = lw$



$A = bh$

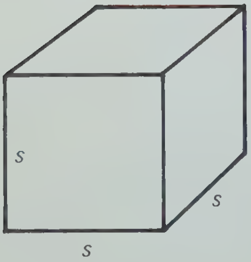


$A = \pi r^2$

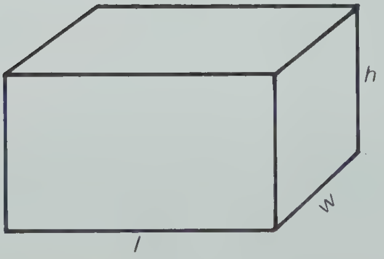


area of a trapezoid: $A = \frac{1}{2}(a + b)h$

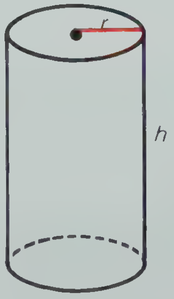
Volume



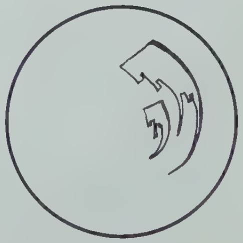
$V = s^3$



$V = lwh$



$V = \pi r^2 h$

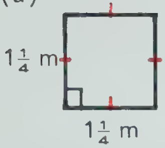


$V = \frac{4}{3}\pi r^3$

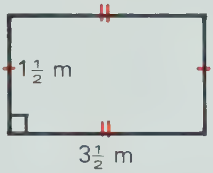
EXERCISE 10-6

1. Calculate the perimeters of the following:

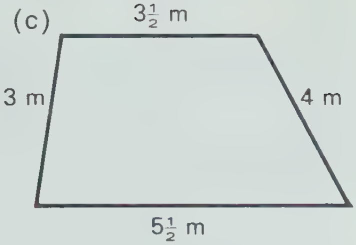
(a)



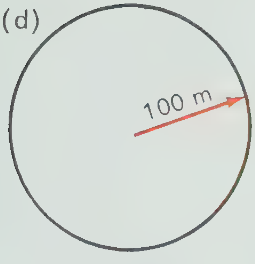
(b)



(c)



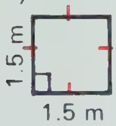
(d)



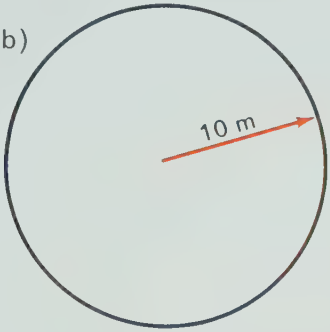
$\pi \doteq 3.14$

2. Calculate the area of the following:

(a)

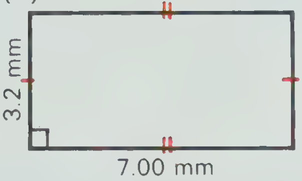


(b)

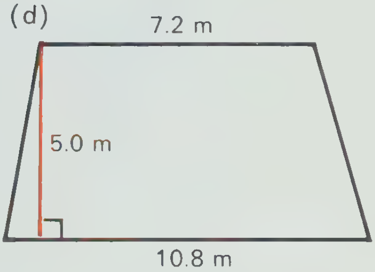


$\pi \doteq 3.14$

(c)

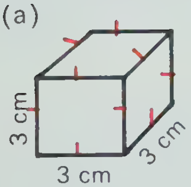


(d)

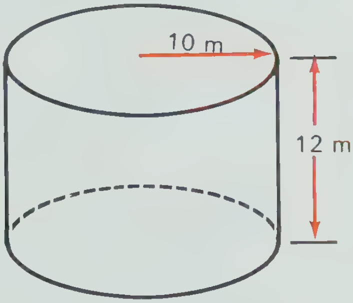


3. Calculate the volume of the following:

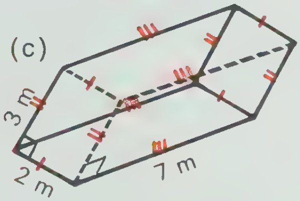
(a)



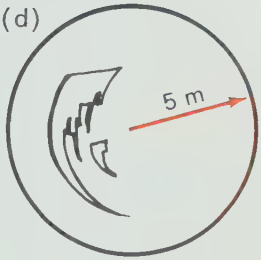
(b)



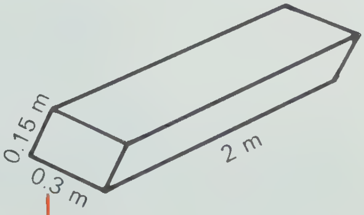
(c)



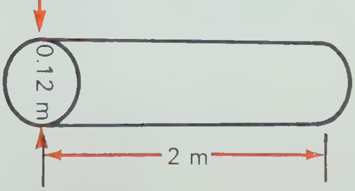
(d)



4. Calculate the area of the sheetmetal required to make the rectangular duct at right (no allowance for seam).



5. Calculate the area of sheetmetal required to make the sound duct (allow 0.01 m for seam).

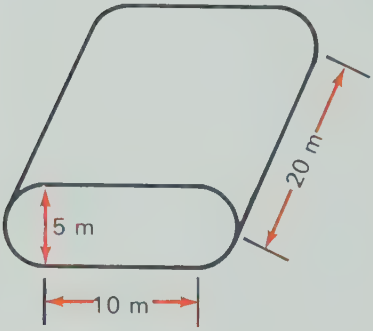


6. Calculate the minimum amount of cove moulding required to trim the ceiling of a room 3.2 m by 4.5 m. Allow for 0.2 m of material.

7. (a) Find the length of metal moulding required to trim a square table, with sides 1.12 m each.
(b) Calculate the cost of the moulding at 25¢/m.

8. Find the length of a steel band required to go around a circular tank if the outside diameter of the tank is 3 m and 0.25 m is required for fastening.

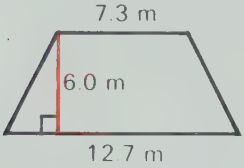
9. Find the length of strap iron required to go around the given figure. Allow 30 cm for fastening.



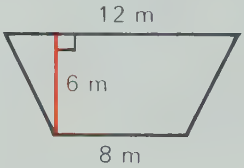
($P = \pi d + 2l$)

10. Find the area of the following trapezoids.

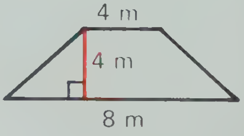
(a)



(b)

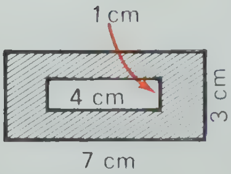


(c)

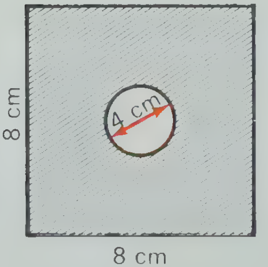


11. Find the area of the following shapes.

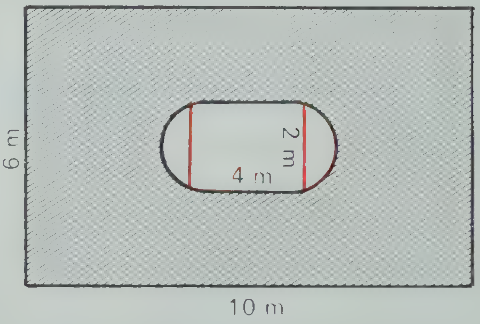
(a)



(b)

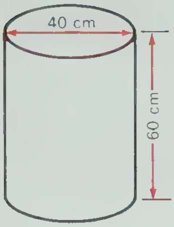


(c)

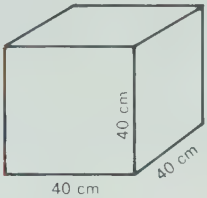


12. Calculate the volume of the following.

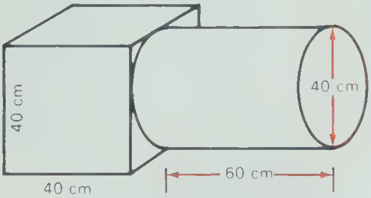
(a)



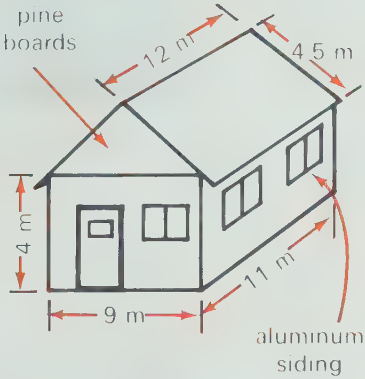
(b)



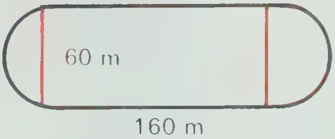
(c)



13. (a) Calculate the total roof area. What is the least number of squares of roof shingles that you should order (1 square = 10 m²)?
(b) Calculate the total area of aluminum siding required for the walls. (Ignore windows and doors when calculating.)



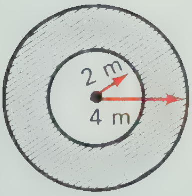
14. (a) Calculate the number of square metres of astroturf required to cover the interior of the field and track illustrated at right.



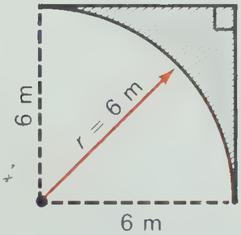
(b) What is the cost to the nearest $1000 of covering the field at $25 per square metre?

15. Find the area of the following:

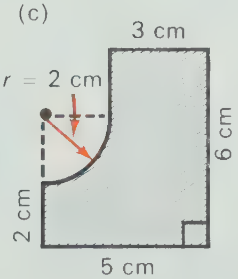
(a)

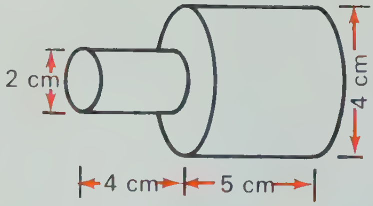


(b)



(c)



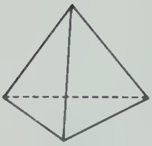


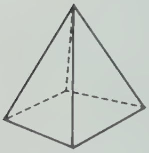
16. (a) Calculate the number of cubic centimetres of material in the given pin.
(b) If the material has a density of 1200 kg/m^3 calculate the mass of 100 pins to the nearest kilogram.

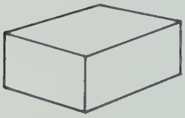
17. What is the volume of a spehrical gas tank 10 m in diameter?

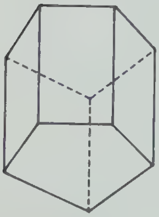
10.7 PRISMS, PYRAMIDS, AND CONES

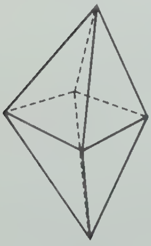
A **polyhedron** is a three-dimensional figure bounded by polygons, called faces. Prisms and pyramids are examples of polyhedrons.

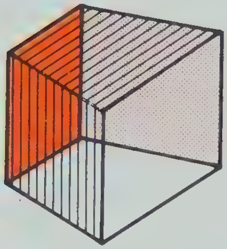

tetrahedron

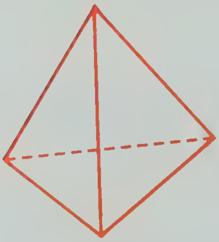

pentahedron

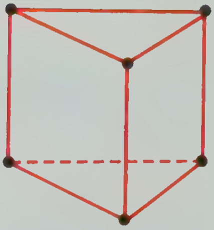

hexahedron


heptahedron


octahedron


faces

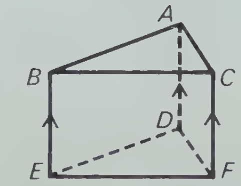

edges


vertices

INVESTIGATION 10-7

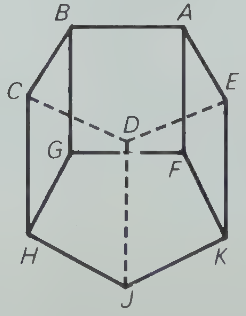
1. **Prisms:** A prism is a polyhedron with one pair of opposite faces that are congruent polygons (bases) and sides that are parallelograms.

(i)



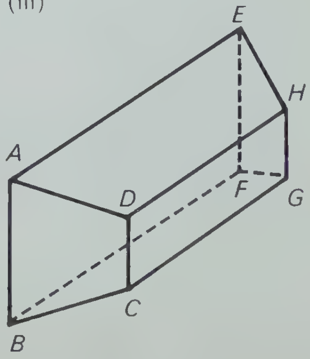
$\triangle ABC \cong DEF$

(ii)



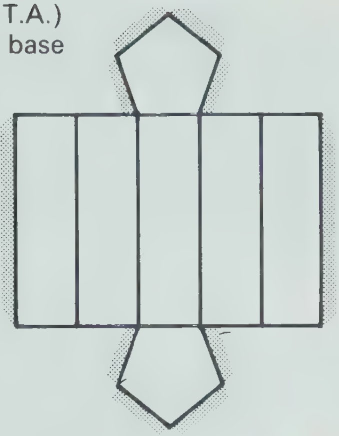
$ABCDE \cong FGHJK$

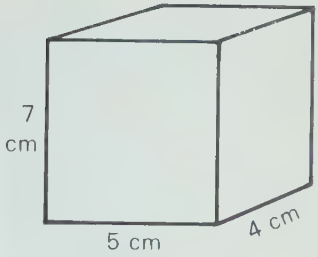
(iii)



$ABCD \cong EFGH$

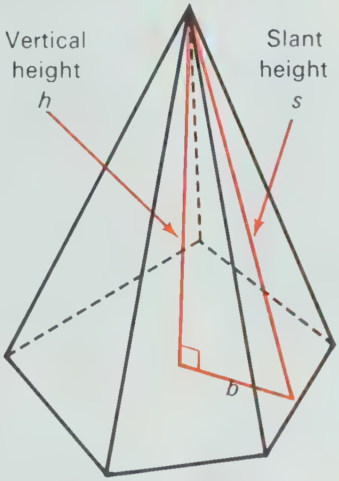
(a) Find the lateral area (L.A.), total area (T.A.) and volume of a prism with a rectangular base 4 cm by 5 cm, and 7 cm in height.
(b) Construct the prism in part (a).
(c) Construct a prism 15 cm high with pentagons having sides 5 cm as bases.





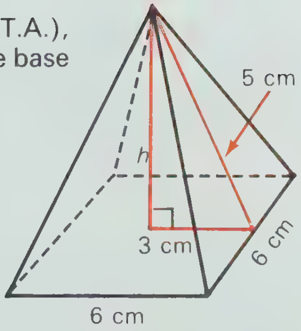
Lateral Area is the area of the sides.
Total Area is the lateral area plus the area of the base(s).

2. **Pyramids:** A pyramid has a polygon as base and the laterial faces are triangles with a common vertex not contained in the base. A **regular** pyramid has a regular polygon for its base and the laterial faces are congruent isosceles triangles. The vertical height and the slant height of a pyramid are not the same.



$$s^2 = h^2 + b^2$$

(a) Find the lateral area (L.A.), total area, (T.A.), and the volume of the pyramid with square base 6 cm by 6 cm and slant height 5 cm.



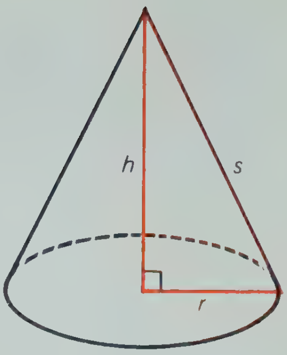
(b) Construct the pyramid in part (a).
(c) Construct a pyramid with slant height 6 cm having a regular hexagon with sides 2 cm as base.

$h^2 + 3^2 = 5^2$
$V = \frac{1}{3}(\text{area of base}) \times (\text{height})$

3. **Cones:** A cone is a solid with a circular base and sides which taper to a point called the **vertex**. If the line through the vertex, perpendicular to the base, passes through the centre of the base then the figure is called a **right circular cone.** The slant height, s, the vertical height, h, and the radius of the base, r, can form the sides of a right triangle, and

$$s^2 = h^2 + r^2$$

(a) Find the lateral area (L.A.) and the volume (V) of a right circular cone with radius of the base 5 cm and slant height 13 cm.
(b) Construct a cone with radius of the base 3 cm and slant height 5 cm. (Hint: find the circumference of the base.)

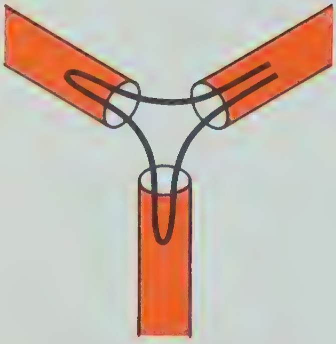


$s^2 = h^2 + r^2$
L.A. $= \pi rs$
$V = \frac{1}{3}$(area of base) $\times$ (height)
$= \frac{1}{3}\pi r^2 h$

4. If V is the number of vertices, F is the number of faces and E is the number of edges, calculate the value of $V + F - E$ for the polyhedrons constructed in questions 1 and 2. Observe the $V + F - E$ column in the table and state a probable conclusion. Test this conclusion by constructing another prism and another pyramid.

Figure	V	F	E	V + F − E

10.8 CONSTRUCTING THE REGULAR POLYHEDRONS



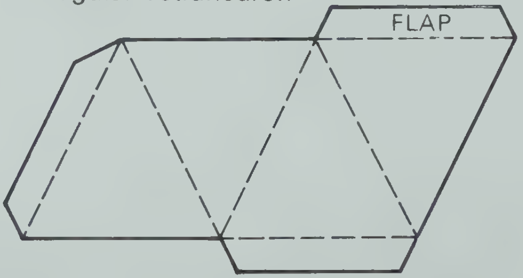
Paper clips holding straws at vertex

A regular polyhedron is one whose faces are congruent regular polygons. Although the faces could be equilateral triangles, squares, regular pentagons, hexagons, or octagons, it is only possible to construct five regular polyhedrons.

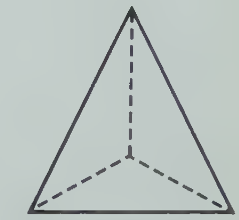
INVESTIGATION 10-8

Construct the following polyhedrons.

1. Regular Tetrahedron

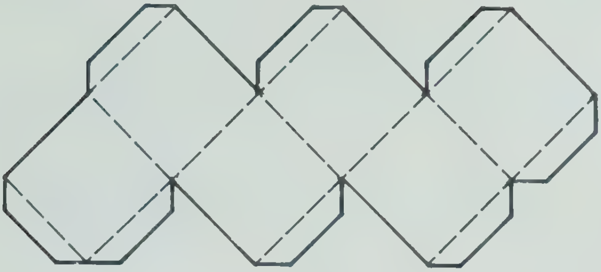


net

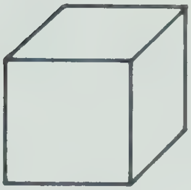


solid

2. Regular Pentahedron (Cube)

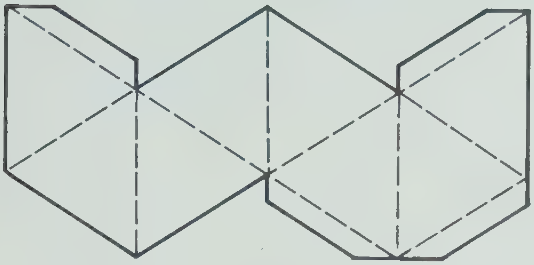


net

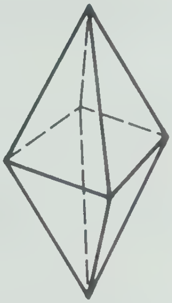


solid

3. Regular Octahedron



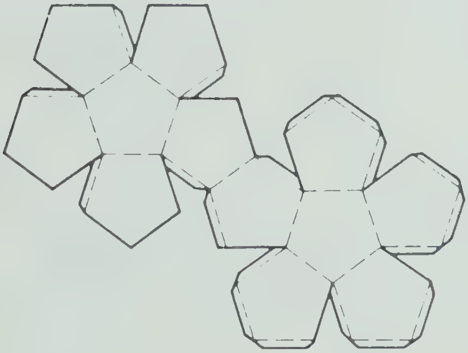
net

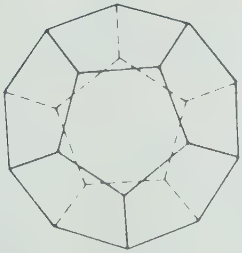


solid

Photograph supplied by the Ontario Ministry of Industry and Tourism.

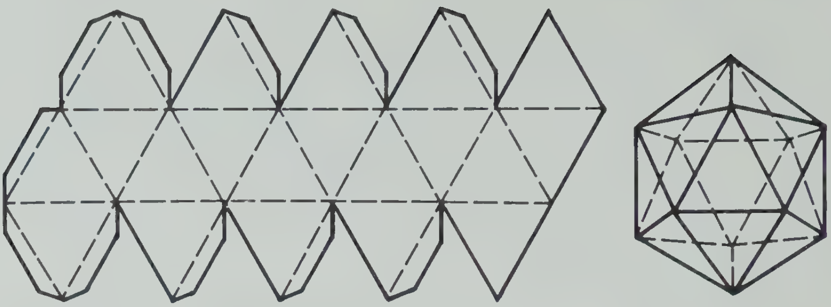
4. Regular Dodecahedron



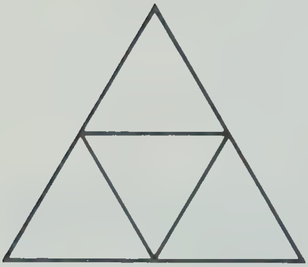


The nets show flaps for gluing when the models are made from bristol board. The models can also be built using straws and paper clips, pipe cleaners, or coat hangers.

5. Regular Icosahedron


net

solid



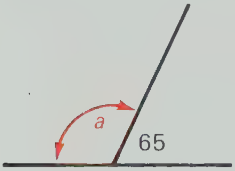
6. The preceding questions suggested nets for each of the regular polyhedrons. Another net for the tetrahedron is shown here. Can you find alternate nets for the other regular polyhedrons?

7. Find the value of $V + F - E$ for the regular polyhedrons you have constructed. What do you conclude from your results?

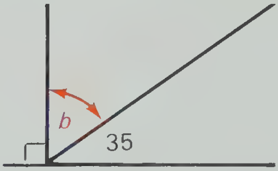
REVIEW EXERCISE

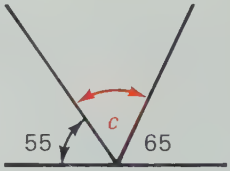
1. State the measure of the indicated angles:

(a)



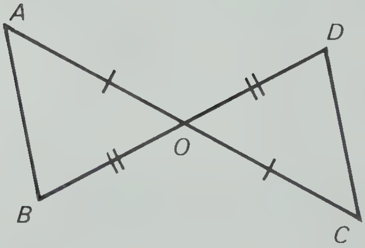
(b)



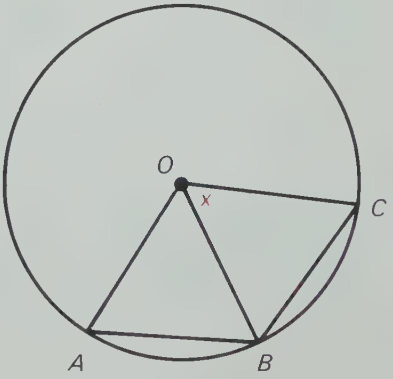


2. State why the following pairs of triangles are congruent:

(a)

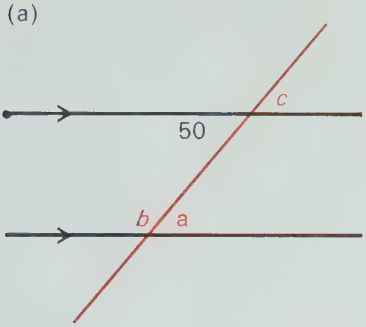


(b)

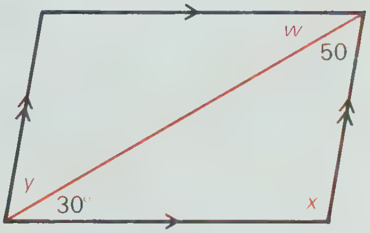


3. Calculate the measure of the indicated angles:

(a)

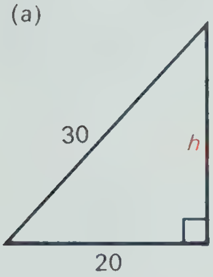


(b)

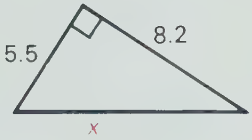


4. Calculate the length of the unknown side:

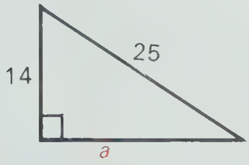
(a)



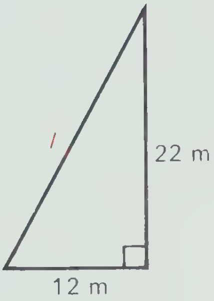
(b)



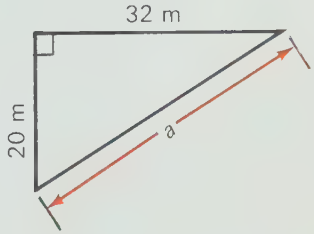
(c)



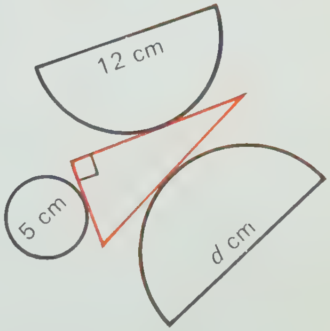
5. Calculate the length of guy wire to anchor a 22 m tower at a point 12 m from the base. Allow 3 m for fastening.



6. Calculate the length of brace required to secure a rafter as in the diagram.

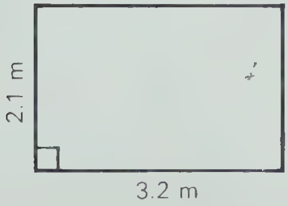


7. What size of pipe is equivalent to 2 pipes, 5 cm and 12 cm in diameter, so that the pressure is the same?

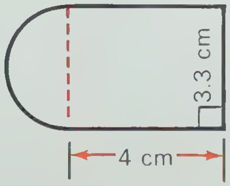


8. Calculate the perimeter of each of these figures.

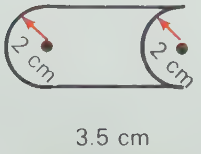
(a)



(b)

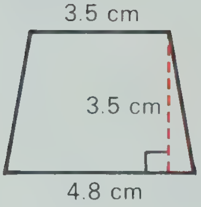


(c)

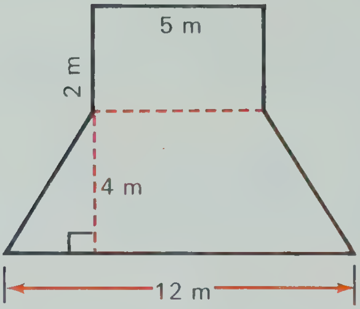


9. Calculate the area of each of:

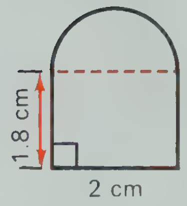
(a)



(b)

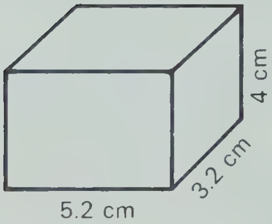


(c)

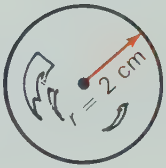


10. Calculate the volume of:

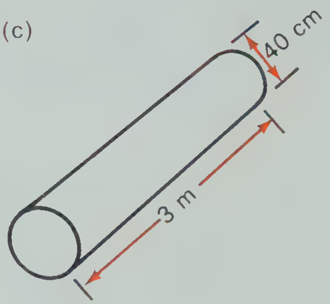
(a)



(b)

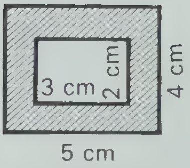


(c)

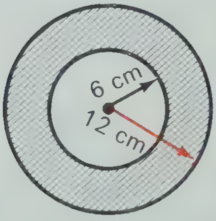


11. Find the area of each of:

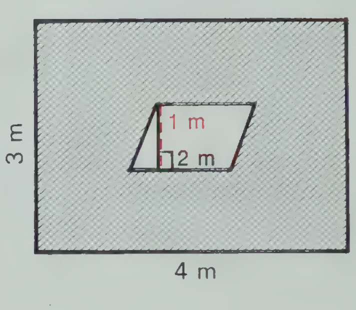
(a)



(b)



(c)



REVIEW AND PREVIEW TO CHAPTER 11

EXERCISE

1. Simplify to three significant figures:

(a) $\frac{5}{13}$ (b) $\frac{7}{24}$ (c) $\frac{7}{25}$ (d) $\frac{28}{4.5}$
(e) $\frac{5.5}{8}$ (f) 3.5×2.7 (g) 6.8×4.6 (h) 3.8^2
(i) 4.2×8.1 (j) 1.73×1.41

2. Solve the following equations to three significant figures:

Example: $\frac{x}{3.5} = \frac{4.7}{2.8}$

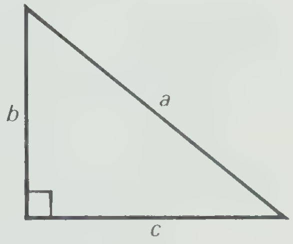
$$x = 3.5 \times \frac{4.7}{2.8}$$
$$\doteq 5.88 \text{ (by slide rule)}$$

(a) $\frac{a}{5.7} = \frac{8.2}{12.5}$ (b) $\frac{h}{2.7} = \frac{6.5}{2.3}$ (c) $\frac{h}{6.2} = \frac{37}{44}$

(d) $\frac{d}{140} = \frac{4}{3}$ (e) $\frac{x}{53} = \frac{2.7}{4.1}$ (f) $\frac{m}{6.5} = 0.337$

(g) $\frac{n}{58} = 0.246$ (h) $\frac{p}{62} = 0.331$ (i) $\frac{h}{135} = 1.73$

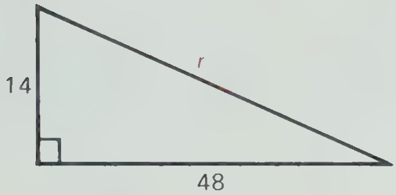
(j) $\frac{h}{2.6} = 0.866$



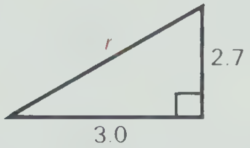
$a^2 \quad b^2 + c^2$

3. Use the Pythagorean relation to find the hypotenuse in each of the following:

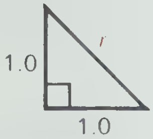
(a)



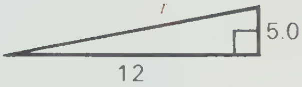
(b)



(c)

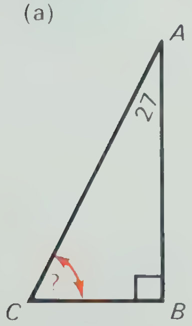


(d)

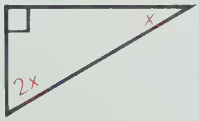


4. Calculate the measure of the indicated angles in the following:

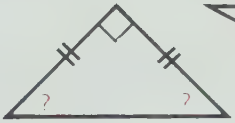
(a)



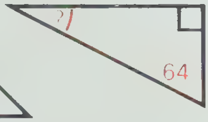
(b)



(c)



(d)



CHAPTER 11

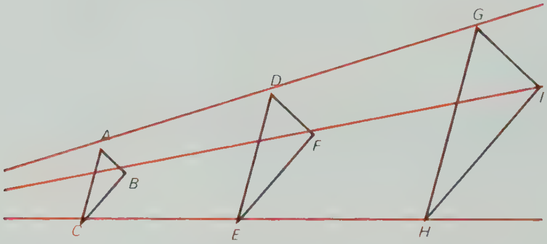
Trigonometry

Trigonometry is a branch of mathematics used more than 2000 years ago by astronomers, navigators, and surveyors to determine distances to inaccessible objects and to determine unmeasurable heights. Today, trigonometry continues to grow in importance not only in the space program but also in the day to day work of skilled machinists, draftsmen and other modern day technologists.

11.1 SIMILAR TRIANGLES ~

$\sim$ is similar to
$=$ is equal to
$\cong$ is congruent to

The images on a colour slide and on the projection screen are an example of similar figures. In this figure the triangles *ABC*, *DEF*, and *GHI* are similar triangles.



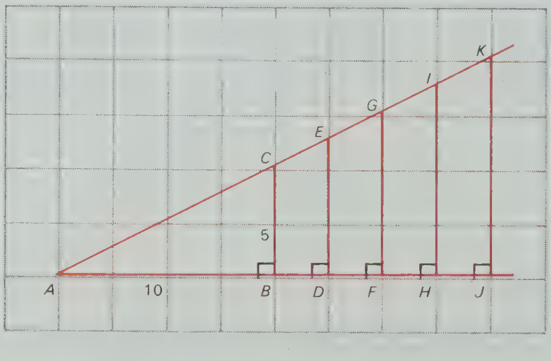
Scale drawings, blueprints, and map making are applications of similar figures.

In this section, we shall restrict our study of similar figures to triangles, and in particular, right triangles.

INVESTIGATION 11-1

To study the ratios of the sides of similar right triangles.

1.



In the accompanying diagram, several similar right triangles are drawn on squared paper so that the lengths of the sides can be read in terms of units ($AB = 10$ units). From the geometry of the figure, state the relation between the angles at C, E, G, I, and K.

2. (a) Complete the following table in your notebook by filling in the lengths.

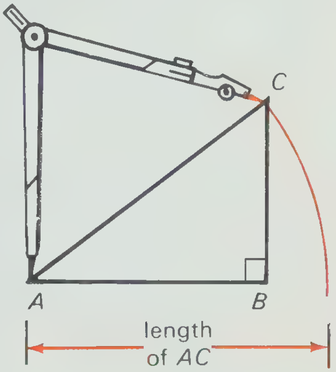
$\triangle ABC$	$\triangle ADE$	$\triangle AFG$	$\triangle AHI$	$\triangle AJK$
$AB = 10$ units	$AD =$	$AF =$	$AH =$	$AJ =$
$BC = 5$ units	$DE =$	$FG =$	$HI =$	$JK =$
$\frac{BC}{AB} = \frac{5}{10} = \frac{1}{2}$	$\frac{DE}{AD} =$	$\frac{FG}{AF} =$	$\frac{HI}{AH} =$	$\frac{JK}{AJ} =$

(b) How are the ratios of the corresponding sides related?

3. (a) Complete the following table as above.

$\triangle ABC$	$\triangle ADE$	$\triangle AFG$	$\triangle AHI$	$\triangle AJK$
$AB = 10$	$AD =$	$AF =$	$AH =$	$AJ =$
$BC = 5$	$DE =$	$FG =$	$HI =$	$JK =$
$AC \doteq 11.2$	$AE =$	$AG =$	$AI =$	$AK =$
$\frac{BC}{AC} =$	$\frac{DE}{AE} =$	$\frac{FG}{AG} =$	$\frac{HI}{AI} =$	$\frac{JK}{AK} =$
$\frac{AB}{BC} =$	$\frac{AD}{AE} =$	$\frac{AF}{AG} =$	$\frac{AH}{AI} =$	$\frac{AJ}{AK} =$

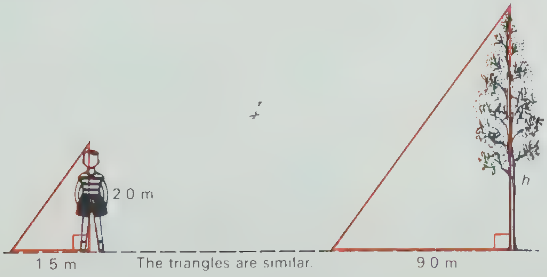
Use your compasses to measure AC, AE, AG, AI, AK



(b) How are the ratios of the corresponding sides of the triangles related?

EXAMPLE 1. *The shadow of a tree is 9 m at the same time of day that the shadow of a 2 m man is 1.5 m. Find the height of the tree.*

Solution We express the data in diagrams.



Let the height of the tree in metres be h:

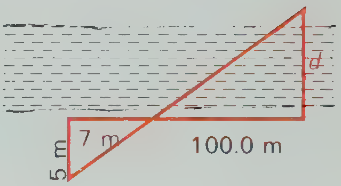
$$\frac{h}{9.0} = \frac{2.0}{1.5}$$

$$h = \frac{2.0 \times 9.0}{1.5}$$

$$= 12$$

$\therefore$ the tree is 12 m high (to two figures).

EXAMPLE 2. *Calculate the distance, d, across a river from data given in the accompanying diagram.*



Solution From the diagram,

$$\frac{d}{100} = \frac{5.00}{7.00}$$

$$d = \frac{100 \times 5.0}{7.0}$$

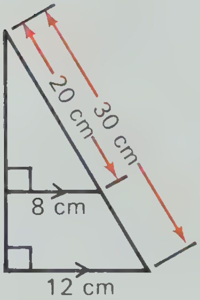
$$d \doteq 71.4$$

$\therefore$ the river is 71.4 m wide to 3 figures.

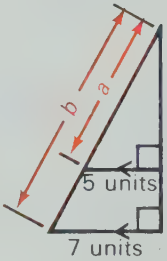
EXERCISE 11-1

1. State the proportions between sides in the following pairs of similar triangles.

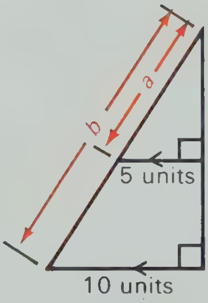
(a)



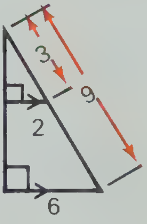
(b)



(c)

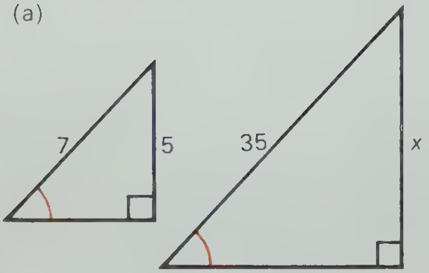


(d)

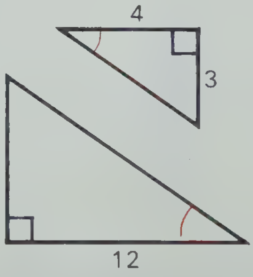


2. Calculate the values, in units, of the variables in the following pairs of similar triangles.

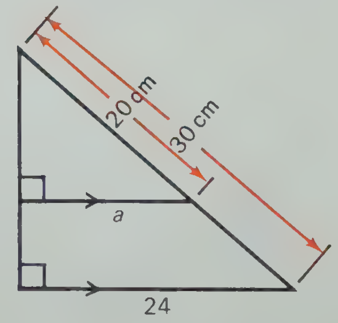
(a)



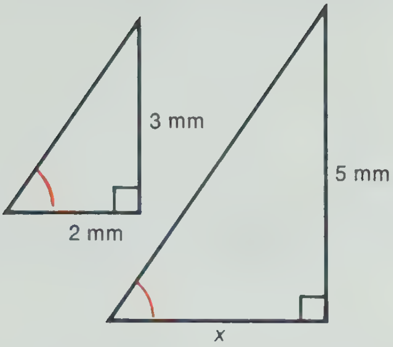
(b)



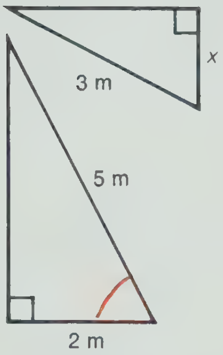
(c)



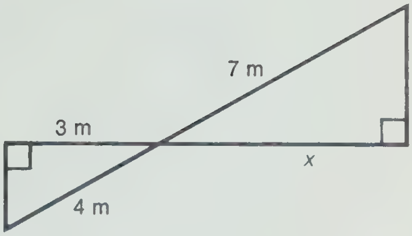
(d)



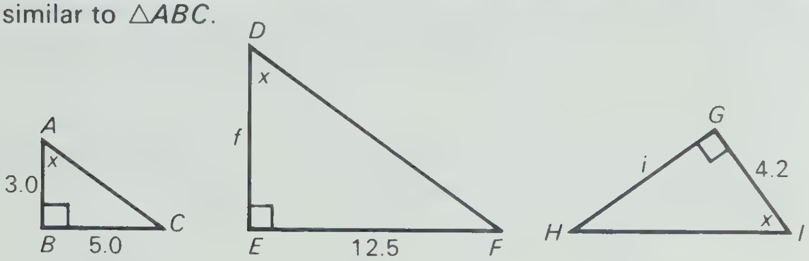
(e)



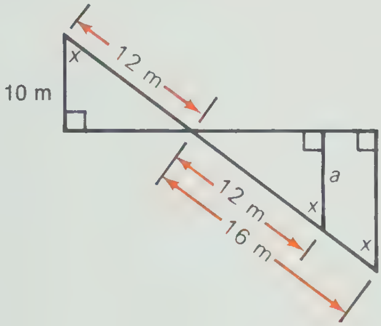
(f)



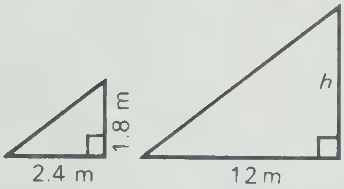
3. Find the values, in units, of *f* and *i* if $\triangle DEF$ and $\triangle GHI$ are similar to $\triangle ABC$.



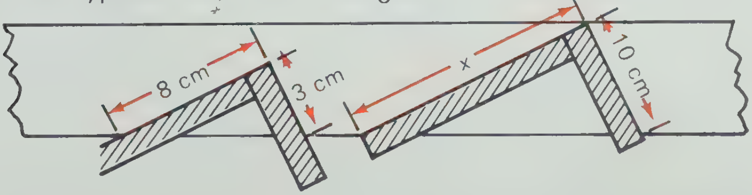
4. Calculate the values of *a* and *b* given that all three triangles are similar.

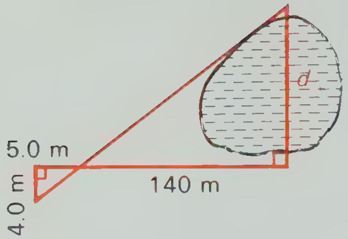


5. A flagpole casts a 12 m shadow when a man 1.8 m tall casts a 2.4 m shadow. Calculate the height of the flagpole.



6. In the accompanying diagram, a carpenter's square is shown in two positions forming a pair of similar right triangles. Calculate
(a) the dimension marked *x*.
(b) the hypotenuse of both triangles.

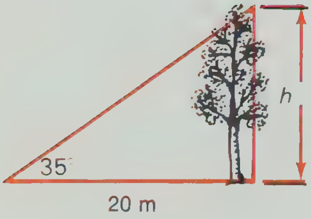




7. Use the data in the diagram to calculate the distance across the pond.

8. Use similar triangles to calculate the height of your school, and the distance across a football field. Describe your method.

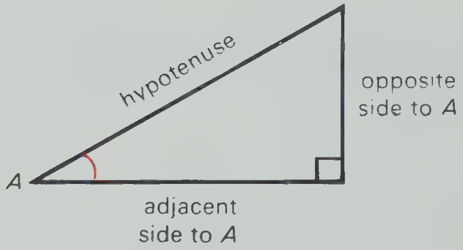
11.2 THE TANGENT RATIO



In the previous section, you were able to calculate lengths, heights, and distances, given two triangles, by setting up a proportion. How would you calculate the height of a tree which casts a shadow 20 m long when the **angle of elevation** of the sun is 35°?

This problem cannot be solved by the method of the previous section. In order to perform such calculations, mathematicians have developed a branch of mathematics called **trigonometry.**

The sides of a right triangle are labelled as follows:



The ratio $\dfrac{\text{opposite side to } A}{\text{adjacent side to } A}$ is called the **tangent ratio.**

We write

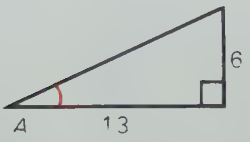
$$\text{tangent of } A = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

or simply

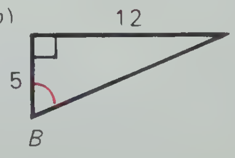
$$\boxed{\tan A = \frac{\text{opposite}}{\text{adjacent}}}$$

EXAMPLE 1. *Calculate the tangent ratio of the indicated angle in the following:*

(a)



(b)



Insert signs of operation to make a true statement:
6 ▨ 2 ▨ 3 ▨ 5 = 10

Solution
(a) The length of the side opposite to A is 6 units. The length of the side adjacent to A is 13 units.

$$\tan A = \frac{\text{opp.}}{\text{adj.}}$$

$$\therefore \quad \tan A = \frac{6}{13}$$

$$\doteq 0.462$$

(b)

$$\tan B = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan B = \frac{12}{5} = 2.400$$

EXAMPLE 2. *Calculate (a) tan A and (b) tan B from the given diagram.*

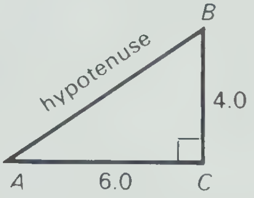
Solution

(a) $$\tan A = \frac{4.0}{6.0}$$

$$\doteq 0.67$$

(b) $$\tan B = \frac{6.0}{4.0}$$

$$= 1.500$$



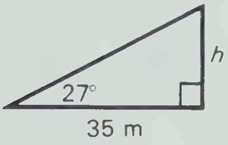
Note that while side BC is opposite to $\angle A$, BC is adjacent to $\angle B$. Similarly AC is opposite to $\angle B$, and adjacent to $\angle A$.

Starting with the word "seam" and changing one letter at a time can you reach the word "molt" in four changes?

seam
1.
2.
3.
4. molt

EXAMPLE 3. *Calculate the height of a tree which casts a 35 m shadow when the angle of elevation of the sun is 27° ($\tan 27° \doteq 0.510$).*

Solution Let the height of the tree be h metres.



$$\frac{h}{35} = \tan 27°$$

$$h = 35 \times \tan 27°$$

$$\doteq 35 \times 0.510$$

$$\doteq 17.9$$

$\therefore$ the height of the tree is 17.9 m

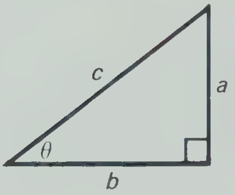
Calculations as in example 3 are possible because the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$ formed in the right triangle does not change as long as the angle does not change. In order to perform calculations quickly and accurately, a table of the values of the tangent ratio is provided here.

θ°	tan θ
0	0.0000
1	0.0175
2	0.0349
3	0.0524
4	0.0699
5	0.0875
6	0.1051
7	0.1228
8	0.1405
9	0.1584
10	0.1763
11	0.1944
12	0.2126
13	0.2309
14	0.2493
15	0.2679
16	0.2867
17	0.3057
18	0.3249
19	0.3443
20	0.3640
21	0.3839
22	0.4040
23	0.4245
24	0.4452
25	0.4663
26	0.4877
27	0.5095
28	0.5317
29	0.5543
30	0.5774
31	0.6009
32	0.6249
33	0.6494
34	0.6745
35	0.7002
36	0.7265
37	0.7536
38	0.7813
39	0.8098
40	0.8391
41	0.8693
42	0.9004
43	0.9325
44	0.9657

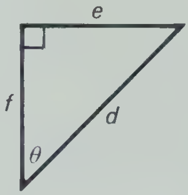
EXERCISE 11-2

A 1. Identify each side of the following triangles as the hypotenuse, or as the side opposite or adjacent to the indicated angle:

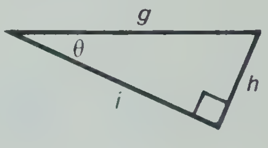
(a)



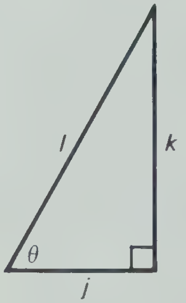
(b)



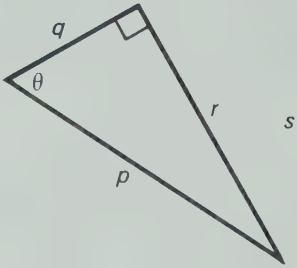
(c)



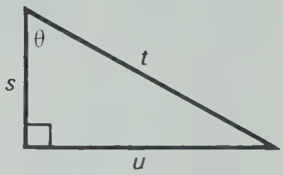
(d)



(e)

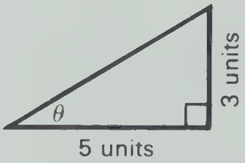


(f)

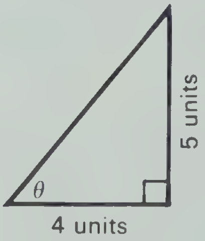


2. State tan θ in each of the following:

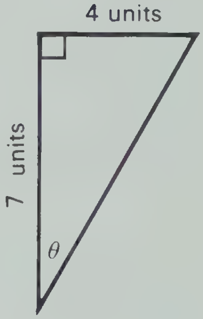
(a)



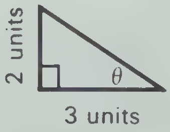
(b)



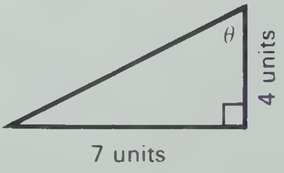
(c)



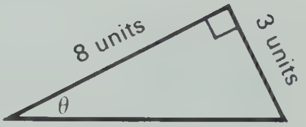
(d)



(e)

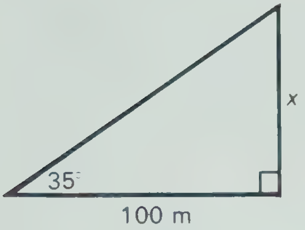


(f)

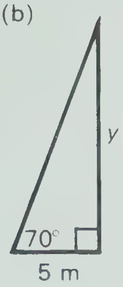


B **3.** Calculate the lengths of the indicated sides using the tangent ratio.

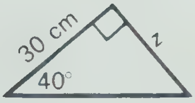
(a)



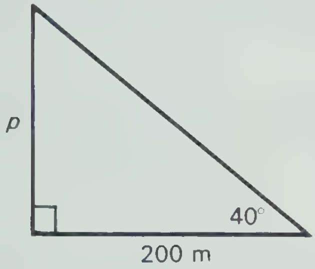
(b)



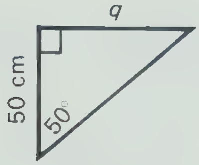
(c)



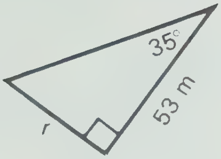
(d)



(e)

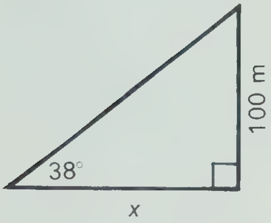


(f)

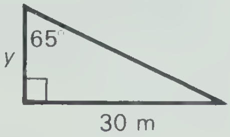


4. Calculate the values of the variables.

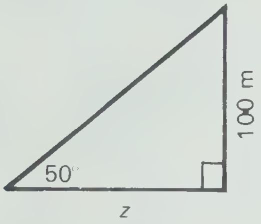
(a)



(b)



(c)



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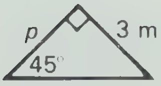

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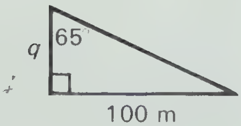

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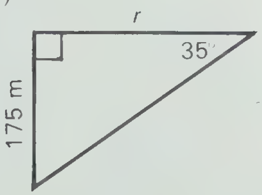


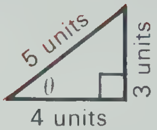
θ°	tan θ
45	1.0000
46	1.0355
47	1.0724
48	1.1106
49	1.1504
50	1.1918
51	1.2349
52	1.2799
53	1.3270
54	1.3764
55	1.4281
56	1.4826
57	1.5399
58	1.6003
59	1.6643
60	1.7321
61	1.8041
62	1.8807
63	1.9626
64	2.0503
65	2.1445
66	2.2460
67	2.3559
68	2.4751
69	2.6051
70	2.7475
71	2.9042
72	3.0777
73	3.2709
74	3.4874
75	3.7321
76	4.0108
77	4.3315
78	4.7046
79	5.1446
80	5.6713
81	6.3138
82	7.1154
83	8.1443
84	9.5144
85	11.4301
86	14.301
87	19.081
88	28.636
89	57.290
90	

Starting with the word "move" and changing one letter at a time, can you reach the word "part" in four changes?

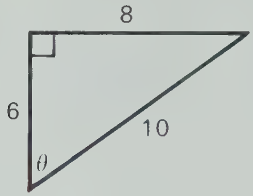
move
1.
2.
3.
4. part

5. Calculate the value of tan θ in each of the following triangles, and then read the value of θ to the nearest degree from the table.

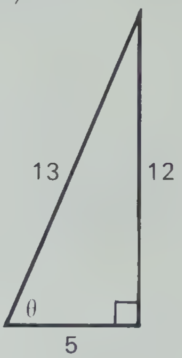
(a)



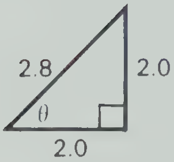
(b)



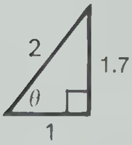
(c)



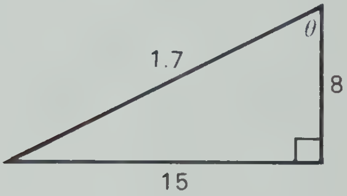
(d)

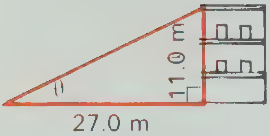


(e)

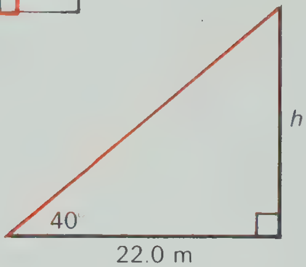


(f)

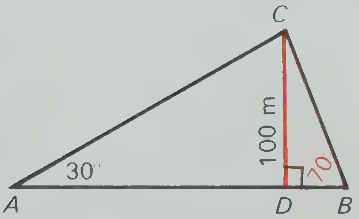




6. A building 11.0 m high casts a 27.0 m shadow. Calculate the angle of elevation of the sun.

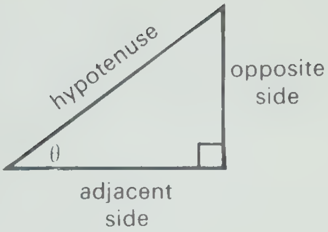


7. A flagpole casts a shadow of 22.0 m when the angle of elevation of the sun is 40°. Calculate the height of the flagpole.



8. Calculate the distance *AB* from the data given in the diagram. Hint: find *AD* and *BD* separately.

11.3 THE SINE AND COSINE RATIOS



In the previous section, we formed the ratio $\frac{\text{opposite side}}{\text{adjacent side}}$ and called it the tangent of θ (tan θ). We now define two more ratios, sine and cosine, as follows:

$$\text{sine of } \theta = \sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

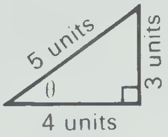
$$\text{cosine of } \theta = \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

EXAMPLE 1. *State sin θ and cos θ from the given diagram.*

Solution

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{4}{5}$$



EXAMPLE 2. *Calculate a and b to the nearest centimetre from the data in the diagram, given that sin 25° = 0.423, and cos 25° = 0.906.*

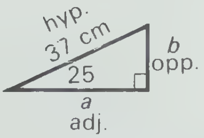
$\frac{b}{37} = \frac{\text{opp}}{\text{hyp}}$

$\frac{a}{37} = \frac{\text{adj}}{\text{hyp}}$

Solution

$$\frac{b}{37} = \sin 25^\circ \qquad \frac{a}{37} = \cos 25^\circ$$

$$\begin{aligned} b &= 37 \times \sin 25^\circ & a &= 37 \times \cos 25^\circ \\ &\doteq 37 \times 0.423 & &\doteq 37 \times 0.906 \\ &\doteq 15.7 & &\doteq 33.5 \end{aligned}$$

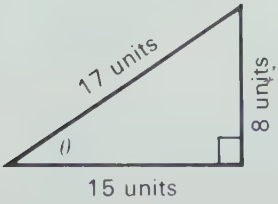


$\therefore$ in the given triangle a is 34 cm and b is 16 cm

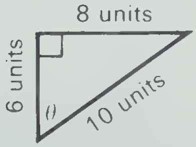
EXERCISE 11-3

A 1. State the value of (i) sin θ, (ii) cos θ, and (iii) tan θ in the following:

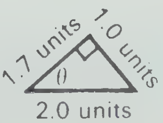
(a)

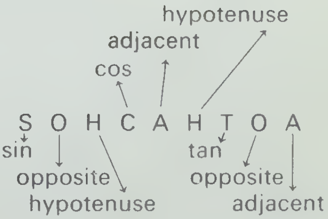


(b)

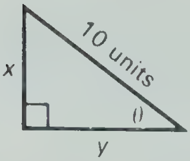


(c)

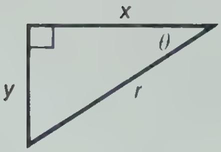




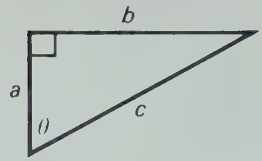
(d)



(e)

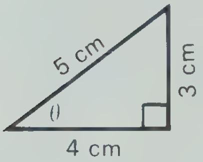


(f)

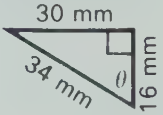


2. State the value of (i) sin θ, (ii) cos θ, and (iii) tan θ in each of the following:

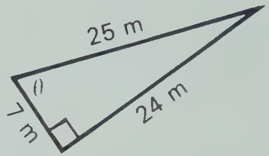
(a)



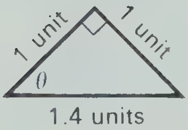
(b)



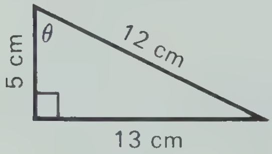
(c)



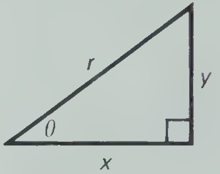
(d)



(e)



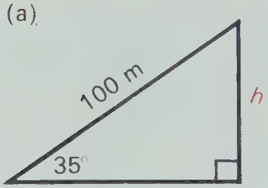
(f)



$\sqrt{\sqrt{10\ 000}} = ?$

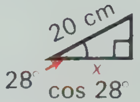
B **3.** Calculate the length of the indicated side using the data given in each case:

(a)



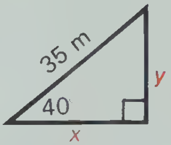
sin 35° ≐ 0.574

(b)



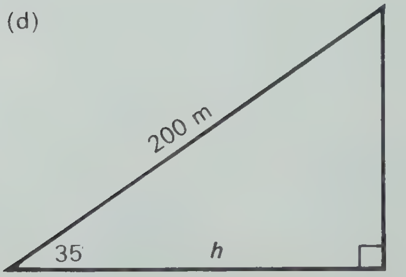
cos 28° ≐ 0.883

(c)



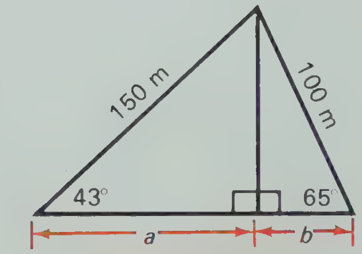
sin 40° ≐ 0.643
cos 40° ≐ 0.766

(d)



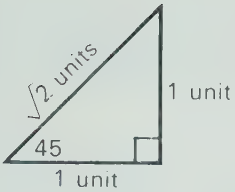
cos 35° ≐ 0.819

(e)

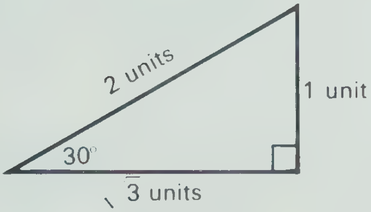


cos 43° ≐ 0.731
cos 65° ≐ 0.423

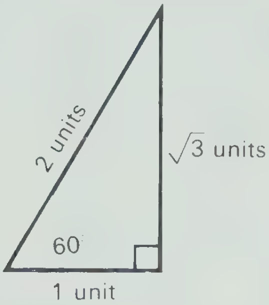
4. State the value of
(a) sin 45°
(b) cos 45°
(c) tan 45°
using the given diagram.



5. State the value of
(a) sin 30°
(b) cos 30°
(c) tan 30°
using the given diagram.



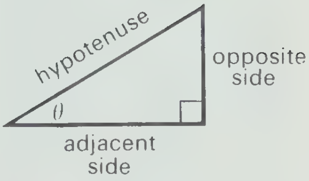
6. State the value of
(a) sin 60°
(b) cos 60°
(c) tan 60°
using the given diagram.



11.4 RIGHT TRIANGLES

We have defined the trigonometric ratios **sine, cosine,** and **tangent** as ratios of the sides of a right triangle.

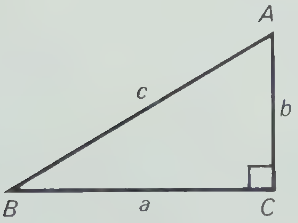
In $\triangle ABC$, the side opposite $\angle A$ is a units long, the side opposite $\angle B$ is b units long, and the side opposite $\angle C$ is c units long. When the meaning is clear, the letter corresponding to the vertex can mean the measure of the angle at that vertex.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$



From the diagram

$$\sin A = \frac{a}{c} \qquad \sin B = \frac{b}{c}$$

$$\cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

$$\tan A = \frac{a}{b} \qquad \tan B = \frac{b}{a}$$

and $a^2 + b^2 = c^2$.

In order to use these ratios efficiently, tables of values of sin θ, cos θ, and tan θ have been prepared and these are found on pages 282 and 283. A portion of the table is reprinted in the margin of pages 232 and 233.

θ	$\sin\theta$	$\cos\theta$
0	0.0000	1.0000
1	0.0175	0.9998
2	0.0349	0.9994
3	0.0523	0.9986
4	0.0698	0.9976
5	0.0872	0.9962
6	0.1045	0.9945
7	0.1219	0.9925
8	0.1392	0.9903
9	0.1564	0.9877
10	0.1736	0.9848
11	0.1908	0.9816
12	0.2079	0.9781
13	0.2250	0.9744
14	0.2419	0.9703
15	0.2588	0.9659
16	0.2756	0.9613
17	0.2924	0.9563
18	0.3090	0.9511
19	0.3256	0.9455
20	0.3420	0.9397
21	0.3584	0.9336
22	0.3746	0.9272
23	0.3907	0.9205
24	0.4067	0.9135
25	0.4226	0.9063
26	0.4384	0.8988
27	0.4540	0.8910
28	0.4695	0.8829
29	0.4848	0.8746
30	0.5000	0.8660
31	0.5150	0.8572
32	0.5299	0.8480
33	0.5446	0.8387
34	0.5592	0.8290
35	0.5736	0.8192
36	0.5878	0.8090
37	0.6018	0.7986
38	0.6157	0.7880
39	0.6293	0.7771
40	0.6428	0.7660
41	0.6561	0.7547
42	0.6691	0.7431
43	0.6820	0.7314
44	0.6947	0.7193

EXAMPLE 1. *Calculate a and b using data given in the diagram.*

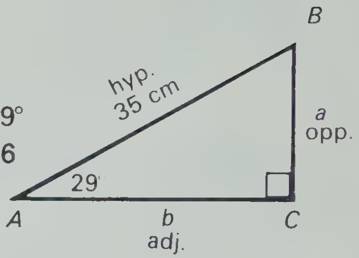
Solution

$$\frac{a}{35} = \sin 29^\circ \qquad \frac{b}{35} = \cos 29^\circ$$

$$a = 35 \times \sin 29^\circ \qquad b = 35 \times \cos 29^\circ$$

$$\doteq 35 \times 0.4848 \qquad \doteq 35 \times 0.8746$$

$$\doteq 17.0 \text{ cm} \qquad \doteq 30.6 \text{ cm}$$



We say that a triangle is **solved** when we can state the measure of the **three angles** and the lengths of the **three sides**.

EXAMPLE 2. *Solve $\triangle ABC$, given $\angle A = 90^\circ$, $b = 35$ units and $c = 45$ units.*

Solution

(i) $\tan B = \dfrac{35}{45}$

$\doteq 0.7778$

$\angle B \doteq 38^\circ$ from tables

(ii) $\angle C = 90^\circ - 38^\circ$

$= 52^\circ$

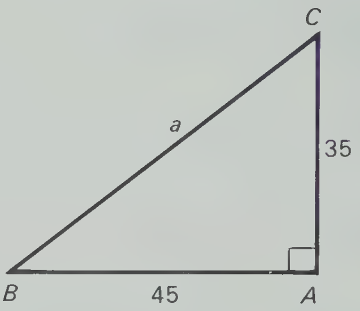
(iii) $\dfrac{35}{a} = \sin 38^\circ$

$35 = a \times \sin 38^\circ$

$\dfrac{35}{\sin 38^\circ} = a$

$\dfrac{35}{0.6157} = a$

$57 \doteq a$



$\therefore$ $a = 57$ units, $\angle B = 38^\circ$, and $\angle C = 52^\circ$.

EXERCISE 11-4

1. Read the following values from the tables:

(a) sin 25° (b) tan 80° (c) tan 45° (d) cos 30° (e) cos 60°
(f) sin 45° (g) cos 45° (h) tan 28° (i) cos 32° (j) sin 65°
(k) cos 38° (l) sin 20° (m) sin 30° (n) sin 50° (o) tan 12°

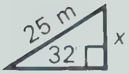
2. State the measure of each angle to the nearest degree.

(a) $\tan A = 0.6009$ (b) $\sin B = 0.9903$ (c) $\cos C = 0.2590$
(d) $\sin D = 0.250$ (e) $\sin E = 0.695$ (f) $\tan F = 0.810$
(g) $\sin G = 0.707$ (h) $\cos H = 0.809$ (i) $\tan I = 1.600$
(j) $\tan H = 0.250$ (k) $\cos K = 0.707$ (l) $\sin L = 0.500$
(m) $\cos M = 0.309$ (n) $\tan N = 11.400$ (o) $\sin \theta = 1.000$

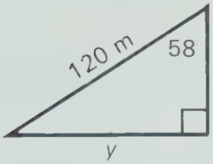
3, Examine the tables on pages 282 and 283 and complete the following statement: "As the angle, θ, increased from 0° to 90°
(a) $\sin\theta$ increases from ▨ to ▨,
(b) $\cos\theta$ decreases from ▨ to ▨,
(c) $\tan\theta$ increases from ▨ to ▨".

4. Use the sine ratio to find the length of the indicated side:

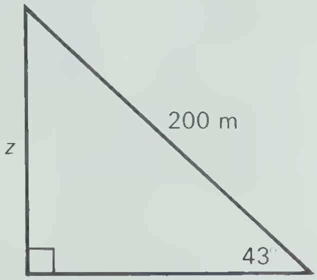
(a)



(b)

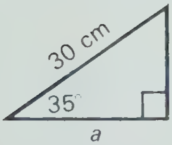


(c)

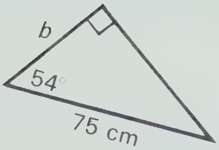


5. Use the cosine ratio to find the lengths of the indicated side:

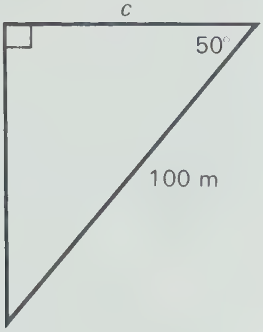
(a)



(b)

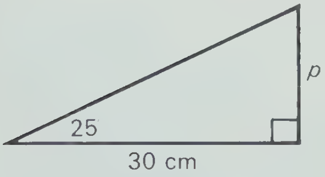


(c)

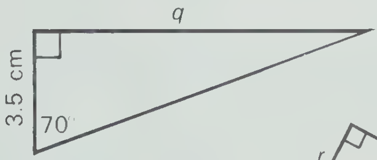


6. Use the tangent ratio to find the length of the indicated side:

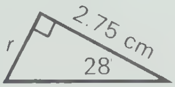
(a)



(b)

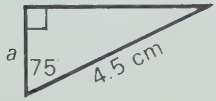


(c)

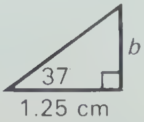


7. Find the length of the indicated side:

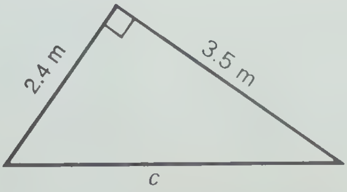
(a)



(b)



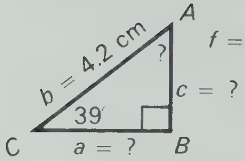
(c)



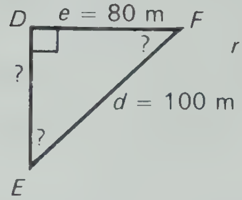
θ°	$\sin\theta$	$\cos\theta$
45	0.7071	0.7071
46	0.7193	0.6947
47	0.7314	0.6820
48	0.7431	0.6691
49	0.7547	0.6561
50	0.7660	0.6428
51	0.7771	0.6293
52	0.7880	0.6157
53	0.7986	0.6018
54	0.8090	0.5878
55	0.8192	0.5736
56	0.8290	0.5592
57	0.8387	0.5446
58	0.8480	0.5299
59	0.8572	0.5150
60	0.8660	0.5000
61	0.8746	0.4848
62	0.8829	0.4695
63	0.8910	0.4540
64	0.8988	0.4384
65	0.9063	0.4226
66	0.9135	0.4067
67	0.9205	0.3907
68	0.9272	0.3746
69	0.9336	0.3584
70	0.9397	0.3420
71	0.9455	0.3256
72	0.9511	0.3090
73	0.9563	0.2924
74	0.9613	0.2756
75	0.9659	0.2588
76	0.9703	0.2419
77	0.9744	0.2250
78	0.9781	0.2079
79	0.9816	0.1908
80	0.9848	0.1736
81	0.9877	0.1564
82	0.9903	0.1392
83	0.9925	0.1219
84	0.9945	0.1045
85	0.9962	0.0872
86	0.9976	0.0698
87	0.9986	0.0523
88	0.0994	0.0349
89	0.9998	0.0175
90	1.0000	0.0000

8. Solve the following triangles:

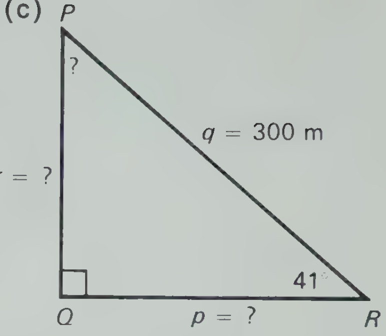
(a)



(b)



(c)



9. Solve △*ABC*, given

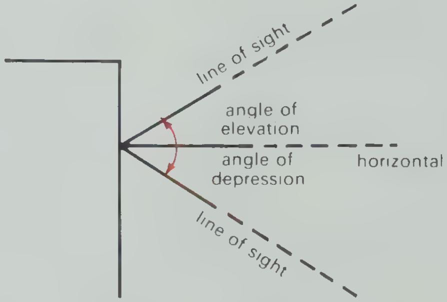
$$\angle A = 90°, \quad b = 7.0 \text{ m}, \quad c = 3.0 \text{ m}$$

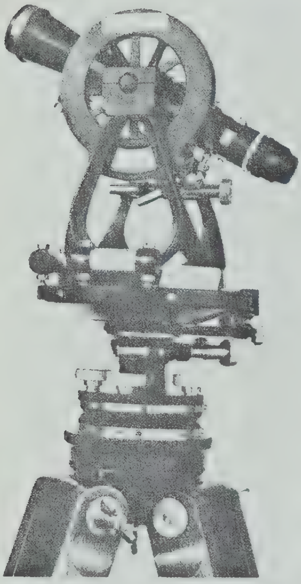
10. Solve △*DEF*, given

$$\angle D = 35°, \quad \angle E = 90°, \quad e = 20 \text{ m}$$

11.5 APPLICATIONS OF TRIGONOMETRY

Trigonometry can be used to solve problems which can be expressed as a diagram containing triangles. As we saw in earlier sections, we can calculate the height of a flagpole without actually climbing the pole. The figure shows some of the terminology which will be used in the work which follows:

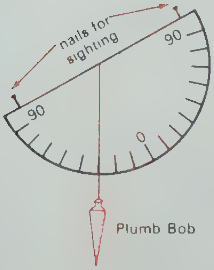



Courtesy Hughes-Owens

A Theodolite

When measuring angles, a surveyor uses an instrument called a **transit,** or **theodolite.** In practice this means that the height of the instrument (H.I.) must be taken into account.

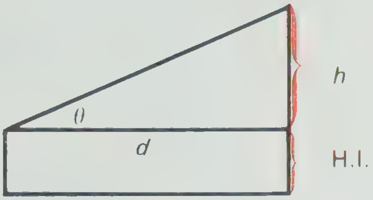
A simple measuring device can be constructed using a blackboard protractor. Two nails serve as sights, and a plumb bob is attached to the centre. Note that the scale on the protractor has been changed so that when you are sighting along the horizontal, the angle of elevation will read 0°.



The investigations which follow can be performed using a commercial transit, or a student-built transit to measure the angles.

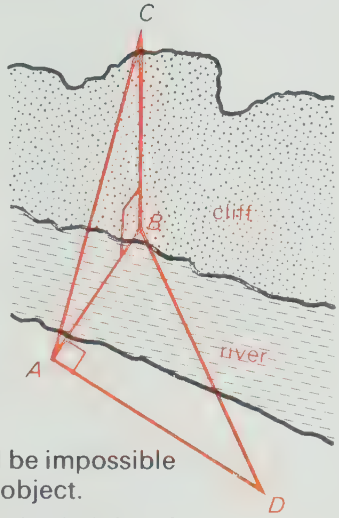
INVESTIGATION 11-2

1. Select inaccessible heights such as the height of a flagpole, tree, or building near your school.
2. Measure and record the values of the angle of elevation θ, distance to the object d, and the height of the instrument $H.I.$, on a similar diagram in your books.
3. Calculate the dimension h, and then the actual height of the object $h + H.I.$



INVESTIGATION 11-3

In the accompanying diagram, a surveyor wishes to find the height of BC. He sets up his transit at A and measures $\angle CAB$. Because he cannot measure AB, he proceeds to find AB indirectly by laying off a base line AD as in the diagram. He then sets up his instrument at D and measures $\angle ADB$, and DA. Explain why he can now calculate AB and then BC.

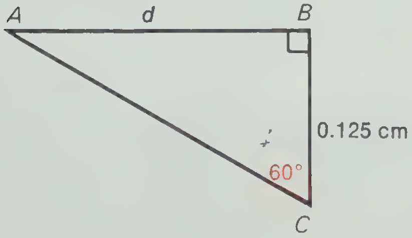


1. Select a height you wish to measure, where it will be impossible to measure the distance from the instrument to the object.
2. Lay off a base line to acquire data to calculate the height of the object you have selected. Be sure to add the height of the instrument $H.I.$

11.6 SHOP PROBLEMS

EXAMPLE 1. *Each section of a sharp V thread is composed of equilateral triangles. Calculate the depth of cut, d, if the pitch of the thread is 0.250* cm.

Solution



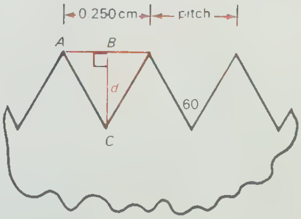
From the diagram,

$$\frac{d}{0.125} = \tan 60^\circ$$

$$= 0.125 \times \tan 60^\circ$$

$$\doteq 0.125 \times 1.732$$

$$\doteq 0.217 \text{ (by slide rule)}$$



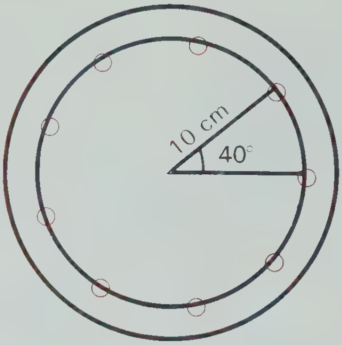
∴ the depth of the cut is 0.217 cm.

Starting with the word "lane" and changing one letter at a time, can you reach the word "dirt" in four changes?

lane
1.
2.
3.
4. dirt

EXAMPLE 2. *Calculate the distance between the centres of the holes on a 20 cm bolt circle containing nine holes.*

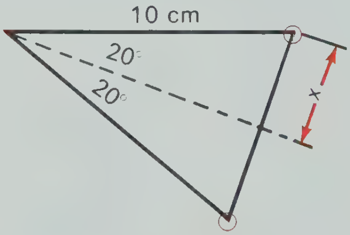
Solution



The radius of the circle is 10 cm; the angle at the centre can be found when the number of holes is known.

Angle at centre $= \frac{360^\circ}{9} = 40^\circ$.

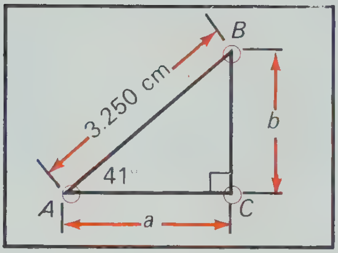
If x represents one half the distance between the holes (often called chordal distance), then



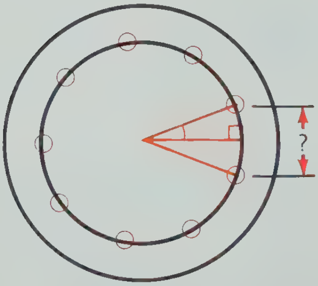
$$\frac{x}{10} = \sin 20^\circ$$
$$x = 10 \times \sin 20^\circ$$
$$x \doteq 10 \times 0.342$$
$$= 3.42$$
$$3.42 \times 2 = 6.84$$

∴ the distance between centres is 6.84 cm.

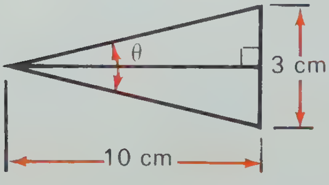
EXERCISE 11-6



1. Three holes are to be located in a rectangular plate as shown. When hole A has been bored, the table carrying the plate is moved horizontally the distance a so that hole C can be bored. The table is then moved vertically so that hole B can be bored. Find the dimensions a and b (centimetres).

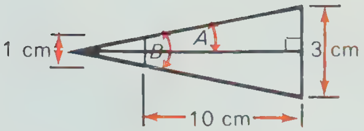


2. Ten holes are to be evenly spaced on a 20 cm bolt circle. Calculate the distance between centres of two adjacent holes, AB.

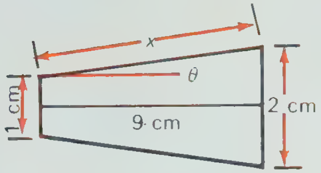


3. Find the indicated angle in the wedge as shown. Use a right triangle.

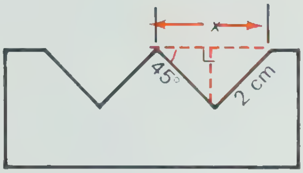
4. Calculations for tapers are similar to those for wedges. Calculate the angles marked *A* and *B* to the nearest degree for the conical taper to be turned in a lathe.



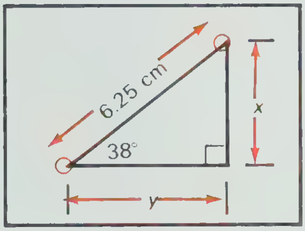
5. (a) Find the angle marked θ.
(b) Find the length marked *x*.



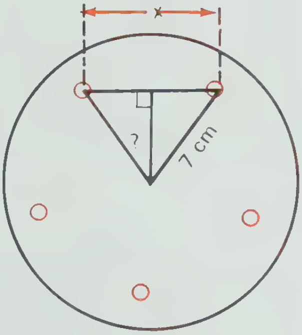
6. Calculate the measurement *x* in the given diagram of a template.



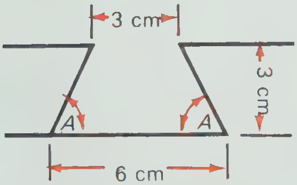
7. In the given jig-plate, two holes must be drilled as shown. Calculate the distances marked *x* and *y* if the distance between centres is 6.25 cm.



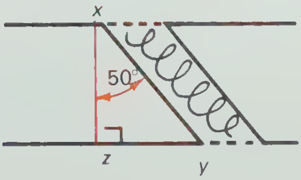
8. Calculate the distance between the centres of two adjacent holes on a five-hole bolt circle with diameter 14 cm.



9. Find the indicated angles, *A*, in the dovetail slide in the diagram.



10. If the length of the hole *xy* is 4.00 cm and the hole is at an angle of 50°, calculate the thickness of the work *xz*.

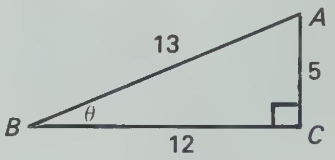


11.7 RECIPROCAL TRIGONOMETRIC RATIOS

Many problems in trigonometry are solved using the reciprocals of the primary trigonometric ratios.

Primary Ratios	**Reciprocal Ratios**
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	cosecant $\theta = \csc\theta = \frac{\text{hyp}}{\text{opp}}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	secant $\theta = \sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	cotangent $\theta = \cot\theta = \frac{\text{adj}}{\text{opp}}$

EXAMPLE 1. *State the six trigonometric ratios of the angle θ in ABC.*



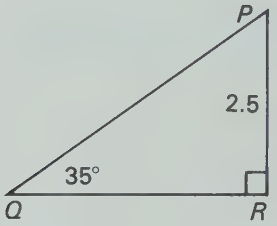
Solution

$\sin\theta = \frac{5}{13}$ $\quad\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\quad\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad\cot\theta = \frac{12}{5}$

EXAMPLE 2. *Determine the value of r in △ PQR correct to the nearest tenth of a unit.*



Solution

Two methods may be used.

(i)

$$\sin 35^{\circ} = \frac{2.5}{r}$$
$$0.5736 = \frac{2.5}{r}$$
$$0.5736 \times r = 2.5$$
$$r = \frac{2.5}{0.5736}$$
$$r = 4.358$$

(ii)

$$\frac{r}{2.5} = \csc 35^{\circ}$$
$$\frac{r}{2.5} = 1.743$$
$$r = 2.5 \times 1.743$$
$$r = 4.358$$

the value of r is 4.4 units.

The calculation using cosecant θ involves less manipulation.

EXERCISE 11-7

A 1. Using tables or calculators, state the following values:

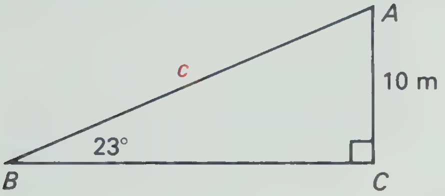
(a) csc 46° (b) cot 89° (c) sec 5° (d) cot 72°
(e) tan 30° (f) csc 45° (g) sec 18° (h) cot 90°
(i) sin 55° (j) csc 90° (k) sec 80° (l) cot 8°

2. State the measure of the angle θ for each of the following statements.

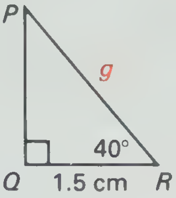
(a) $\sec \theta = 2.559$ (b) $\csc \theta = 1.325$ (c) $\cot \theta = 0.7265$
(d) $\cot \theta = 57.2900$ (e) $\csc \theta = 2.669$ (f) $\tan \theta = 0.5317$

B 3. Determine the length of the indicated side in each triangle (correct to the nearest tenth of a unit).

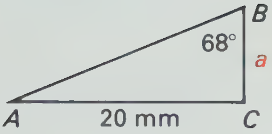
(a)



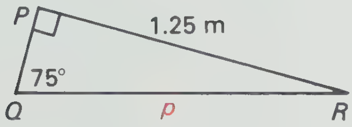
(b)



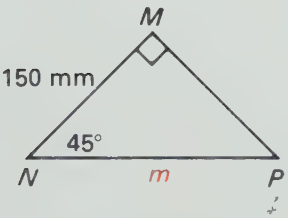
(c)



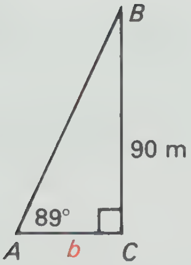
(d)

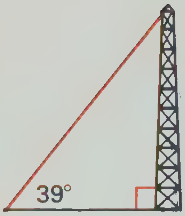


(e)

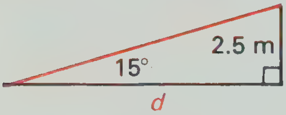


(f)





4. A supporting guy wire makes an angle of 39° with the ground. If the height of the tower is 95 m, calculate the length of the guy wire to the nearest tenth of a metre.

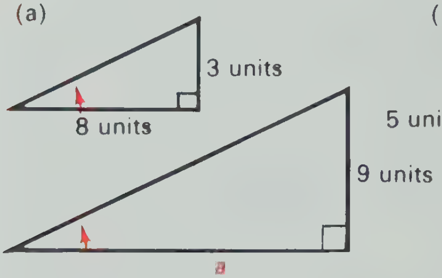


5. A ramp to a loading platform must reach a height of 2.5 m. If the ramp angle is 15°, calculate the horizontal distance the ramp must extend, correct to the nearest tenth of a metre.

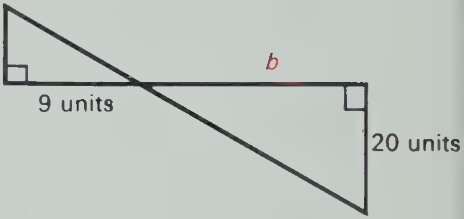
REVIEW EXERCISE

B 1. Calculate the indicated lengths using similar triangles:

(a)

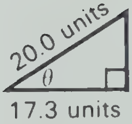


(b)

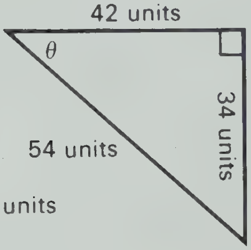


2. State the six trigonometric ratios for each of the following.

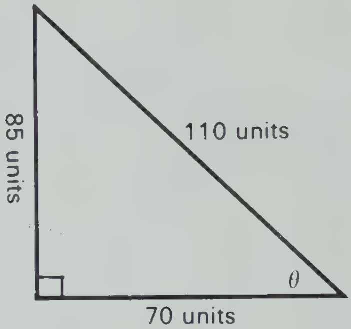
(a)



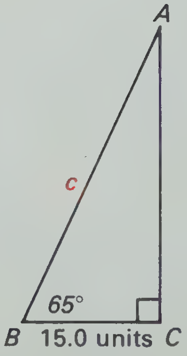
(b)



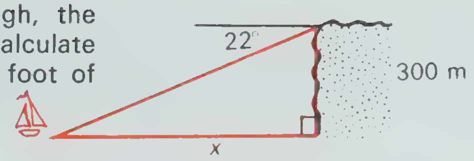
(c)



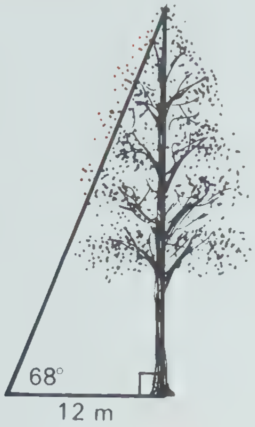
3. In the diagram at the right,
(a) solve for *b* using the tangent ratio
(b) solve for *c* using the secant ratio.



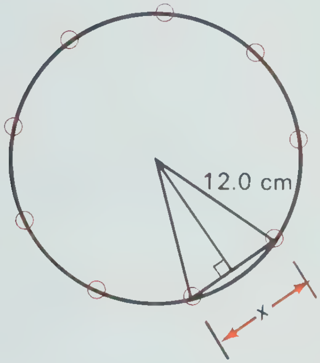
4. From the top of a cliff 300 m high, the angle of depression of a boat is 22°. Calculate the distance, x, from the boat to the foot of the cliff.



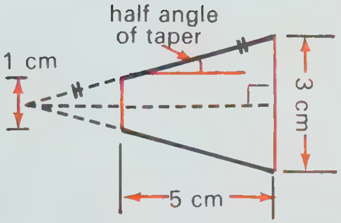
5. The shadow of a tree is 12 m when the angle of elevation of the sun is 68°. Calculate the height of the tree.



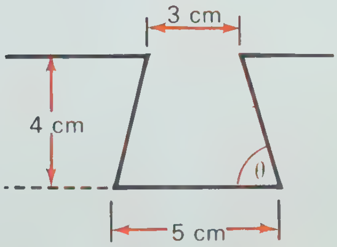
6. Calculate the distance between the centres of two adjacent holes on nine-hole bolt circle with diameter 24.0 cm.



7. Calculate the angle of taper from the diagram.



8. Calculate the angles in the dovetail in the diagram.

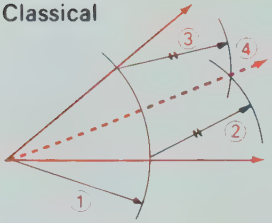


REVIEW AND PREVIEW TO CHAPTER 12

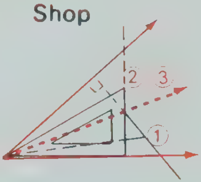
BASIC CONSTRUCTIONS

To bisect a given angle.

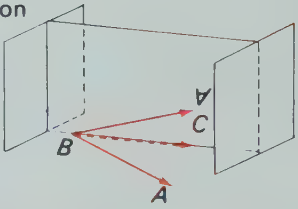
Classical



Shop

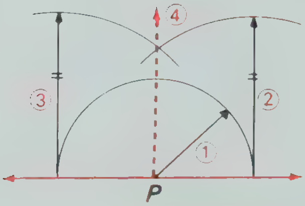


Reflection

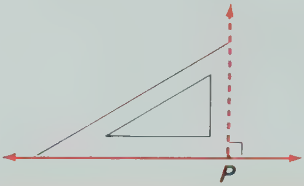


To construct a perpendicular to a given line at a point on the line.

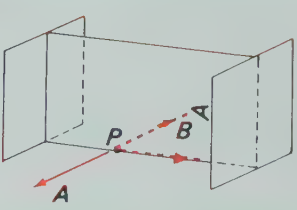
Classical



Shop

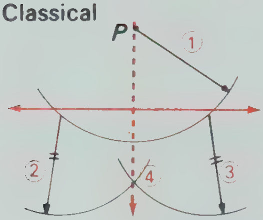


Reflection

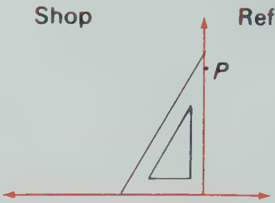


To construct a perpendicular to a given line from a point outside the line.

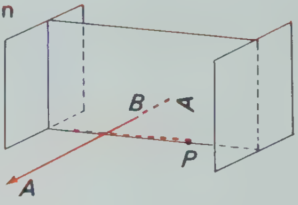
Classical



Shop

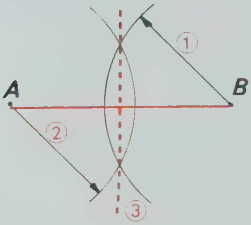


Reflection

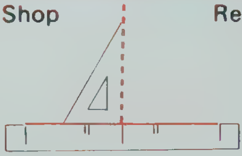


To construct the right bisector of a line segment.

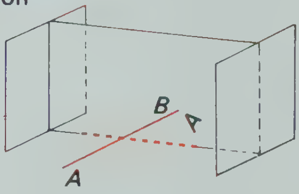
Classical



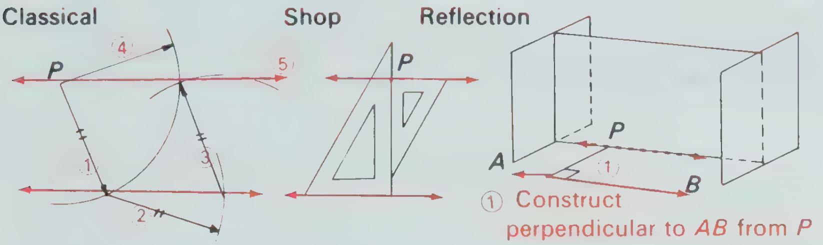
Shop



Reflection



To construct a line parallel to a given line through a point outside the line.

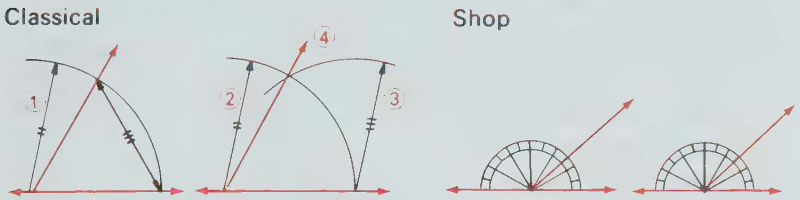


① Construct perpendicular to *AB* from *P*

Starting with the word "fast" and changing one letter at a time, can you reach the word "lope" in four changes?

fast
1.
2.
3.
4. lope

To construct an angle equal to a given angle at a point on a given line.



EXERCISE

1. Use the fundamental constructions to draw the following angles.
(a) 90° (b) 45° (c) $22\frac{1}{2}°$ (d) 135° (e) $67\frac{1}{2}°$

2. (a) Construct a rectangle with dimensions 4 cm by 7 cm.
(b) Bisect any two of the angles.
(c) Are the angle bisectors also the diagonals of the rectangle?

3. (a) Construct a parallelogram with dimensions 5 cm by 7 cm and one angle 45°.
(b) Draw the two diagonals.
(c) Determine by measurement whether the two diagonals bisect each other.

4. (a) Construct a right triangle with the sides containing the right angle 4 cm and 3 cm.
(b) Draw the right bisector of the hypotenuse.
(c) With compasses and centre at the midpoint of the hypotenuse, draw the circle through the three vertices of the triangle.

Insert signs of operation to make a true statement:
5 ▨ 5 ▨ 1 ▨ 2 = 0

CHAPTER 12

Geometric Vectors

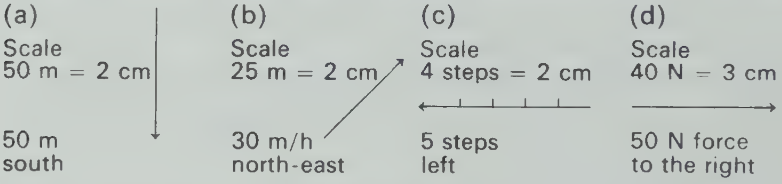
Quantities such as 50 m, 30°C, $3\ \text{cm}^2$, 17 yrs, and 20 km/h relate to the **magnitude** of a measurement and are called **scalar quantities,** or simply **scalars.** Other quantities such as 50 km to the south or 10 m to the left have **direction** as well as magnitude and are called **vector quantities,** or simply **vectors.**

Identify the following as scalars or vectors:

1. 10 s
2. 5 km south
3. 14 a old
4. 20 cm
5. 30 km/h west wind
6. 45°C
7. 30 m straight up
8. $36 50
9. 5 steps to the left
10. 50 N force acting down
11. 47 students
12. 300 m/s

12.1 DIRECTED LINE SEGMENTS

Since a vector has magnitude and direction, we can represent a vector geometrically by a **directed line segment** as below.



In each case the length of the line segment is proportional to the **magnitude** and the arrowhead indicates the **direction.** Most vector diagrams will be scale drawings, so you must select a suitable scale for your diagrams.

EXAMPLE 1. *Using a suitable scale, make a vector diagram for each of the following:*

(a) 35 m east

(b) 30 m/s east

(c) 620 N force downwards

(d) 125 km south-west

Solution

(a) Scale: 25 m → 2 cm

$1\text{ m} \rightarrow \frac{2}{25}\text{ cm}$

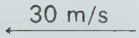
$35\text{ m} \rightarrow 35 \times \frac{2}{25} = 2.8\text{ cm}$

Direction: east requires arrowhead pointing right.

(b) Scale: 10 m/s → 1 cm

$1 \text{ m/s} \rightarrow \frac{1}{10} \text{ cm}$

$30 \text{ m/s} \rightarrow 3 \text{ cm}$

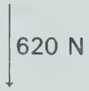


Direction: east wind blows from east to west and requires arrowhead pointing left.

(c) Scale: 500 N → 2 cm

$1 \text{ N} \rightarrow \frac{2}{500} \text{ cm}$

$620 \text{ N} \rightarrow 620 \times \frac{2}{500} = 2.25 \text{ cm}$

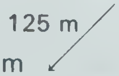


Direction: downwards requires arrowhead pointing down.

(d) Scale: 100 km → 2 cm

$1 \text{ km} \rightarrow \frac{2}{100} \text{ cm}$

$125 \text{ km} \rightarrow 125 \times \frac{2}{100} = 2.25 \text{ cm}$



Direction: south-west requires arrowhead pointing downwards to the left at 45°.

EXERCISE 12-1

Time Zones:
If it is 03:00 in Paris, what time is it 4 h later in Los Angeles?

1. Calculate the length of the directed line segment representing the following vectors using the scale: 20 km/h = 1 cm

(a) 120 km/h south
(b) 90 km/h south-east
(c) 30 km/h north
(d) 45 km/h north
(e) 75 km/h west

2. Calculate the length of the directed line segment representing the following vectors using the scale: 2 N = 1 cm

(a) 10 N left
(b) 15 N to the right
(c) $7\frac{1}{2}$ N to the right
(d) 12.5 N upwards
(e) 4 N downwards

3. Calculate the length of the directed line segment representing the following vectors using the scale: 20 N = 1 cm

(a) 60 N down
(b) 40 N left
(c) 75 N right
(d) 25 N left
(e) 110 N right

4. Using the scale: 20 N = 1 cm, calculate the lengths of the directed line segments, then make vector diagrams for each of the following:

(a) 80 N to the right
(b) 100 N downward
(c) 50 N to the left
(d) 120 N upward
(e) 80 N upward to the right at 30°

5. For each of the following vectors,

(i) state a suitable scale
(ii) calculate the length of the directed line segment
(iii) make a vector diagram

(a) 60 m/s to the north-west
(b) 45 N to the left
(c) 450 km south
(d) 60 paces west
(e) a 12 N force up a 30° incline
(f) 30 steps upward
(g) a 24 N force upward to the left at 50°

6. Make a vector diagram for each of the following:

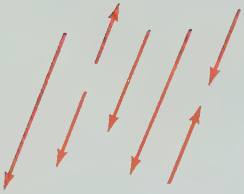
(a) a 40 m/s north wind
(b) the wind blows south-east at 15 m/s
(c) a ship has sailed 500 km in a direction 30° south of west
(d) a cable car is being hauled up a track which is inclined at 25° to the horizontal by a force of 7000 N

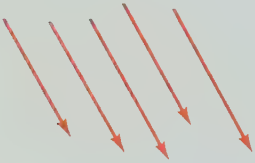
12.2 COLLINEAR VECTORS

Vectors that act **along the same line** are called **collinear** vectors.

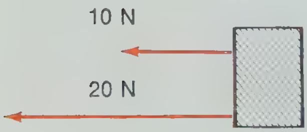
Vectors that have both the same **magnitude** and the same **direction** are called **equal** vectors.

In this section we will study the effect of two or more vectors acting in the same line.

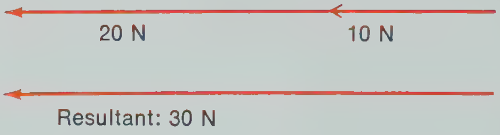

Collinear Vectors


Equal Vectors

EXAMPLE 1. *Two forces, 20 N and 10 N, both acting to the left, are applied to an object. What single force has the same effect as this pair of forces?*



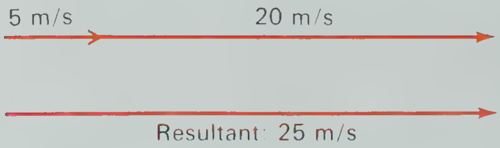
Solution We make a vector diagram:
Scale: 5 N = 2 cm



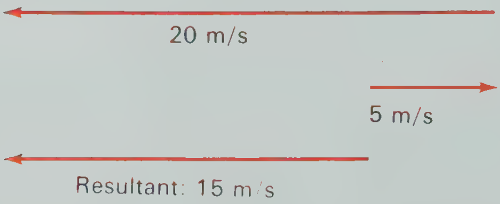
The single force that has the same effect as the 20 N and 10 N force is a 30 N force acting in the same direction. This single force is called the **resultant** of the two forces.

EXAMPLE 2. *A boat is capable of travelling 20 m/s in still water. Find the speed of the same boat in a stream with a current of 5 m/s if the boat is going (a) downstream, (b) upstream.*

Solution Scale: 5 m/s = 2 cm
(a) Downstream:



The downstream speed is 25 m/s.
(b) Upstream:



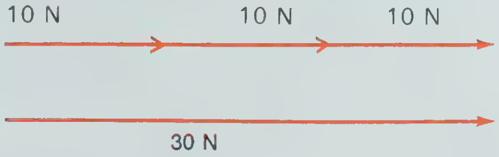
The upstream speed is 15 m/s.

While it was simple to add the collinear vectors of Example 1 or Example 2(a), we note that the vectors of Example 2(b) have **opposite directions.** Adding vectors with opposite directions in this example is equivalent to subtracting the 5 m/s vector from the 20 m/s vector. Note that the direction of the resultant is the direction of the larger vector.

Two people stood facing in opposite directions, yet each could see the other without turning round. How?

EXAMPLE 3. *What is the resultant of 3 forces of 10 N each, all acting to the right?*

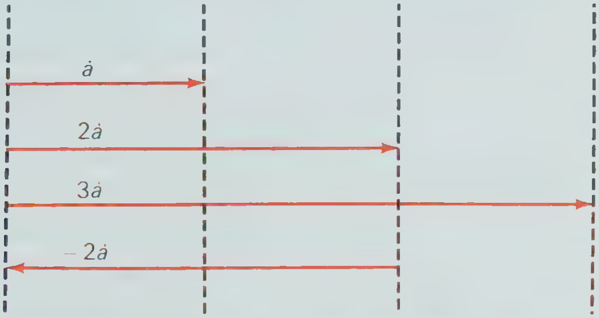
Solution Scale: 5 N = 1 cm



The resultant is 30 N to the right.

The above diagram suggests that a vector can be multiplied by a scalar—in this case, multiplying the vector quantity "10 N to the right" by the scalar, 3, resulted in another vector quantity "30 N to the right." We use the symbols $\vec{a}$, $\vec{b}$, $\vec{v}$, . . . , (a small letter with an $^\rightarrow$ above) to indicate a vector. We read these as "vector *a*, ,vector *b*, vector *v*, ..."

Consider the following diagram.



The multipliers 2, 3, and −2 are scalars. Note that when a vector is multiplied by a positive scalar the direction remains the same; when a vector is multiplied by a negative scalar the direction is reversed.

If we multiply a vector by 0, then we have

$$0 \cdot \vec{v} = \vec{0}$$

where $\vec{0}$ is called the **zero** or **null** vector. Hence the scalar can be any real number.

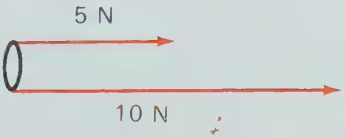
Starting with the word "port" and changing one letter at a time, can you reach the word "site" in four changes?

port
1.
2.
3.
4. site

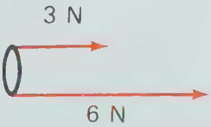
EXERCISE 12-2

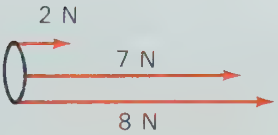
A **1.** Find the magnitude and direction of the resultant of each of the following:

(a)

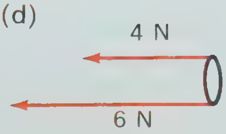


(b)

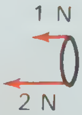




(d)

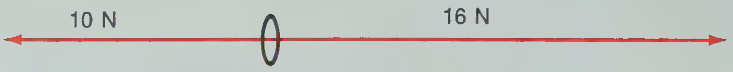


(e)

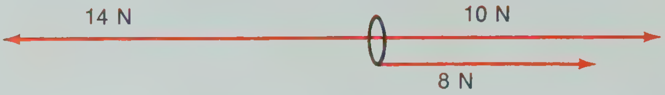


2. Find the magnitude and direction of the resultant of each of the following systems of forces.

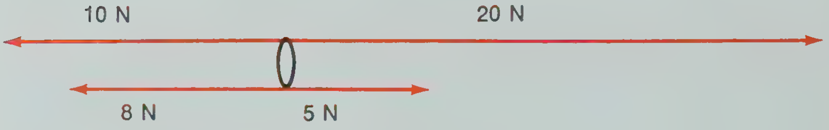
(a)



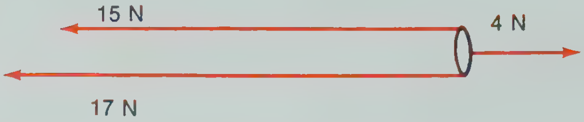
(b)



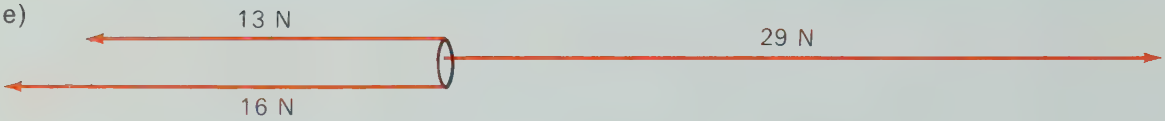
(c)



(d)



(e)



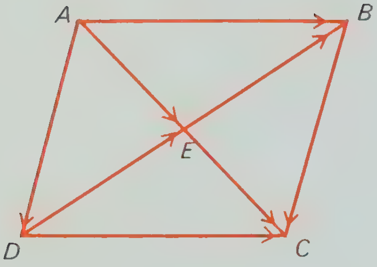
3. Find the magnitude and direction of the resultants of the following systems of forces:

(a) two 5 N forces acting to the right and one 7 N force acting to the left.

(b) two forces 18 N and 7 N acting in a northerly direction, and two forces, 5 N and 12 N acting in a southerly direction.

4. If $\vec{v}$ is a vector representing a force 20 N to the left, select a suitable scale and draw the following:

(a) $2\vec{v}$ (b) $3\vec{v}$ (c) $-2\vec{v}$ (d) $1.5\vec{v}$ (e) $0 \cdot \vec{v}$



5. $\overrightarrow{AB}$ represents the vector from A to B

(a) What does $\overrightarrow{BA}$ represent?

(b) Add $\overrightarrow{AB} + \overrightarrow{BA}$.

(c) Name all the vectors in the diagram.

(d) Name a vector equal to each of

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{BC}$ (iii) $\overrightarrow{AE}$ (iv) $\overrightarrow{EB}$

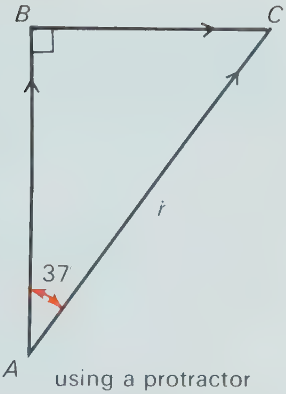
(e) Name two vectors collinear with $\overrightarrow{AC}$.

12.3 ADDITION OF VECTORS

In the previous section we dealt only with collinear vectors—acting in the same line. Using a **protractor** we can make scale diagrams to add non-collinear vectors.

EXAMPLE 1. *If you take 40 paces north and then 30 paces east, draw the vector diagram and find the resultant to determine how far you are from the starting point. Find your direction from the starting point.*

Solution Scale: 10 paces = 1 cm

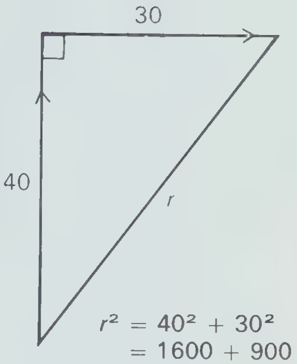


Length of $AC \doteq 5$ cm

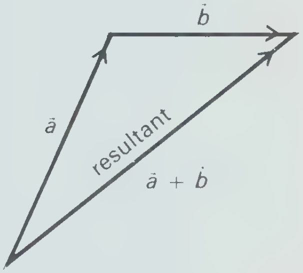
magnitude of $\vec{r} \doteq 5 \times 10$
$\doteq 50$
$\angle A = 37°$ (by measurement)

∴ You are 50 paces from your starting point in a direction 37° east of north.

Example 1 suggests that we can add two non-collinear vectors by arranging them so that the tail of one meets the arrowhead of the other. The resultant is found as the third side of the triangle that is formed. This is called the **triangle method.**

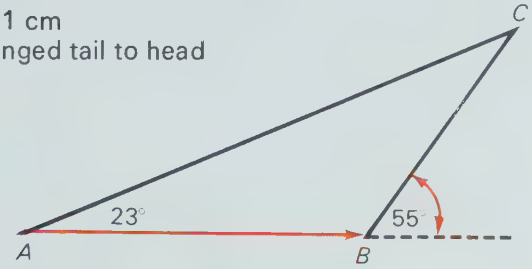


$$r^2 = 40^2 + 30^2$$
$$= 1600 + 900$$
$$= 2500$$
$$r = 50$$



EXAMPLE 2. *Use the triangle method to find the single force which is equivalent to the forces of 40 N and 30 N acting at a point with their lines of action forming an angle of 55°.*

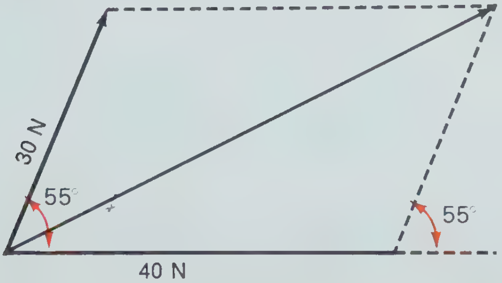
Solution Scale: 10 N = 1 cm
The vectors are arranged tail to head



Length of $AC \doteq 6.2$ cm
Magnitude of resultant $\doteq 6.2 \times 10 \doteq 62.5$ N
$\angle A \doteq 23°$

∴ The resultant has a magnitude of 62.5 N and is inclined 23° to the 40 N force.

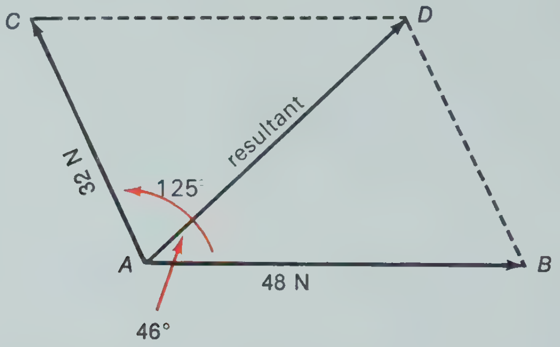
If the three vectors of Example 2 are arranged as in Example 2, we can complete the diagram to form a parallelogram.



This suggests that we can also add two non-collinear vectors using a **parallelogram method.**

EXAMPLE 3. *Use the parallelogram method to find the magnitude and direction of the resultant two forces, 32 N and 48 N, acting at 125° to each other.*

Solution Scale: 8 N = 1 cm



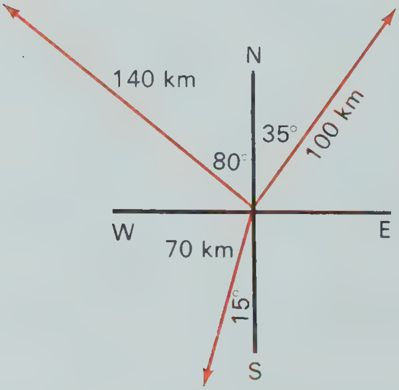
$$\begin{aligned} \text{The length of } AD &\doteq 5 \text{ cm} \\ \text{The magnitude of the resultant} &\doteq 8 \times 5 \\ &\doteq 40 \text{ N} \end{aligned}$$

in a direction 46° to the 48 N force.

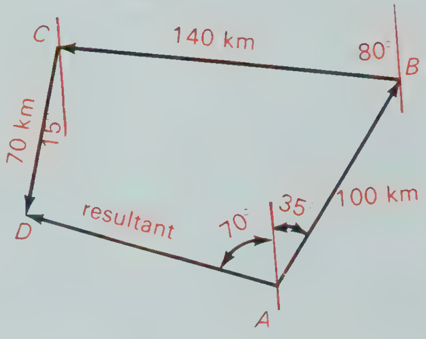
EXAMPLE 4. *A ship sails on the following course: 100 km at 35° east of north, then 140 km at 80° west of north and finally 70 km at 15° west of south. Find the single course that is equivalent to these three courses.*

Solution Scale: 20 km = 1 cm

(i) Here are the vectors



(ii) We arrange the vectors tail to head.



$$\begin{aligned} \text{The length of } AD &\doteq 5.35 \text{ cm} \\ \text{The magnitude of the resultant} &\doteq 20 \times 5.25 \\ &\doteq 105 \text{ km} \end{aligned}$$

in a direction 70° west of north.

This is often called the **polygon method** because the diagram includes a polygon (*ABCD*).

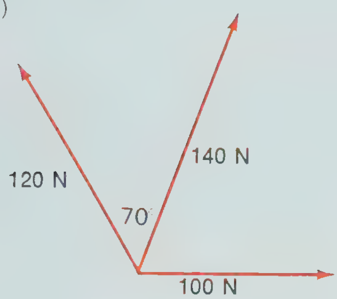
EXERCISE 12-3

B **1.** With suitable scales, make vector diagrams to find the magnitude and direction of the resultant of each of the following pairs of vectors.

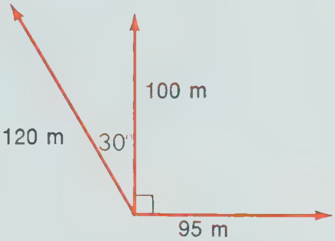
(a) 3 m and 2 m at 75° to each other.
(b) 40 N and 60 N at 90° to each other.
(c) 100 km/h and 20 km/h at 45° to each other.
(d) 35 m/s and 70 m/s at 40° to each other.
(e) 30 paces south and 40 paces south-west.
(f) 20 N and 20 N at 90° to each other.
(g) 30 m/s and 20 m/s at 40° to each other.
(h) 30 N and 50 N at 180° to each other.
(i) 40 N and 60 N at 120° to each other.
(j) 50 km north and 30 km south-east.

2. Make your own vector diagrams to find the magnitude of the resultant of each of the following systems of vectors.

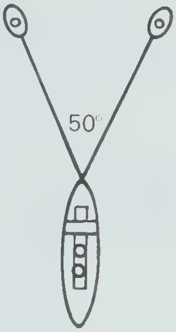
(a)



(b)



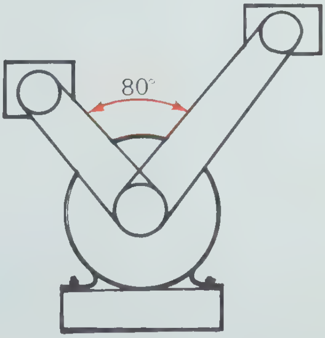
3. Two tugs tow an ocean liner. If the tension in each tow line is 7.5 kN, and the angle between the lines is 50°, find the magnitude of the total force acting on the liner.

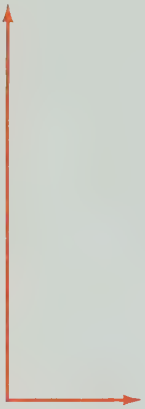


4. A man attempts to row a boat at 10 km/h directly across the 4 km/h current of a river. What is his true speed and direction?

5. A ship sails 300 km north and 125 km west, then develops engine trouble. Using a suitable scale, make a vector diagram to find the course a rescue ship would take to reach the stricken ship from the same port.

6. Two machines are driven by belts from the same shaft of a motor. The tensions (forces) in the belts cause forces of 8 and 12 N at 80° on the shaft of the motor. Make a vector diagram and find the total force on the motor shaft.





7. Two forces of 8 N and 10 N act at an angle of 75° with each other. Use a vector diagram to find the magnitude of their resultant, and the angle that it makes with the 8 N force.

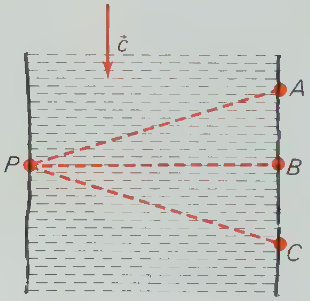
8. Arrange three forces of 7, 9, and 13 N in a triangle so that the 13 N force is the resultant of the other two. Find the angles between the three forces.

9. A motorboat which has a speed of 15 km/h in still water sets out to cross a stream which has a current of 5 km/h. If the boat heads directly across, find its actual speed and the direction that it takes.

12.4 NAVIGATION PROBLEMS

1 nautical mile is 1852 m, or 1′ of latitude.

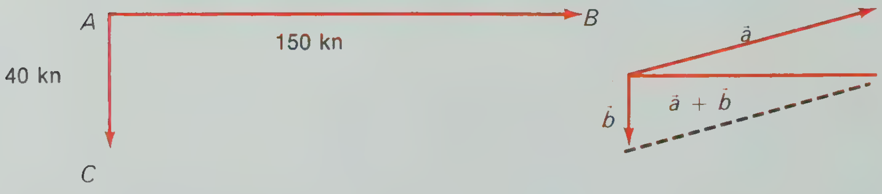
In the problems that we have done to this point, we have seen how winds in the air and currents in water alter the speed and direction of the craft. Suppose you wish to cross a river from a point P on the left side to the right side. Note that the direction of the current is given by the vector $\vec{c}$.



If you head directly across, intending to reach B, the current carries the craft to C. In what direction would you aim the craft to reach B?

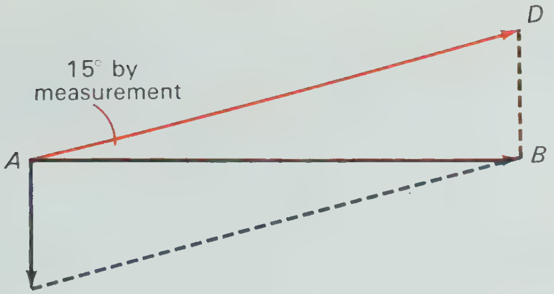
EXAMPLE 1. *An aircraft wishes to fly at 150 kn in an easterly direction. What course should the pilot set in order to overcome a 40 kn north wind?*

Solution Scale: 40 kn = 1 cm



$\overrightarrow{AB}$ represents the desired course 150 kn east.
$\overrightarrow{AC}$ represents the 40 kn north wind.

We must find $\overrightarrow{AD}$ so that $\overrightarrow{AB}$ is the resultant of adding $\overrightarrow{AD}$ and $\overrightarrow{AC}$. A navigator does this by drawing a parallelogram where $\overrightarrow{AC}$ is one side and $\overrightarrow{AB}$ is one diagonal. The other side is the course he sets.



① draw this arc with centre B and radius AC
② draw this arc with centre A and radius AB

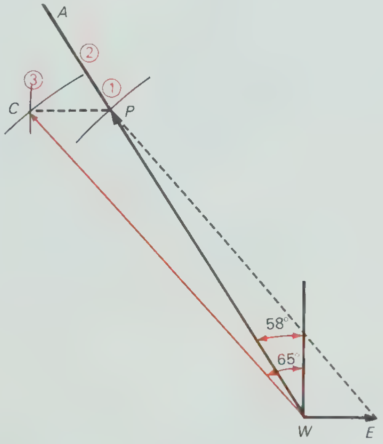
The two arcs intersect at D, so that $\overrightarrow{AD}$ is the required course (note that $ACBD$ is a parallelogram).

$$
\begin{aligned}
\text{The length of } AD &\doteq 3.9 \text{ cm}\\
\text{The magnitude of } \overrightarrow{AD} &\doteq 40 \times 3.9\\
&\doteq 156 \text{ kn}
\end{aligned}
$$

∴ She sets a course 15° north of east at 156 kn.

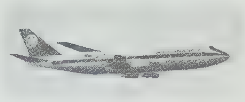
EXAMPLE 2. *Prince Albert is north 58° west of Winnipeg. Find the course that must be set to fly from Winnipeg to Prince Albert when there is a 35 kn west wind and the cruising speed of the aircraft is 180 kn. What will be the ground speed?*

Solution Scale: 16 kn = 1 cm



$\overrightarrow{WE}$ represents the wind.
The line WA gives the direction to Prince Albert.
With compasses, radius 11.3 cm (180 kn) and centres W and E draw arcs ① and ②;
Arc ① cuts WA at P. With centre P and radius WE draw arc ③ to cut arc ② at C.
$\overrightarrow{WC}$ represents the required course, 65° west of north, and the ground speed is approximately 145 kn.

Time zones:
If it is 18:00 in Moscow, what time was it 4 h earlier in Honolulu?



Pictures courtesy of Air Canada.

EXERCISE 12-4

Answer the following questions using vector diagrams.

1. A pilot wishes to fly at 105 kn toward the west while there is a 30 kn south wind. Find the course he must set and the air speed.

2. A boat travels at 20 kn across a stream with a 4 kn current. Find the direction the boat must take in order to travel directly across the current, and also the actual speed of the boat relative to the ground.

3. (a) What course must a pilot set to fly north if his air speed is 150 knots and there is a 50 kn west wind?
(b) Find the ground speed.

4. (a) In what direction would you head a motorboat that travels at 20 kn relative to the water to cross an 8 kn current?
(b) What is the speed of the boat relative to the land?

5. (a) Find the course required to fly in a direction 30° west of north at 200 kn while there is a 50 kn west wind.
(b) What air speed should the aircraft have?

6. (a) Find the course required to fly in a south-east direction while the wind blows 30° west of north at 20 kn, if the cruising speed of the aircraft is 150 kn.
(b) What is the ground speed?

7. A small aircraft flies at 120 kn in a direction 50° west of south while there is a 20 kn south-east wind. Find the ground speed and direction relative to the ground.

Time zones:
What happens to the date when you cross the International Date line going from (i) east to west (ii) west to east?

8. A pilot whose aircraft cruises at 150 kn wishes to fly in a direction 30° east of south while there is a 40 kn east wind. Find the course he must set and determine the ground speed.

9. Plot the course and calculate the ground speed of an aircraft cruising at 150 kn if there is a 30 kn east wind and you wish to travel north-east.

10. A pilot wishes to fly his aircraft directly east. What course must he set and what will be the ground speed if there is a 50 kn north-east wind and the air speed of the aircraft is 150 kn?

REVIEW EXERCISE

1. (a) State the difference between a vector and a scalar.
(b) Give three examples of each.

2. If $\vec{v}$ represents a vector 1 cm long, draw $\vec{v}$, $3\vec{v}$, $-2\vec{v}$, and $1\frac{1}{2}\vec{v} + 2\vec{v}$.

3. State a suitable scale for each and draw the following vectors:
(a) 9 N to the right
(b) 16 N to the left
(c) 30 N, 45° down and to the right
(d) 35 N, 45° upwards to the right

4. Using ruler, compasses, and protractor, find the resultant of the following systems of forces, using the triangle method.
(a) 10 N and 15 N acting at 30° to each other.
(b) 8 N and 12 N acting at 70° to each other.
(c) 20 N and 30 N acting at 90° to each other.
(d) 6 N and 5 N acting at 80° to each other.
(e) 12 N and 16 N acting at 120° to each other.

5. Find the resultant of each of the following using the parallelogram method:
(a) 9 N and 7 N acting at 130° to each other.
(b) 10 N and 20 N acting at 90° to each other.
(c) 7 N and 8 N acting at 45° to each other.
(d) 6 N and 5 N acting at 150° to each other.
(e) 11 N and 12 N acting at 80° to each other.

6. Two boys are pulling a log with forces of 10 N and 16 N respectively. The angle between the two forces is 55°. Find the magnitude of the resultant and the angle that it makes with the 10 N force.

7. Two tow trucks pull a car with forces of 800 N and 700 N respectively with an angle of 20° between the tow chains.
(a) Find the magnitude of the resultant.
(b) If the car moves, in what direction does it move relative to the more powerful truck?

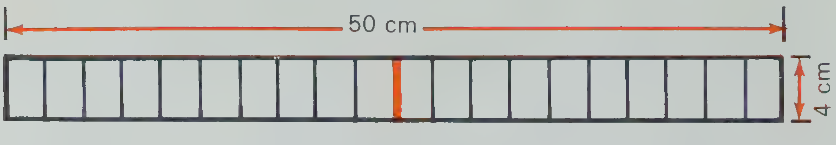
8. Arrange, if possible, the following systems of forces so that the larger force is the resultant of the other two.
(a) 9 N, 10 N, 12 N
(b) 6 N, 10 N, 5 N
(c) 4 N, 4 N, 8 N
(d) 5 N, 6 N, 12 N

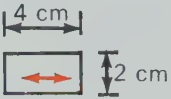
9. An aircraft with a cruising speed of 150 kn is to fly in a direction 30° south of east relative to the ground, while there is a 25 kn south-east wind. Find the required course for the flight and determine the ground speed.

10. A ship is towed into harbour by two tugs, each pulling with a force of 4 kN. If the angle between the tow lines is 35°, make a vector diagram and find the magnitude of the resultant force.

FIGHTING FORCES

Two pair of dice, two players and a piece of bristol board 4 cm by 50 cm are required for this game. Indicate the centre of the strip with a heavy line. Draw lighter lines every centimetre along the strip as shown below.





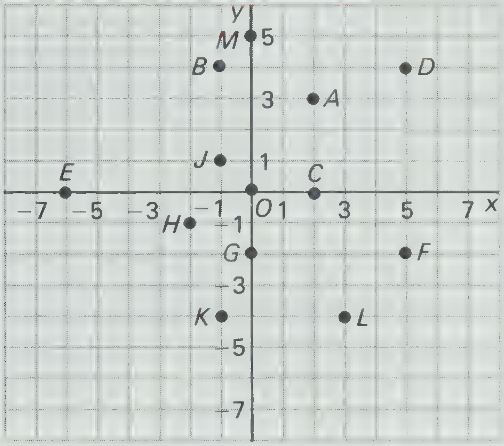
marker

Cut another piece of bristol board 2 cm by 4 cm and make a marker as above right.

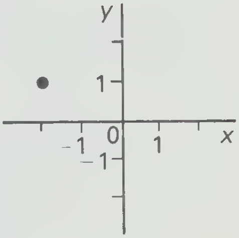
X and **Y** represent two forces pulling in opposite directions. Place the marker on the heavy centre line of the 50 cm strip. One player plays force **X** and the other force **Y**. The players roll their dice simultaneously. If player **X** rolls 7 and player **Y** rolls 9, the resultant is 2 in favour of **Y**. Player **Y** then moves the marker 2 units. The dice are rolled again and the game continues until the marker reaches one end of the strip.

REVIEW AND PREVIEW TO CHAPTER 13

1. State the coordinates of each point in the diagram at right.



2. In your notebook, set up the axes as shown, showing a large dot on the point (− 2,1).



Plot the points below, joining each point to the following point.

Start (− 3,0), (− 4.5,0.5), (− 4,1), (− 3,1.5), (− 2,2), (− 1,2.5), (0,3.5), (2,4), (3,4), (2,3), (1,2), (2,1.5), (3,1), (6,1.5), (5,1), (4,0.5), (3,0), (4,− 1.5), (3,− 1), (2,− 0.5), (1,− 1), (0,− 1.5), (1,− 3), (0,− 2.5), (− 1,− 2), (− 2,− 2.5), (− 3,− 2), (− 4,− 1), (− 4.5,− 0.5), (− 3,0).

3. Determine which ordered pairs satisfy the given equation.

(a) $y = 2x - 3$; $A\,(1,-2)$, $B\,(0,3)$, $C\,(4,5)$
(b) $3x + 2y = 12$; $P(1,4)$, $Q\,(4,0)$, $R(-2,9)$
(c) $4x - y = 8$; $M(0,8)$, $N(1,4)$, $P(1,-4)$
(d) $x = 3y + 2$; $S(-1,-1)$, $T(-4,-2)$, $V(5,1)$
(e) $y - 3x = 6$; $P(-3,-3)$, $W(0,-2)$, $R(2,12)$
(f) $x = 5$; $A\,(5,5)$, $B\,(0,5)$, $C\,(5,0)$

4. Determine the x- and y-intercepts for the graph of each equation.

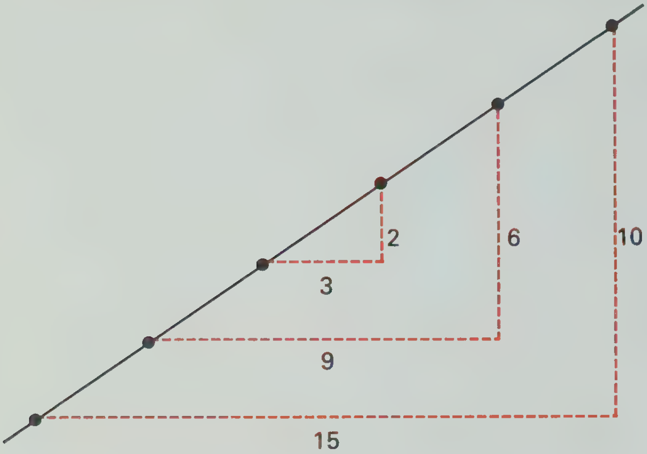
(a) $y = x - 1$
(b) $2x + y = 10$
(c) $x - y = 5$
(d) $3x + 4y = 12$
(e) $4x - 5y = 40$
(f) $x = -2$
(g) $x - y = 0$
(h) $y = 8$

5. Rearrange each of the formulas to solve for the indicated variable.

(a) $d = vt$; v
(b) $c = 2\pi r$; r
(c) $P = a + b + c$; b
(d) $I = Prt$; P
(e) $y = mx + b$; b
(f) $v = u + at$; u
(g) $V = \pi r^2 h$; h
(h) $A = \frac{1}{2}bh$; b
(i) $P = 2l + 2w$; l
(j) $A = P + Prt$; t

CHAPTER 13

Coordinate Geometry



13.1 SLOPE

The word *slope* refers to the measure of the "steepness" of many things — a roof, a mountain, a road, or a line.

For all of these,

$$\boxed{\text{slope} = \frac{\text{rise}}{\text{run}}}$$

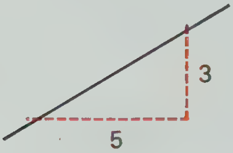
where, between any two points, the rise is the vertical distance, and the run is the horizontal distance.

$\frac{2}{3} = \frac{6}{9}$
$= \frac{10}{15}$
.
.
.

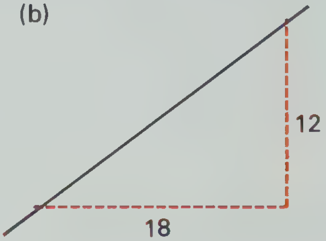
The line above has a slope of $\frac{2}{3}$.

EXAMPLE 1. *State the slope of each line.*

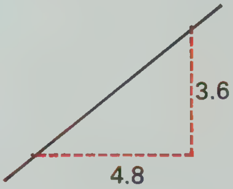
(a)



(b)



(c)



Solution

(a) *slope is* $\frac{3}{5}$

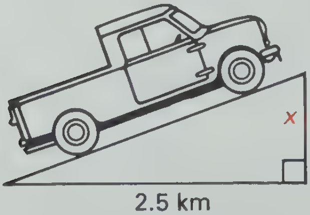
(b) *slope is* $\frac{12}{18}$
$= \frac{2}{3}$

(c) *slope is* $\frac{3.6}{4.8}$
$= \frac{3}{4}$

EXAMPLE 2. *The grade of a roadway is 5%. For a section with a 2.5 km run, calculate the rise in metres.*

Solution *A grade or slope of 5% can be written as* $\frac{5}{100}$. *Let x represent the rise in metres.*

2.5 km = 2500 m

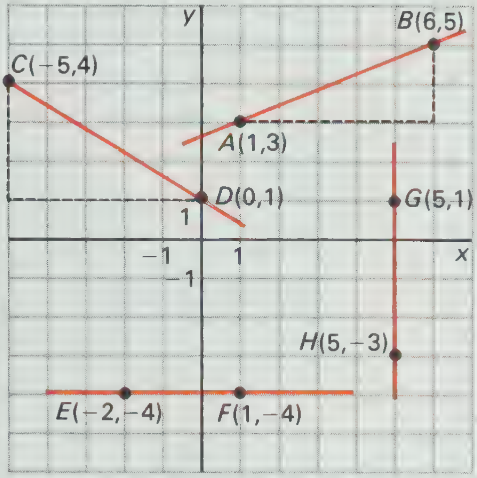


$$\frac{x}{2500} = \frac{5}{100}$$

$\therefore 100x = 12\,500$
$x = 125$
$\therefore$ *the rise will be 125 m*

We can calculate the slope of a line when given the coordinates of two points on the line.

EXAMPLE 3. *Calculate the slope, m, of each of the line segments shown.*



Solution

(i) For AB

rise of AB is $5 - 3 = 2$
run of AB is $6 - 1 = 5$
$\therefore m = \frac{2}{5}$

OR

rise of BA is $3 - 5 = -2$
run of BA is $1 - 6 = -5$
$\therefore m = \frac{-2}{-5}$
$= \frac{2}{5}$

(ii) For CD

rise of CD is $4 - 1 = 3$
run of CD is $-5 - 0 = -5$
$\therefore m = -\frac{3}{5}$
$= -\frac{3}{5}$

(iii) For EF

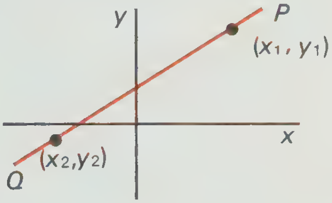
rise of EF is $-4 - (-4) = 0$
run of EF is $1 - (-2) = 3$
$\therefore m = \frac{0}{3}$
$= 0$

(iv) For GH

rise of GH is $1 - (-3) = 4$
run of GH is $5 - 5 = 0$
$\therefore m = \frac{4}{0}$ *which is undefined.*

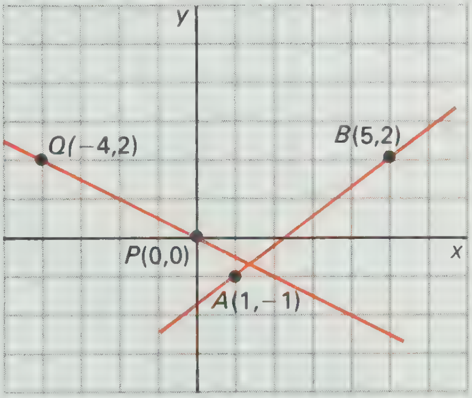
The slope of a line through any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$



EXAMPLE 4. *Calculate the slopes of the lines through the following points. Sketch each line.*

(a) $A(1, -1)$ and $B(5, 2)$
(b) $P(0, 0)$ and $Q(-4, 2)$
(c) $M(2.4, 4.8)$ and $N(1.2, 3.0)$



slope $AB \Leftrightarrow m_{AB}$

Solution

(a) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} = \frac{2 - (-1)}{5 - 1}$

$= \frac{3}{4}$

(b) $m = \frac{y_2 - y_1}{x_2 - x_1}$

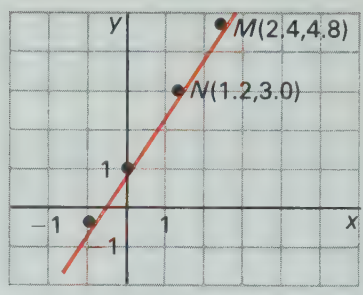
$m_{PQ} = \frac{2 - 0}{-4 - 0}$

$= \frac{2}{-4}$ ← *this indicates that the run from P to Q is a directed distance to the left.*

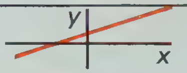
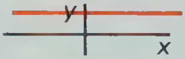
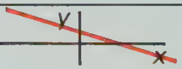
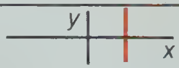
$= -\frac{1}{2}$

A line sloping up to the right has a positive slope. A line sloping up to the left has a negative slope.

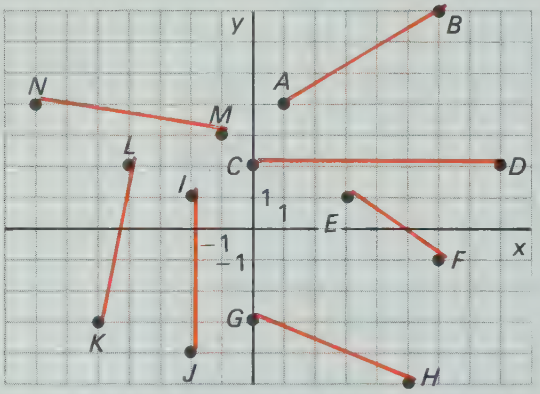
$$\frac{1.8}{1.2} = \frac{1.8 \times 10}{1.2 \times 10}$$
$$= \frac{18}{12}$$
$$= \frac{3}{2}$$



(c) $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{MN} = \frac{4.8 - 3.0}{2.4 - 1.2}$$
$$= \frac{1.8}{1.2}$$
$$= \frac{3}{2}$$

Slope	Sketch	Description
positive		up to the right
zero		horizontal
negative		up to the left
undefined		vertical

EXERCISE 13-1



A 1. (a) For each line segment shown, state the rise and the run.

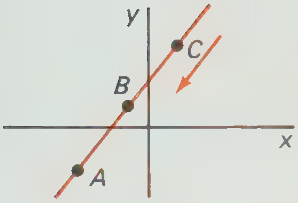
(b) State the slope of each line segment.

B 2. Calculate the slope of the line passing through each pair of points.

(a) $A(2,3)$, $B(5,9)$
(b) $P(4,0)$, $Q(6,4)$
(c) $M(0,0)$, $N(4,3)$
(d) $C(2,5)$, $D(4,5)$
(e) $T(4,-3)$, $S(5,2)$
(f) $A(-1,-3)$, $B(5,4)$
(g) $H(-3,1)$, $K(0,2)$
(h) $J(-4,-3)$, $K(-1,5)$
(i) $A(-6,0)$, $B(4,-5)$
(j) $P(1,-5)$, $Q(-3,-2)$
(k) $M(1.5,2)$, $N(3,3)$
(l) $E(0.3,2)$, $F(0.7,1)$
(m) $F(4,3.6)$, $G(6,5)$
(n) $H(2.5, 3.2)$, $J(2.4,3.3)$
(o) $A(10,4.2)$, $B(8.5,4.8)$
(p) $S(-1.6,1.8)$, $T(2,-1.8)$

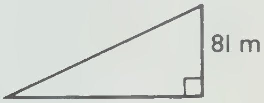
3. (a) Plot the points $P(-3,-1)$, $Q(0,1)$ and $R(6,5)$.
(b) Find the slope of PQ.
(c) Find the slope of QR.
(d) How are the points related?

> Three or more points on the same line are said to be *collinear*.
>
> To show that points are collinear, show that the slopes of the line segments are equal.



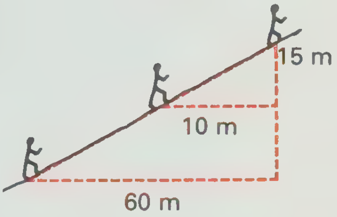
4. Determine if the three points in each case are collinear.
(a) $P(0,7)$, $Q(1,1)$, $R(2,-5)$ (b) $A(-8,-2)$, $B(0,0)$, $C(4,1)$
(c) $D(-1,0)$, $E(0,1)$, $F(2,3)$ (d) $M(2,5)$, $N(-1,-2)$, $P(-2,-3)$

5. The grade of a road is 3%.
(a) Calculate the vertical climb in metres for a horizontal distance of 1.5 km.
(b) Calculate the run in kilometres necessary to reach the top of an 81 m hill.



6. Two Olympic ski runs are to be compared. One has a vertical drop of 450 m for a horizontal run of 1.2 km, and the other a drop of 625 m for a run of 1.8 km. Which hill has the greater slope?

7. John, Pierre and Sue are climbing a hill. Sue is 15 m higher than Pierre and 10 m ahead horizontally. If Sue is 60 m horizontally ahead of John, how much higher is Sue than John?



13.2 SLOPE OF A LINE

If the equation of a line is known, the slope may be calculated by finding two points on the line.

EXAMPLE 1. *Find the slope of the line $3x - 2y = 12$*

Solution

When $x = 0$, $y = -6$
When $y = 0$, $x = 4$
$\therefore$ two points are $A(0,-6)$ and $B(4,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - (-6)}{4 - 0}$$

$$= \tfrac{6}{4}$$

$$= \tfrac{3}{2}$$

EXAMPLE 2. *Express the following equations in the form $y = mx + b$. State the value of m and b.*

(a) $3x + y = 5$ *(b)* $4x - 2y = 6$

Solution

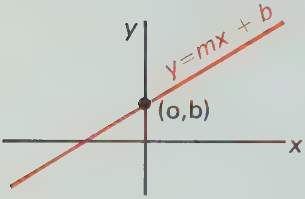
(a)
$$3x + y = 5$$
$$\therefore y = -3x + 5$$
$$\therefore m = -3$$
$$b = 5$$

(b)
$$4x - 2y = 6$$
$$-2y = -4x + 6$$
$$y = 2x - 3$$
$$\therefore m = 2$$
$$b = -3$$

Investigation 13-1

1. Complete the table below in your notebook. Use the two points (one of which must have $x = 0$) to calculate the slope.

	Equation	Equation in the form $y = mx + b$	m	b	Two Points $(0,▨)$ $(▨, ▨)$	Slope $\frac{y_2 - y_1}{x_2 - x_1}$
(a)	$y - 5x = 2$					
(b)	$4x + y = 8$					
(c)	$x - y = 5$					
(d)	$10x - 5y = 15$					
(e)	$x - 2y = 5$					
(f)	$4x + 3y = 12$					



2. State the relationship between (i) m and the slope, (ii) b and the y-coordinate of the point $(0,▨)$.

The y-intercept of a line is equal to the *y-coordinate* of the point at which the line intersects the y-axis.

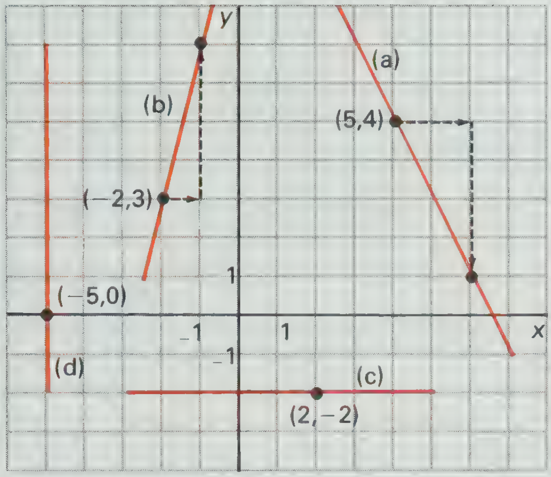
The slope and y-intercept of a line may be found by rearranging the equation into the form.

$$y = \underset{\text{slope}}{\underset{\uparrow}{m}}x + \underset{y\text{-intercept}}{\underset{\uparrow}{b}}$$

We can sketch a line if we know a point and the slope.

EXAMPLE 3. *Using the given information, graph each line*
(a) *through $(5,4)$ with slope $-\frac{3}{2}$*
(b) *through $(-2,3)$ with slope 4*
(c) *through $(2,-2)$ with slope 0*
(d) *through $(-5,0)$ with an undefined slope*

Solution



EXERCISE 13-2

1. Determine the slope and y-intercept of each line by first expressing the equation in the form $y = mx + b$.

(a) $3x + y = 5$

(b) $4x + 2y = 6$

(c) $2x - y = 6$

(d) $y - 5x = 2$

(e) $x + y + 5 = 0$

(f) $3x + 4y = 2$

(g) $5y - 2x = 3$

(h) $2x + 6y = 5$

(i) $3x - 12y = 4$

(j) $x - 3y = 4$

(k) $5x + 5y = 2$

(l) $3x - 3y - 1 = 0$

2. Express the equation $y = 5$ in the form $y = mx + b$. State the slope and y-intercept of this line.

3. Use the following points and slopes to sketch the lines.

(a) $m = \frac{1}{2}$; $P(0,5)$

(b) $m = \frac{2}{5}$; $P(1,2)$

(c) $m = -\frac{2}{3}$; $P(2,0)$

(d) $m = \frac{1}{4}$; $P(-1,4)$

(e) $m = 0$; $P(2,-3)$

(f) $m = -\frac{1}{3}$; $P(-2,-4)$

(g) $m = -\frac{5}{4}$; $P(0,0)$

(h) $m = 0$; $P(-1,6)$

4. Graph the line with an undefined slope through the given point.

(a) $P(2,3)$ (b) $Q(5,-1)$ (c) $A(-1,2)$ (d) $K(-3,-5)$

5. Find the equation of each line in the form $y = mx + b$.

(a) $m = 3, b = 5$

(b) $m = \frac{1}{2}, b = -2$

(c) the slope is $\frac{3}{4}$ and the y-intercept is 6

(d) with slope -3 and through $(0,4)$

(e) $m = \frac{2}{3}$ and through $(0,-1)$

(f) $m = -5$ and through $(0,0)$

(g) $m = 0.6, b = 1.8$

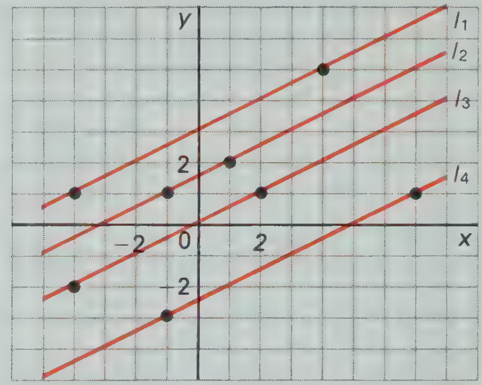
(h) $m = -2.5, b = 400$

13.3 PARALLEL LINES

Investigation 13-2

1. Using the points indicated on each line, complete the table below in your notebooks.

line	slope
l_1	
l_2	
l_3	
l_4	



2. How are the lines in the diagram related?

Parallel lines have equal slopes.

EXAMPLE 1. *Show that the line through A(− 3,5) and B(1,0) is parallel to the line through P(− 1,− 1)and Q(− 5,4).*

Solution

$$m_{AB} = \frac{0-5}{1-(-3)} = -\tfrac{5}{4} \qquad m_{PQ} = \frac{4-(-1)}{-5-(-1)} = -\tfrac{5}{4}$$

Since the slopes are equal, the lines are parallel.

EXAMPLE 2. *If the line through M(2,1) and N(5,3) is parallel to the line through P(4,7) and Q(x,− 1), calculate the value of x.*

Solution

$$\begin{aligned} MN \quad & PQ \\ \therefore m_{MN} &= m_{PQ} \\ \frac{3-1}{5-2} &= \frac{-1-7}{x-4} \\ \frac{2}{3} &= \frac{-8}{x-4} \\ \therefore 2x-8 &= -24 \\ 2x &= -16 \\ x &= -8 \end{aligned}$$

If equations of the line are known, the slopes may be compared by expressing the equations in the form $y = mx + b$.

EXAMPLE 3. *Determine if the lines represented by the equations $3x + y = 5$ and $6x + 2y = 7$ are parallel.*

Solution

$$\begin{aligned} 3x + y &= 5 \\ \therefore y &= -3x + 5 \\ \therefore m &= -3 \end{aligned} \qquad \begin{aligned} 6x + 2y &= 7 \\ 2y &= -6x + 7 \\ y &= -3x + \tfrac{7}{2} \\ \therefore m &= -3 \end{aligned}$$

Since the slopes are equal, the lines are parallel.

EXERCISE 13-3

A **1.** State the slope of lines parallel to the given line.

(a) $y = 5x + 2$	(b) $y = \frac{2}{3}x + 5$
(c) $y = -x + 6$	(d) $y = 8$
(e) $y - 2x = 3$	(f) $y + 5x = 8$
(g) $x + y = 5$	(h) $7x + y = 5$

2. State a value which may be substituted in each case to make the slope equal to $\frac{2}{3}$.

(a) $\dfrac{▨ - 1}{8 - 5}$	(b) $\dfrac{4 - ▨}{2 - (-1)}$	(c) $\dfrac{5 - ▨}{1 - 4}$
(d) $\dfrac{9 - 1}{▨ - (-2)}$	(e) $\dfrac{6 - 8}{4 - ▨}$	(f) $\dfrac{11 - 9}{▨ - (-3)}$

B **3.** Determine which pairs of line segments are parallel.

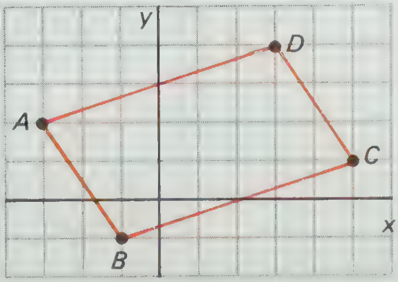
(a) $A(-1,2)$ $B(0,1)$ and $C(2,4)$ $D(0,6)$
(b) $R(-1,1)$ $P(4,3)$ and $S(1,-2)$ $T(8,0)$
(c) $P(-2,1)$ $Q(2,-1)$ and $S(4,3)$ $T(7,0)$
(d) $M(5,9)$ $N(8,1)$ and $O(0,0)$ and $P(3,-8)$
(e) $A(-4,0)$ $B(-2,6)$ and $C(-9,2)$ $D(-7,7)$
(f) $H(-5,-10)$ $J(-3,-14)$ and $K(-9,-8)$ $L(-7,-12)$

4. (a) Determine the slopes of the lines through $P(-2,4)$, $Q(3,4)$, and $M(7,0)$, $N(2,0)$.
(b) How are these two lines related?
(c) What is the slope of a line parallel to the x-axis?

5. (a) Describe the slopes of the lines through $A(4,6)$, $B(4,-2)$ and $C(0,9)$, $D(0,1)$.
(b) How are these two lines related?
(c) Describe the slope of a line parallel to the y-axis.

6. Determine if the given pair of equations define parallel lines.
(a) $4x + y = 8$ and $12x + 3y = 15$
(b) $5x + 10y = 2$ and $y = \frac{1}{2}x - 1$
(c) $3x + y = 1$ and $y = -3x + 1$
(d) $4x + 3y = 12$ and $8x - 6y = 24$
(e) $y = -5$ and $y + 2 = 0$
(f) $2x + 8y - 5 = 0$ and $x = 4y + 5$

7. *ABCD* looks like a parallelogram. Use slopes to show that the opposite sides are parallel.

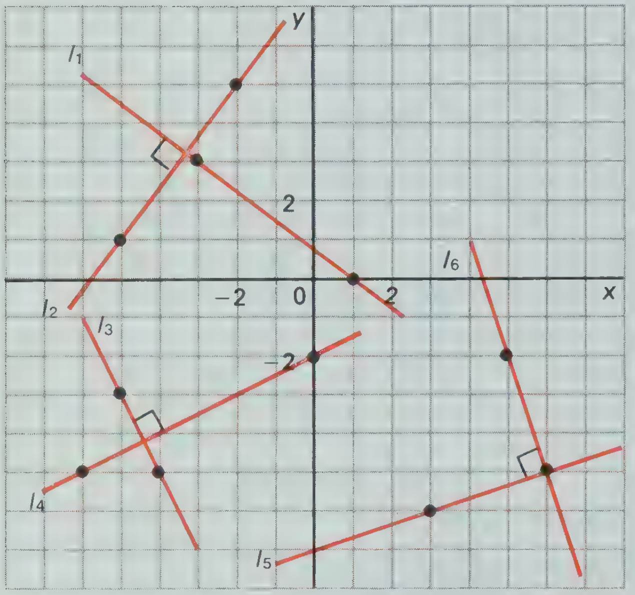


8. If the lines through each pair of points are parallel, find the value of the unknown coordinate.

(a) $A(2,5)$, $B(4,y)$ and $C(0,2)$, $D(2,3)$
(b) $P(4,-2)$, $Q(5,1)$ and $S(2,y)$, $T(1,0)$
(c) $M(5,4)$, $N(x,8)$ and $R(0,0)$, $S(1,-4)$
(d) $A(4,3)$, $M(1,5)$ and $P(x,0)$, $Q(0,6)$
(e) $F(-1,-3)$, $G(5,2)$ and $H(-3,y)$, $K(3,4)$
(f) $A(x,9)$, $B(5,0)$ and $W(-2,-2)$, $T(8,7)$

13.4 PERPENDICULAR LINES

Investigation 13-3



For each *pair* of perpendicular lines, calculate the slopes and complete the table below in your notebook.

line	slope	$m_1 \times m_2$
l_1		
l_2		
l_3		
l_4		
l_5		
l_6		

Two lines are perpendicular if the product of their slopes is − 1.

EXAMPLE 1. *Show that the line through $A(0,2)$ and $B(-3,-4)$ is perpendicular to the line through $P(2,-4)$ and $Q(-8,1)$.*

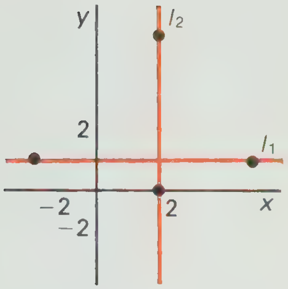
Solution

$$m_{AB} = \frac{-4-2}{-3-0} = \frac{-6}{-3} = 2 \qquad m_{PQ} = \frac{1-(-4)}{-8-2} = \frac{5}{-10} = -\tfrac{1}{2}$$

Since $2 \times \left(-\tfrac{1}{2}\right) = -1$, $AB \perp PQ$.

$\perp$ means "is perpendicular to"

The lines illustrated at right are perpendicular, but the slope of l_2 is undefined, and the slope of l_1 is zero. The product of these slopes cannot be found.



EXAMPLE 2. *State the slope of a line (i) parallel to, and (ii) perpendicular to the line $3x + 2y = 6$.*

Solution

Find the slope of $3x + 2y = 6$ by rearranging the equation to the form $y = mx + b$.

$3x + 2y = 6$

$2y = -3x + 6$

$y = -\frac{3}{2}x + 3$

∴ *the slope is $-\frac{3}{2}$. (i) a parallel line will have a slope of $-\frac{3}{2}$.*
(ii) a perpendicular line will have a slope of $\frac{2}{3}$ since $-\frac{3}{2} \times \frac{2}{3} = -1$.

EXERCISE 13-4

A **1.** State the slope of a line perpendicular to the line having the given slope.

(a) 3 (b) $\frac{2}{5}$ (c) $-\frac{1}{4}$ (d) $\frac{5}{6}$ (e) -1
(f) $1\frac{1}{2}$ (g) -5 (h) 1 (i) 0 (j) undefined

2. For two perpendicular lines, the product of the slopes is -1. Describe a situation when this rule will not work.

3. State the slope of the line perpendicular to each of the following lines.

(a) $y = 3x + 2$ (b) $y = -5x + 1$ (c) $y = \frac{1}{2}x$
(d) $y = -\frac{3}{2}x - 3$ (e) $x + y = 5$ (f) $3x + y = 9$
(g) $y - 5x = 3$ (h) $\frac{1}{2}x + y = 8$ (i) $2y = 4x - 5$
(j) $y = 3 - 4x$ (k) $y = \frac{x}{4} - 1$ (l) $y = \frac{3}{8}x + 2$

B **4.** Determine the slope of the line perpendicular to the line through the given points.

(a) $A(5,7)$ $B(4,9)$ (b) $P(-1,5)$ $Q(2,8)$
(c) $C(0,4)$ $D(5,2)$ (d) $M(4,-2)$ $N(-3,1)$
(e) $R(2,3)$ $T(-2,0)$ (f) $S(-5,2)$ $Q(-3,-3)$
(g) $H(1,-6)$ $K(4,2)$ (h) $P(1,5)$ $T(0,4)$

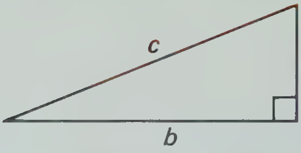
5. Using slopes, show that the triangle formed by the three points is a right-angled triangle. Name the right angle.

(a) $P(3,1)$, $Q(1,7)$, $R(5,5)$
(b) $A(-3,4)$, $B(-6,0)$, $C(5,-2)$
(c) $M(0,1)$, $N(6,1)$, $O(6,-2)$

6. State, with reasons, whether the lines are parallel or perpendicular.

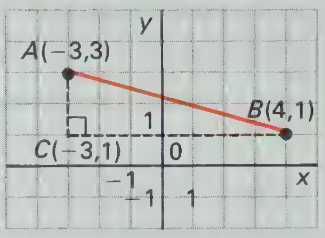
(a) $2x + y = 6$ and $x - 2y = 16$
(b) $y - 3x = 2$ and $6x - 2y = 1$
(c) $y = 5x - 3$ and $10x - 2y = 3$
(d) $x + y = 5$ and $x - y = 2$

13.5 LENGTH OF A LINE SEGMENT



$(c^2 = a^2 + b^2)$

The Pythagorean Theorem may be used to calculate the length of a line segment.



EXAMPLE 1. *Calculate the length of AB in the diagram at left.*

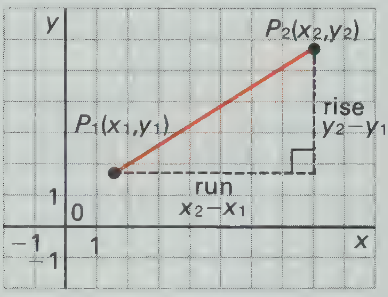
Solution *The rise and run shown in the diagram form a right-angled triangle ACB.*

$$\begin{aligned}\therefore AB^2 &= AC^2 + CB^2\\ &= 2^2 + 7^2\\ &= 4 + 49\\ &= 53\end{aligned}$$

$$\begin{aligned}\therefore AB &= \sqrt{53}\\ &\doteq 7.28\end{aligned}$$

In the same way a formula for calculating length can be stated.

$$(P_1P_2)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



The length of a line segment P_1P_2 is

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 2. *Calculate the length of the line segment joining P(4, − 2) and Q (0, − 4).*

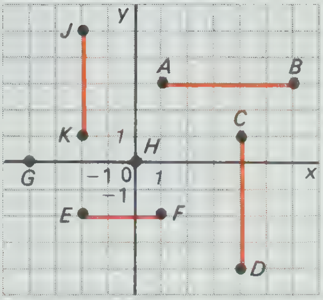
Solution

$$\begin{aligned}l &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\\ &= \sqrt{(0 - 4)^2 + (-4 - (-2))^2}\\ &= \sqrt{(-4)^2 + (-2)^2}\\ &= \sqrt{16 + 4}\\ &= \sqrt{20}\\ &= 2\sqrt{5}\\ &\doteq 4.47\end{aligned}$$

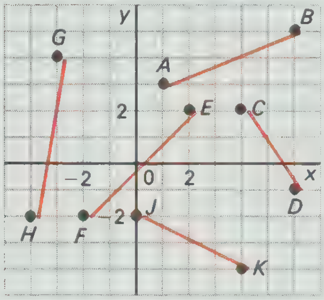
$$\begin{aligned}\sqrt{20} &= \sqrt{4} \times \sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

EXERCISE 13-5

A 1. State the length of the line segments shown.



B 2. Calculate the length of each line segment shown.



3. Calculate the length of each line segment.

(a) $A(3,1)$, $B(6,5)$

(b) $C(0,0)$, $D(-6,4)$

(c) $P(3,4)$, $Q(5,4)$

(d) $M(2,3)$, $N(1,1)$

(e) $E(-3,5)$, $F(2,1)$

(f) $X(-5,1)$, $Y(-1,-1)$

(g) $T(0,-4)$, $W(-7,0)$

(h) $H(-3,-6)$, $K(-1,4)$

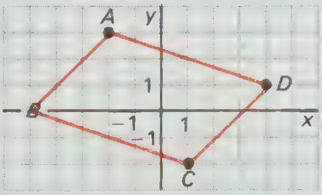
(i) $A(-8,0)$, $B(-8,4)$

(j) $N(3,0)$, $P(-5,-6)$

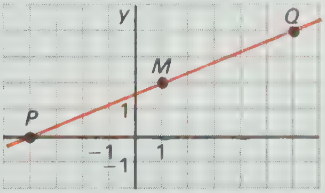
(k) $R(4,2)$, $S(3,-7)$

(l) $T(2,-5)$, $W(-1,5)$

4. Calculate the lengths of the sides of the parallelogram $ABCD$.



5. To show that $M(1,2)$ is the midpoint of PQ, calculate the lengths of PM and QM.



6. Show that $M(-2,0)$ is the midpoint of the line segment joining $A(-4,3)$ and $B(0,-3)$.

13.6 APPLICATIONS

The following examples illustrate some of the uses of graphs and equations in the solving of problems. In each case, the slope has a significant meaning.

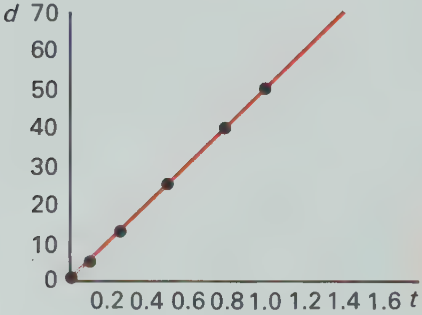
EXAMPLE 1. *In town, Karen drives at an average speed of 50 km/h. The chart shows the distance travelled and time taken.*

distance in kilometres	5	12.5	25	40	50
time in hours	0.1	0.25	0.5	0.8	1.0

(*a*) *Construct a graph using the ordered pairs (t,d).*
(*b*) *Using two of the points, calculate the slope.*
(*c*) *Determine the equation of the graph in the form* $y = mx + b$.
(*d*) *Using the graph, estimate the time to travel 35 km.*
(*e*) *Use the slope to determine the distance travelled in 1.4 h.*

Solution

(*a*)

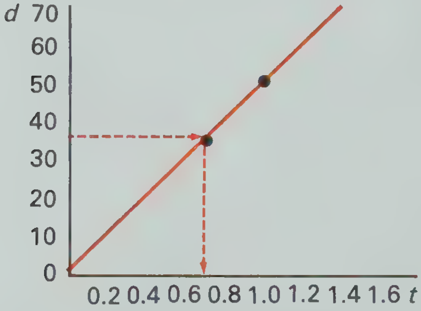


(*b*) *Using points (0.5,25) and (0.8,40)*

$$m = \frac{40 - 25}{0.8 - 0.5}$$
$$= \frac{15}{0.3}$$
$$= 50$$

Note that the slope is equal to the speed in kilometres per hour.

(*c*) *Since* $m = 50$ *and* $b = 0$, *the equation is* $d = 50t$.



For a distance of 35 km, the time would be 0.7 h. Determining a value which lies between known values is called *interpolation.*

(*d*) *Using the points (1,50) and (1.4, d)*

$$\frac{d - 50}{1.4 - 1} = 50$$
$$\frac{d - 50}{0.4} = 50$$
$$d - 50 = 20$$
$$d = 70$$

$\therefore$*for a time of 1.4 h, the distance would be 70 km. Determining a value which lies outside known values is called* *extrapolation.*

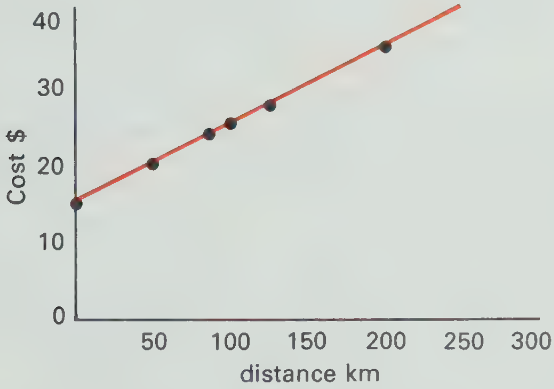
EXAMPLE 2. *Sam often rented a car from the U-Drive rental agency when he was in town. The chart shows the amount he was charged for several rentals, and the number of kilometres driven.*

distance in kilometres	50	100	125	200	90
cost $	20	25	27.50	35	24

(a) Graph this relationship.
(b) Use the graph to extrapolate the cost for 0 km.
(c) Calculate the slope of the line. What does this slope represent?
(d) Express the relationship as an equation in the form $c = md + b$.
(e) Use this equation to interpolate the cost for a distance of 150 km.

Solution

(a)



(b) When extended, the line passes through the point (0,15). Therefore, for 0 km, the cost is $15.00.

(c) Using the points (50,20) and (100,25)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 20}{100 - 50} = \frac{5}{50} = 0.1$$

The slope represents the cost per kilometre for the rental. For each kilometre, Sam is charged 10 cents.

(d) Since the line passes through (0,15), the y-intercept is 15.
$\therefore$ *the equation is* $c = 0.1d + 15$

(e) When $d = 150$,

$$C = 0.1\,(150) + 15 = 15 + 15 = 30$$

$\therefore$ *the cost is $30.00*

EXERCISE 13-6

1. For each pair of points, calculate the slope.

(a)	*A* (2.0,1.5), *B* (3.0, 2.0)	(b)	*M* (1.3,0), *N* (5.8,9.0)
(c)	*X* (4.2,6.0), *Y* (2.2,8.0)	(d)	*P* (5.6,13.3), *Q* (5.8,15.5)
(e)	*R* (5.1,7.3), *S* (5.0,7.5)	(f)	*W* (9.0,7.9), *R* (19.0,8.4)

2. Plot each pair of points. Extend the straight line through each pair of points until it intersects the *y*-axis. State the approximate value of the *y*-intercept.

(a)	*A* (2,4), *B* (4,8)	(b)	*P* (3,6), *Q* (6,7)
(c)	*M* (3,2.5), *N* (6,3)	(d)	*S* (3.5,1), *T* (10.5,3)
(e)	*X* (3,5), *Y* (6,2)	(f)	*C* (4.5,4.5), *D* (9,6)
(g)	*A* (3,10), *B* (6,10.5)	(h)	*H* (3.5,4.5), *K* (10.5,5.5)

3. The interest paid for borrowing money depends on the amount borrowed. The table below shows the amount of interest to be paid for one year on several amounts of money borrowed.

Interest $	18	24	42	60	96
Amount $	150	200	350	500	800

(a) Draw a graph using these values.
(b) Calculate the slope of the straight line passing through these points. Express this slope as a percent.
(c) Using your graph, interpolate the annual interest for amounts of $175 and $400.
(d) Using your graph, extrapolate the amount for annual interest payments of $12 and $99.

4. After several taxi trips, Nancy set up a table showing the number of kilometres travelled, and the cost for each trip.

distance in kilometres	10	35	25	75	15
cost $	3	8	6	16	4

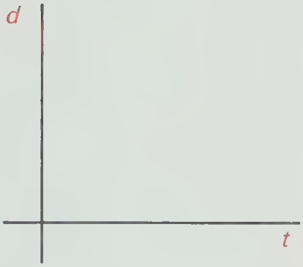
(a) Graph this relationship.
(b) Use the graph to extrapolate the cost for 0 km.
(c) Calculate the slope of the line. What does this slope represent?
(d) Express this relationship as an equation in the form

$$c = md + b$$

(e) Use the equation to interpolate the cost for a trip of 60 km.

5. At a constant speed, Lori made up a chart showing distance travelled in km and time taken in hours.

distance in kilometres	45	60	75	90	105
time in hours	0.2	0.4	0.6	0.8	1

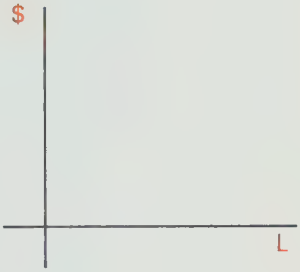


(a) Draw a graph using these values.
(b) Extend the line to determine how far she had travelled before she began the chart. State the "*d*" intercept for this line.
(c) Determine the slope of the line. At what speed was she travelling?
(d) Express this relationship in the form

$$d = mt + b$$

6. For predicting gasoline costs, George made up a table of values for some of his recent purchases.

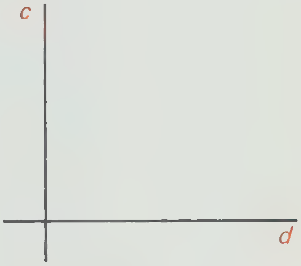
cost $	3.44	6.87	4.58	9.16	1.83
litres L	15	30	20	40	8



(a) Draw a graph using these values.
(b) Calculate the slope of this line. What is the cost per litre of gasoline?
(c) Calculate the cost of 25 L of gasoline at this price.

7. A class experiment determined the following values when circumference and diameter of a circle were measured.

circumference in centimetres	7.8	31.7	14.2	46.9	80.0
diameter in centimetres	2.5	10.1	4.5	15.0	25.5



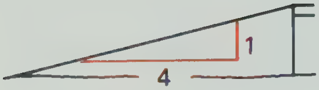
(a) Draw a graph using these values.
(b) Calculate the slope of this line. What symbol is used to represent this constant value?
(c) Express this relationship as a formula in the form $c = md$.
(d) For a circular object of your choice, measure the diameter. Use your formula to calculate the circumference, and check your results by measurement.

REVIEW EXERCISE

1. Calculate the slope of the line through each pair of points.
(a) (5,3), (8,9)
(b) (3,− 1), (7,3)
(c) (4,2), (3,5)
(d) (8,6), (− 2,5)
(e) (− 1,0), (4,− 2)
(f) (5,− 1), (5,3)
(g) (0,2), (0,− 4)
(h) (8,− 3), (4,− 3)
(i) (4,2.5), (5,3)
(j) (2,1.3), (1,2.8)
(k) (3.7,− 2), (4.2,3)
(l) (1.2,7.9), (1.5,8.1)

2. Using slopes, determine if the points are collinear.

(a) $A(7,2)$, $B(5,3)$, $C(-1,6)$
(b) $P(-1,-5)$, $Q(1,0)$, $R(4,6)$
(c) $W(-6,0)$, $T(2,4)$, $V(-2,2)$
(d) $X(1,-3)$, $Y(0,0)$, $Z(-3,9)$



3. The slope of a ramp is 1:4, which means a 1 unit rise for each 4 unit run.

(a) How far back must a ramp begin to reach a height of 3.5 m?
(b) What height may be reached from a distance of 30.0 m?

4. Use the slope and point given to graph the line. State the coordinates of a second point on the line.

(a) $m = \frac{2}{3}$; $P(0,4)$
(b) $m = -\frac{1}{2}$; $P(4,3)$
(c) $m = 5$; $P(-1,-5)$
(d) $m = -\frac{5}{2}$; $P(2,-1)$
(e) $m = 0$; $P(-1,3)$
(f) $m = 0.75$; $P(-5,0)$

5. Express each equation in the form $y = mx + b$. State the slope and y-intercept.

(a) $2x + y = 7$
(b) $3y = 15x - 6$
(c) $x + y = 10$
(d) $3x + 2y = 6$
(e) $y - 4x - 3 = 0$
(f) $2x - y = 5$
(g) $5x = y - 2$
(h) $y - 3x = 0$

6. Calculate the value of the unknown coordinate if the slope of the line segment is equal to the given slope.

(a) $A(3,2)$, $B(4,y)$; $m = 5$
(b) $P(2,y)$, $Q(5,8)$; $m = \frac{2}{3}$
(c) $M(-1,3)$, $N(4,y)$; $m = 2$
(d) $S(5,0)$, $T(x,1)$; $m = \frac{1}{2}$
(e) $C(x,7)$; $D(-2,3)$; $m = -4$
(f) $V(0,y)$, $W(5,1)$; $m = -\frac{3}{5}$
(g) $X(-4,5)$, $Y(3,y)$; $m = 0$
(h) $P(-1,1)$, $Q(x,-2)$; $m = -1.5$

7. Determine if the line segments are parallel or perpendicular.

(a) $A(1,0)$, $B(5,3)$ and $C(-1,3)$, $D(3,6)$
(b) $P(-4,-1)$, $Q(2,-3)$ and $T(2,-1)$, $V(3,2)$
(c) $M(-1,1)$, $N(5,4)$ and $R(3,3)$, $S(2,5)$
(d) $X(0,0)$, $Y(6,-2)$ and $T(3,1)$, $W(9,-1)$
(e) $A(0,-1)$, $C(-4,-3)$ and $D(2,-3)$, $E(10,1)$

8. Calculate the length of each line segment.

(a) $A(5,7)$, $B(9,10)$
(b) $C(3,12)$, $D(8,0)$
(c) $E(4,3)$, $F(4,-7)$
(d) $G(-2,5)$, $H(8,5)$
(e) $P(-1,1)$, $Q(0,2)$
(f) $R(4,-2)$, $S(1,5)$
(g) $X(-5,0)$, $Y(1,3)$
(h) $T(-8,-1)$, $S(2,3)$
(i) $W(5,-2)$, $D(4,-3)$
(j) $Q(-3,-2)$, $P(-5,-6)$
(k) $A(-7,0)$, $Y(1,1)$
(l) $X(-2,6)$, $Z(1,3)$

9. Calculate the perimeter of $\triangle ABC$ for the points $A(2,4)$, $B(-10,-1)$, and $C(2,-1)$.

10. In a science experiment, the mass and volume of several pieces of chalk were found.

mass g	6	3	9	2	1
volume cm^3	3	1.5	4.5	1	0.5

(a) Draw a graph using these values.
(b) Calculate the slope of this line. This value is the density of chalk in g/cm^3.
(c) Using your graph, interpolate the mass when the volume is 2.5 cm^3.
(d) Using your graph, extrapolate the volume when the mass is 13 g.

TRIGONOMETRIC RATIOS

θ°	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
0	0.0000	1.0000	0.0000	—	1.0000	—
1	0.0175	0.9999	0.0175	57.290	1.0001	57.299
2	0.0349	0.9994	0.0349	28.636	1.0006	28.654
3	0.0523	0.9986	0.0524	19.081	1.0014	19.107
4	0.0698	0.9976	0.0699	14.301	1.0024	14.335
5	0.0872	0.9962	0.0875	11.430	1.0038	11.474
6	0.1045	0.9945	0.1051	9.5144	1.0055	9.5668
7	0.1219	0.9926	0.1228	8.1443	1.0075	8.2055
8	0.1392	0.9903	0.1405	7.1154	1.0098	7.1853
9	0.1564	0.9877	0.1584	6.3137	1.0125	6.3924
10	0.1737	0.9848	0.1763	5.6713	1.0154	5.7588
11	0.1908	0.9816	0.1944	5.1445	1.0187	5.2408
12	0.2079	0.9782	0.2126	4.7046	1.0223	4.8097
13	0.2250	0.9744	0.2309	4.3315	1.0263	4.4454
14	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336
15	0.2588	0.9659	0.2680	3.7320	1.0353	3.8637
16	0.2756	0.9613	0.2867	3.4874	1.0403	3.6279
17	0.2924	0.9563	0.3057	3.2708	1.0457	3.4203
18	0.3090	0.9511	0.3249	3.0777	1.0515	3.2361
19	0.3256	0.9455	0.3443	2.9042	1.0576	3.0715
20	0.3420	0.9397	0.3640	2.7475	1.0642	2.9238
21	0.3584	0.9336	0.3839	2.6051	1.0711	2.7904
22	0.3746	0.9272	0.4040	2.4751	1.0785	2.6695
23	0.3907	0.9025	0.4245	2.3558	1.0864	2.5593
24	0.4067	0.9136	0.4452	2.2460	1.0946	2.4586
25	0.4226	0.9063	0.4663	2.1445	1.1034	2.3662
26	0.4384	0.8988	0.4877	2.0503	1.1126	2.2812
27	0.4540	0.8910	0.5095	1.9626	1.1223	2.2027
28	0.4695	0.8830	0.5317	1.8807	1.1326	2.1300
29	0.4848	0.8746	0.5543	1.8040	1.1433	2.0627
30	0.5000	0.8660	0.5774	1.7320	1.1547	2.0000
31	0.5150	0.8572	0.6009	1.6643	1.1666	1.9416
32	0.5299	0.8481	0.6249	1.6003	1.1792	1.8871
33	0.5446	0.8387	0.6494	1.5399	1.1924	1.8361
34	0.5592	0.8290	0.6745	1.4826	1.2062	1.7883
35	0.5736	0.8192	0.7002	1.4281	1.2208	1.7434
36	0.5878	0.8090	0.7265	1.3764	1.2361	1.7013
37	0.6018	0.7986	0.7536	1.3270	1.2521	1.6616
38	0.6157	0.7880	0.7813	1.2799	1.2690	1.6243
39	0.6293	0.7772	0.8098	1.2349	1.2867	1.5890
40	0.6428	0.7660	0.8391	1.1917	1.3054	1.5557
41	0.6561	0.7547	0.8693	1.1504	1.3250	1.5242
42	0.6691	0.7431	0.9004	1.1106	1.3456	1.4945
43	0.6820	0.7314	0.9325	1.0724	1.3673	1.4663
44	0.6947	0.7193	0.9657	1.0355	1.3902	1.4395
45	0.7071	0.7071	1.0000	1.0000	1.4142	1.4142

TRIGONOMETRIC RATIOS

0°	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
46	0.7193	0.6947	1.0355	0.9657	1.4395	1.3902
47	0.7314	0.6820	1.0724	0.9325	1.4663	1.3673
48	0.7431	0.6691	1.1106	0.9004	1.4945	1.3456
49	0.7547	0.6561	1.1504	0.8693	1.5242	1.3250
50	0.7660	0.6428	1.1917	0.8391	1.5557	1.3054
51	0.7772	0.6293	1.2349	0.8098	1.5890	1.2867
52	0.7880	0.6157	1.2799	0.7813	1.6243	1.2690
53	0.7986	0.6018	1.3270	0.7536	1.6616	1.2521
54	0.8090	0.5878	1.3764	0.7265	1.7013	1.2361
55	0.8192	0.5736	1.4281	0.7002	1.7434	1.2208
56	0.8290	0.5592	1.4826	0.6745	1.7883	1.2062
57	0.8387	0.5446	1.5399	0.6494	1.8361	1.1924
58	0.8481	0.5299	1.6003	0.6249	1.8871	1.1792
59	0.8572	0.5150	1.6643	0.6009	1.9416	1.1666
60	0.8660	0.5000	1.7320	0.5774	2.0000	1.1547
61	0.8746	0.4848	1.8040	0.5543	2.0627	1.1433
62	0.8830	0.4695	1.8807	0.5317	2.1300	1.1326
63	0.8910	0.4540	1.9626	0.5095	2.2027	1.1223
64	0.8988	0.4384	2.0503	0.4877	2.2812	1.1126
65	0.9063	0.4226	2.1445	0.4663	2.3662	1.1034
66	0.9136	0.4067	2.2460	0.4452	2.4586	1.0946
67	0.9205	0.3907	2.3558	0.4245	2.5593	1.0864
68	0.9272	0.3746	2.4751	0.4040	2.6695	1.0785
69	0.9336	0.3584	2.6051	0.3839	2.7904	1.0711
70	0.9397	0.3420	2.7475	0.3640	2.9238	1.0642
71	0.9455	0.3256	2.9042	0.3443	3.0715	1.0576
72	0.9511	0.3090	3.0777	0.3249	3.2361	1.0515
73	0.9563	0.2924	3.2708	0.3057	3.4203	1.0457
74	0.9613	0.2756	3.4874	0.2867	3.6279	1.0403
75	0.9659	0.2588	3.7320	0.2680	3.8637	1.0353
76	0.9703	0.2419	4.0108	0.2493	4.1336	1.0306
77	0.9744	0.2250	4.3315	0.2309	4.4454	1.0263
78	0.9782	0.2079	4.7046	0.2126	4.8097	1.0223
79	0.9816	0.1908	5.1445	0.1944	5.2408	1.0187
80	0.9848	0.1737	5.6713	0.1763	5.7588	1.0154
81	0.9877	0.1564	6.3137	0.1584	6.3924	1.0125
82	0.9903	0.1392	7.1154	0.1405	7.1853	1.0098
83	0.9926	0.1219	8.1443	0.1228	8.2005	1.0075
84	0.9945	0.1045	9.5144	0.1051	9.5668	1.0055
85	0.9962	0.0872	11.430	0.0875	11.474	1.0038
86	0.9976	0.0698	14.301	0.0699	14.335	1.0024
87	0.9986	9.0523	19.081	0.0524	19.107	1.0014
88	0.9994	0.0349	28.636	0.0349	28.654	1.0006
89	0.9999	0.0175	57.290	0.0175	57.299	1.0001
90	1.0000	0.0000	—	0.0000	—	1.0000

ANSWERS

REVIEW AND PREVIEW TO CHAPTER 1

Exercise 1

1. (a) 153.8 (b) 164.885 (c) 23.715 (d) 26.0316 (e) 119.67 (f) 199.67
(g) 29.28 (h) 1206.8 (i) 217.750 (j) 11 405.9
2. (a) 5.7 (b) 17.9 (c) .098 (d) 5072.43 (e) 9.087
(f) 25.037 (g) 475.268 (h) 1.1596 (i) 89.84 (j) 11.439
3. (a) 4.944 (b) 100.609 (c) 20.27 (d) 42.69 (e) 113.917
(f) 60.58 (g) 0.2227 (h) 0.4264 (i) 4.909
4. (a) 8.25 (b) − 1.75 (c) 1.05 (d) 4.15
(e) 0.3285 (f) 8.276 (g) 2.273 (h) 22.67
(i) 405

Exercise 2

1.

n	$100n$	$10n$	$0.1n$	$0.001n$
12	1200	120	1.2	0.012
35.1	3510	351	3.51	0.0351
5.86	586	58.6	0.586	0.005 86
64.3	6430	643	6.43	0.0643
8.395	839.5	83.95	0.8395	0.008 395
3.7	370	37	0.37	0.0037

2. (a) 50.4 (b) 1874.4 (c) 0.98 (d) 5091.24 (e) 1.393 83
(f) 531.0036 (g) 0.003 61 (h) 51.638 36 (i) 24.071 43 (j) 0.522 525 6
3. (a) 18.5 (b) 12.7 (c) 0.0317 (d) 31.6 (e) 0.005 62
(f) 781 (g) 0.008 (h) 0.0532 (i) 58 200 (j) 784
4. (a) 217.887 12 (b) 4.818 44 (c) 15.7696 (d) 6.175 216 (e) 369.6
(f) 3.9 (g) 1.2852 (h) 24.22

Exercise 3

1. 58.4 2. 2.7 h

3.

Length cm	Width cm	Perimeter cm	Area cm²
4.3	7.1	22.8	30.53
1.75	21.3	46.1	37.275
0.9	1.3	4.4	1.17
15	7.3	44.6	109.5
11.1	10.3	42.8	114.33

4. 1.2 mm 5. Area is 2.5434 cm²; circumference is 5.652 cm 6. 747.5 mm and 722.5 mm
7. 455 kg 8. 22.0 9. (a) $289.20 (b) $343.43
10. (a) $5.19 (b) 64.5 l (c) $15\frac{7}{10}$¢/ 11. (a) $2.08 (b) $1.07 (c) 3.80 kg

Exercise 4

1. **(a) $4.63** **(b) $42.75** **(c) $18.71** **(d) $0.70**
(e) $3.78 **(f) $4392** **(g) $73.50** **(h) $63.94**
(i) $42.90 **(j) $57.00** **(k) $23.32** **(l) $66.63**

2. (a) $0.15 (b) $1.63 (c) $3.85 (d) $0.96
(e) $32.64 (f) $56.27 (g) $15.70 (h) $0.87
(i) $12.86 (j) $0.98 (k) $0.96 (l) $0.97
3. $6124.95 4. $9096.59
5. Housing $7517, Transportation $6389.45, Tax $6013.6, Food $5637.75, Clothing $3758.50, Savings $3382.65, Medical $3006.80, Other $1879.25.
6. (a) 50% (b) 33% (c) 33% (d) 22%
(e) 17% (f) 30% (g) 150% (h) 118%

Exercise 1-1

1. (a) 7987 (b) 64.3 (c) 97.67 (d) 22.755 (e) 803
(f) 85.24 (g) 57.15 (h) 14.15 (i) 0.2707 (j) 0.2751
2. (a) 12 013 862 (b) 17.344 (c) 3.722 (d) 87.656 (e) 124.525
(f) 4.303 (g) 0.858 531 (h) 0.447 974 (i) 15.534 (j) 26.910 65
3. (a) 3.5 h (b) 7.75 h (c) 5.6 h (d) 6.25 h
(e) 0.5 h (f) 0.8 h (g) 0.25 h (h) 0.4 h
(i) 3.6 h (j) 8.2 h (k) 5.8 h (l) 1.4 h
4. (a) $160.00 (b) $95.00 (c) $345.50 (d) $290.70
(e) $172.20 (f) $111.00 (g) $52.20 (h) $140.25
5. (a) $7.00 (b) $6.50 (c) $10.80 (d) $7.82
(e) $11.15 (f) $7.25
6. (a) $10 920 (b) $15 600 (c) $8840 (d) $14 144
(e) $19 240 (f) $22 464 (g) $11 760 (h) $8450
(i) $11 640 (j) $26 104
7. (a) $16 000 (b) $16 020 (c) $15 960 (d) $16 380
(e) $16 120 (f) $16 120 (d) is the best rate

Exercise 1-2

1. (a) 1 (b) 1 (c) 6 (d) 0
(e) 13 (f) 3 (g) 4 (h) 14
(i) 25 (j) 13 (k) 16 (l) 20
2. (a) 8 (b) 5 (c) 26 (d) 0
(e) 5 (f) 45 (g) 20 (h) 8
(i) 1 (j) 3 (k) 8 (l) 17
3. (a) 10 (b) 19 (c) 37 (d) 0
(e) 73 (f) 36
4. (a) 16.69 (b) 51.6 (c) 21.045 039 (d) 2.9928
(e) 179.1 (f) 93.25
5. (a) 114.6 (b) 2550 (c) 5050 (d) 265.375
(e) 13 (f) 1023
6. 56, 35, 35, 35

Exercise 1-3

1. (a) $1.25 (b) $84.66 (c) $12.81 (d) $6.29
(e) $4.78 (f) $5.32 (g) $61.21 (h) $8.75
(i) $9.72 (j) $8.52 (k) $8.26 (l) $9.56
2. (a) 6.4 (b) 0.2 (c) 5.2 (d) 62.3
(e) 27.2 (f) 32.3 (g) 69.6 (h) 90.0
(i) 100.0 (j) 8.1 (k) 3.7 (l) 91.5
3. (a) 2750 (b) 60 (c) 560 (d) 220
(e) 6230 (f) 30 (g) 10 (h) 440
(i) 5020 (j) 230 (k) 190 (l) 50
4. (a) 2132 (b) 55 (c) 8 (d) 61
(e) 51 (f) 44 (g) 1 (h) 70
(i) 3 (j) 21 (k) 0 (l) 117

5. (a) 25 400 (b) 165 800 (c) 200 (d) 3200
(e) 500 (f) 700 (g) 1600 (h) 1900
(i) 2100 (j) 400 (k) 300 (l) 425 800
6. (a) 10.73 (b) 5.03 (c) 16.37 (d) 12.41
(e) 0.13 (f) 0.07 (g) 0.05 (h) 3.98
(i) 12.49 (j) 128.46 (k) 465.56 (l) 158.82
7. (a) 39 960 (b) 40 000 (c) 40 000

Exercise 1-4

1. (a) 21 (b) 20 (c) 100 (d) 25
(e) 2 (f) 2 (g) 27 (h) 75
(i) 3 (j) 120 (k) 30 (l) 30
2. (a) 500, 487.305 46 (b) 10, 9.375
(c) 1, 1.058 224 1 (d) 35, 36.534 939
(e) 8, 7.632 349 5 (f) 0.2, 0.181 014
3. (a) \$900 (b) \$1500 (c) \$900 (d) \$600
(e) \$900 (f) \$900
4. (a) \$145 (b) \$178 (c) \$221 (d) \$286 (e) \$191
5. (a) \$1200 (b) \$600
6. (a) \$6000 (b) \$1500
7. (a) 420 000 (b) 1 600 000 (c) 400 000 (d) 400 000
8. (a) \$1000 (b) \$1400 (c) \$1000 (d) \$900

Exercise 1-5

1. (a) 3^4 (b) 4^3 (c) 5^4 (d) $(-3)^5$
(e) 7^5 (f) 9^3 (g) 5^1 (h) 2^6
(i) $(-7)^6$
2. (a) 27 (b) 25 (c) 49 (d) 343 (e) 125
(f) 64 (g) 16 (h) 16 (i) 64 (j) 10 000
(k) 1 (l) 0 (m) 1 (n) $\frac{1}{2}$ (o) 0.001
(p) $\frac{1}{9}$
3. (a) $2^2 \times 3$ (b) $2^2 \times 3^2$ (c) $2^3 \times 10^2$ (d) $2^2 \times 10^2$
(e) $3^2 \times 10$ (f) $2^4 \times 3^2$ (g) 7×2^2 (h) $3^2 \times 5^2$
(i) $2^2 \times 3^3$ (j) $2^2 \times 5^4$ (k) $2^5 \times 3$ (l) $2^4 \times 3^2 \times 5$
4. (a) 25 (b) 169 (c) 625 (d) 289
(e) 91 (f) 1853 (g) 14 167 (h) 3887
5. (a) 4 (b) 9 (c) 16
6. (a) 1.795 86 (b) 3.172 17 (c) 1.480 24 (d) 1.126 49
(e) 1.126 69 (f) 1.181 56 (g) 0.508 349 (h) 0.901 026
(i) 0.589 663

Exercise 1-6

1. (a) approx. (b) approx. (c) exact (d) approx.
(e) approx. (f) exact
2. (a) 3 (b) 4 (c) 1 (d) 3
(e) 5 (f) 2 (g) 3 (h) 2
(i) 5 (j) 3
3. (a) 5.6×10^6 (b) 3, 8.61×10^1 (c) 1250
(d) 2, 3.6×10^1 (e) 0.4316, 4 (f) 5.1×10^0
(g) 1.560×10^{-2} (h) 0.000 320, 3
4. (a) 8.93×10^2 (b) 1.632×10^1 (c) 2.56×10^{-2}
(d) 3.656×10^9 (e) 3.2×10^{-4} (f) 9.91×10^{-1}
(g) $5. \times 10^{-2}$ (h) 4.1×10^5 (i) 6.3×10^{-2}
(j) 2.011×10^2 (k) 1.6×10^3 (l) 3.57×10^{-7}
5. (a) 320 (b) 0.50 (c) 0.000 089
(d) 3261 (e) 0.04 (f) 5.7
(g) 0.210 (h) 0.000 000 001 03

REVIEW EXERCISE

1. **(a)** 47.015 **(b)** 92.9 **(c)** 48.395 **(d)** 418.139
(e) 261.903 75 **(f)** 4.75 **(g)** 6.5 **(h)** 78.625 35
2. **(a)** 0.125 **(b)** 0.25 **(c)** 0.375 **(d)** 0.5
(e) 0.625 **(f)** 0.75 **(g)** 0.875 **(h)** 0.2
(i) 0.4 **(j)** 0.6 **(k)** 0.8 **(l)** 1.5
3. **(a)** 3.75 h **(b)** 5.25 h **(c)** 7.4 h **(d)** 2.1 h
4. **(a)** $5.64 **(b)** $11.20 **(c)** $7.18
5. **(a)** $18 408 / a **(b)** $18 360 / a
6. **(a)** 9 **(b)** 1 **(c)** 15 **(d)** 5
(e) 10.75 **(f)** 69.7 **(g)** 98.55 **(h)** 12.75
(i) 38.44 **(j)** 37.4544
7. **(a)** 3.3, 6.6, 14.1, 91.6
(b) 58 530, 6350, 15 850, 20
(c) 6.13, 3.14, 58.27, 0.03
8. **(a)** 3.14, 585 000, 6160, 0.126, 0.003 73
(b) 25 000 000, 3 190 000, 15.8, 0.382, 0.005 56
(c) 7, 0.04, 50 000 000, 0.6, 2
9. **(a)** 12 **(b)** 5 **(c)** 120
(d) 8 **(e)** 4 **(f)** 20
(g) $200 **(h)** $700 **(i)** $200 **(j)** 5000
10. **(a)** 3 **(b)** 3 **(c)** 4 **(d)** 2
(e) 3 **(f)** 1 **(g)** 2 **(h)** 4
(i) 3 **(j)** 4 **(k)** 4 **(l)** 2
11. **(a)** 4.86×10^1 4.9×10^1 **(b)** 3.62×10^1
7.33×10^{-1} 7.3×10^{-1} 430 000
1.94×10^2 1.9×10^2 1.376×10^3
4.98×10^{-3} 5.0×10^{-3} 0.82
7.87×10^1 7.9×10^1 6.4×10^{-3}
5.85×10^{-1} 5.9×10^{-1} 3.9×10^1
9.10×10^{-4} 9.1×10^{-4} 0.000 047 5
1.86
9.16×10^{-1}

(c)	450 × 56 000	$4.5 \times 10^2 \times 5.6 \times 10^4$	2.52×10^7	25 200 000
	0.39 × 175 000	$3.9 \times 10^{-1} \times 1.75 \times 10^5$	6.825×10^4	68 250
	76 × 0.0035	$7.6 \times 10^1 \times 3.5 \times 10^{-3}$	2.66×10^{-1}	0.266
	0.018 × 0.0063	$1.8 \times 10^{-2} \times 6.3 \times 10^{-3}$	1.134×10^{-4}	0.000 113 4
	71.5 ÷ 4300	$\frac{7.15 \times 10^1}{4.3 \times 10^4}$	1.66×10^{-3}	0.001 66
	56 000 ÷ 1.65	$\frac{5.6 \times 10^4}{1.65 \times 10^0}$	3.39×10^4	33 900
	0.09 ÷ 680	$\frac{9.0 \times 10^{-2}}{6.8 \times 10^2}$	1.32×10^{-4}	0.000 132
	6.6 ÷ 0.0035	$\frac{6.6 \times 10^0}{3.5 \times 10^{-3}}$	1.89×10^3	1890
	0.003 92 ÷ 0.000 062	$\frac{3.92 \times 10^{-3}}{6.2 \times 10^{-5}}$	6.32×10^1	63.2

12. **(a)** 23.876 16 cm **(b)** 47.124 cm
13. **(a)** 140.296 cm^3 **(b)** 42 875 mm^3
14. **(a)** 9.9 cm^2 **(b)** 6.552 cm^3
(c) $38.21 **(b)** 62%
(e) 167.4159 cm^2 **(f)** 15.377 L / 100 km

REVIEW AND PREVIEW TO CHAPTER 2

Exercise 1

1. (a) 18.0 cm (b) 42.6 cm 2. (a) 27.5 cm^2 (b) 95.4 cm^2
3. (a) 89.8 cm (b) 2.69×10^3 cm 4. (a) 153 cm^2 (b) 120 cm^2
5. (a) 3.7×10^2 cm^3 (b) 98.1 m^3 6. (a) 9.16×10^7 m^3 (b) 32.4 cm^3
7. (a) 6.8 m^3 (b) 15 cm^2 8. (a) 95.9 cm^2 (b) 179 cm^2

Exercise 2

1. 1241 2. 50.9
3. 1849.32 4. 13.7
5. 3033.3 6. 2125.9
7. 1726.34 8. 53.2

CHAPTER 2

Exercise 2-1

1. (a) line (b) bar (c) circle (d) line
(e) bar (f) circle
2. (d) Mercury, Venus, and Pluto have no moons.
3. (a) \$9 000 000 (b) \$3 000 000 (c) \$2 000 000
(d) \$2 400 000 (e) \$1 200 000 (f) \$2 400 000
4. (a) \$400 (b) \$750 (c) \$375.00
5. (a) Aug. (b) July (c) Ginger
(d) Diet (e) 2000 cases (f) 2750 cases
(g) 3250 cases (h) 500 cases (i) Ginger
(j) in thousand cases
Jan 8, Feb. 4.75, Mar. 3,
Apr. 5.75, May 6.75, June 9.5,
July 12.75, Aug. 13.5, Sept. 10.5,
Oct. 5.25, Nov. 3.5, Dec. 12.75
(k) Aug., summer heat (l) Mar., cold

Exercise 2-2

1. (a) 9 (b) 9, 10 (c) no mode (d) 21
2. (a) 11, 10, 9 (b) 41, 41.5, no mode
(c) 34, 34, 31 (d) 40.5, 39, 9 and 68
(e) 88.4, 90, 91 (f) 9.6, 9.7, no mode
(g) 128, 124, 110

Exercise 2-3

1. (a) mean (b) mode (c) mean (d) median
(e) mean (f) mean (g) mode
2. 2.18 3. 40
4. (a) 174 (b) 166 (c) 172.3 (d) 170
5. 8
6. (a) \$61 000, \$35 000, \$20 000
(b) median

Exercise 2-4

1. (a) 34 (b) 5, 7, 3, 10, 16 (c) 11, 13, 28
(d) 45-49 kg

Exercise 2-5

1. (c) 27 (d) 45 (e) 40
2. (c) 7 (d) 20

Exercise 2-6

1. (a) 60% of the students had marks equal or lower.
   (b) 92% of the students had marks equal or lower.
2. (a) 10 (b) 40
3. (a) 276 (b) 24
4. 52nd
5. (a) 71st (b) 74th (c) Bob
   (d) hard test vs easy test; strong class vs weak class; large class vs small class

Review Exercise

2. (a) 150, 235, 60
3. (a) June (b) March and April, July and August
   (c) 4 (d) December (e) 68
4. (a) 17, 16, 15
   (b) 128, 131, no mode
   (c) 45, 45, 50
5. (a) 202 (b) 206 (c) 204
   (d) 204 (e) 205
7. (c) 29 (d) 35
8. 60th

REVIEW AND PREVIEW TO CHAPTER 3

Exercise 1

1. (a) 2 (b) 5 (c) 8 (d) 5 (e) 2 (f) 9
   (g) 9 (h) 38 (i) 97
2. (a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) 1 (e) $2\frac{5}{12}$ (f) $1\frac{1}{6}$
   (g) $5\frac{17}{20}$ (h) $\frac{1}{24}$ (i) $2\frac{3}{8}$ (j) $1\frac{13}{18}$ (k) $2\frac{5}{28}$ (l) $\frac{23}{56}$
3. (a) $\frac{3}{4}$ (b) $2\frac{11}{15}$ (c) 1 (d) $3\frac{23}{24}$ (e) 0 (f) $4\frac{7}{15}$

Exercise 2

1. (a) $\frac{10}{21}$ (b) $\frac{1}{10}$ (c) $\frac{4}{25}$ (d) $\frac{5}{6}$ (e) 3 (f) $\frac{2}{3}$
   (g) $3\frac{1}{8}$ (h) 1 (i) $4\frac{1}{8}$
2. (a) $1\frac{7}{8}$ (b) $\frac{15}{16}$ (c) $2\frac{1}{12}$ (d) $\frac{5}{9}$ (e) $1\frac{7}{25}$ (f) 4
   (g) $\frac{7}{16}$ (h) $2\frac{1}{8}$ (i) $\frac{4}{9}$
3. (a) $3\frac{15}{16}$ (b) $\frac{3}{4}$ (c) $\frac{11}{20}$ (d) $1\frac{7}{48}$ (e) $1\frac{7}{8}$ (f) $\frac{13}{24}$
   (g) $1\frac{11}{30}$ (h) $\frac{5}{16}$ (i) $\frac{169}{225}$

Exercise 3

1. (a) 13.31 (b) 192.9 (c) 10.92 (d) 27.2 (e) 2.786
2. (a) 54.48 (b) 19.5 (c) 9.1 (d) 367.1
3. (a) 44.935 (b) 37.05 (c) 463.044 (d) 0.39878
4. (a) 5.63 (b) 2.5 (c) 59.8 (d) 0.715

CHAPTER 3

Exercise 3-1

1.

	Fraction form	Decimal form
(a)	$\frac{1}{4}$	0.25
(b)	$\frac{3}{10}$	0.3
(c)	$-\frac{1}{10}$	-0.1
(d)	$2\frac{1}{2}$	2.5
(e)	$-\frac{3}{4}$	-0.75
(f)	$\frac{3}{100}$	0.03
(g)	$\frac{5}{4}$	1.25
(h)	$4\frac{3}{10}$	4.3
(i)	$\frac{5}{8}$	0.625
(j)	$\frac{73}{1000}$	0.073
(k)	$-\frac{1}{1000}$	-0.001
(l)	$10\frac{1}{5}$	10.2

2.

	Decimal	Period	Length of period
(a)	$0.\dot{2}$	2	1
(b)	$0.\dot{5}\dot{6}$	56	2
(c)	$-0.3\dot{9}\dot{8}$	98	2
(d)	$0.\dot{1}56\dot{9}$	1569	4
(e)	$3.\dot{5}6\dot{7}$	567	3
(f)	$0.93\dot{5}$	5	1
(g)	$0.25\dot{0}$	0	1
(h)	$-1.98\dot{0}$	0	1

$$0.25\dot{0} = 0.25$$
$$-1.98\dot{0} = -1.98$$

3.

	x	$10x$	$100x$	$1000x$
(a)	0.333 . . .	3.33 . . .	33.33 . . .	333.333 . . .
(b)	0.2929 . . .	2.929 29 . . .	29.2929 . . .	292.929 29 . . .
(c)	$0.\dot{5}\dot{6}$	$5.6\dot{5}\dot{6}$	$56.\dot{5}\dot{6}$	$565.6\dot{5}\dot{6}$
(d)	$3.2\dot{5}$	$32.\dot{5}$	$325.\dot{5}$	$3255.\dot{5}$
(e)	$1.5\dot{2}\dot{6}$	$15.\dot{2}\dot{6}$	$152.6\dot{2}\dot{6}$	$1526.\dot{2}\dot{6}$
(f)	0.434 34 . . .	4.3434 . . .	43.4343 . . .	434.343 43 . . .
(g)	0.175 55 . . .	1.7555 . . .	17.555 . . .	175.555 . . .
(h)	$-0.60\dot{3}$	$-6.0\dot{3}$	$-60.\dot{3}$	$-603.\dot{3}$
(i)	0.293 232 . . .	2.932 32 . . .	29.323 2 . . .	293.232 323 . . .

4.

	Decimal	Period	Length of period
(a)	0.8	0	1
(b)	$0.\dot{4}$	4	1
(c)	$1.\dot{6}$	6	1
(d)	$0.\dot{6}$	6	1
(e)	0.85	0	1
(f)	$-0.\dot{2}\dot{7}$	27	2
(g)	$2.\dot{1}\dot{8}$	18	2
(h)	$0.\dot{4}28\ 57\dot{1}$	428 571	6
(i)	$0.3\dot{6}$	6	1
(j)	$0.1\dot{2}\dot{7}$	27	2

5. (a) $\frac{7}{10}$ (b) $\frac{5}{4}$ (c) $-\frac{7}{20}$ (d) $\frac{9}{2}$ (e) $\frac{3}{8}$ (f) $-\frac{59}{25}$
(g) $\frac{1}{400}$ (h) $-\frac{101}{100}$
6. (a) $\frac{5}{9}$ (b) $\frac{7}{9}$ (c) $\frac{3}{11}$ (d) $-\frac{13}{99}$ (e) $\frac{5}{3}$ (f) $\frac{41}{111}$
(g) $-\frac{181}{333}$ (h) $\frac{11}{45}$ (i) $\frac{3}{5}$ (j) $\frac{233}{990}$ (k) $\frac{43}{30}$ (l) $\frac{47}{225}$
7. (a) 6.2 (b) 14.98 (c) 6.6 (d) 0.9 (e) 14.44 (f) 0.19
(g) 15.6 (h) 0.5125 (i) 0.2256 (j) 152.5 (k) 3.35 (l) 2
8. (a) (i) 0.1 (ii) 0.2 (iii) 0.25 (iv) 0.5 (v) 0.6 (vi) 0.75 (vii) 0.9 (b) 25 trips
9. 1.625 km

Exercise 3-2

1. $\overline{Q}$, Q, Q, Q, $\overline{Q}$, Q, Q, Q, Q, Q, Q, $\overline{Q}$
3. (a) 8 (b) 6 (c) 49 (d) 1 (e) 0 (f) 121
(g) $\frac{3}{2}$ (h) $\frac{1}{36}$ (i) 10
4. (a) 12 (b) 15 (c) 18 (d) 8 (e) 16 (f) 28
5. (a) $\overline{Q}$ (b) $\overline{Q}$ (c) 20 (d) 14 (e) $\overline{Q}$ (f) 28
(g) 21 (h) $\overline{Q}$
7. (a) 3 (b) 1 (c) 7
8. (a) $152 (b) 4 m (c) 16 m (d) $10.40
9. $1700
10. no, since $\sqrt{2}$ is an irrational number

Exercise 3-3

1. (a) 2 (b) $\sqrt{15}$ (c) 7 (d) $\sqrt{10}$ (e) $6\sqrt{15}$ (f) $\sqrt{55}$
(g) $2\sqrt{21}$ (h) 6 (i) 10 (j) 18 (k) $\sqrt{14}$ (l) 12
2. (a) $2\sqrt{3}$ (b) $3\sqrt{3}$ (c) $3\sqrt{6}$ (d) $4\sqrt{2}$ (e) $6\sqrt{2}$ (f) $6\sqrt{2}$
(g) $6\sqrt{2}$ (h) $10\sqrt{6}$ (i) $14\sqrt{7}$ (j) $7\sqrt{2}$ (k) $9\sqrt{7}$ (l) $8\sqrt{2}$
3. (a) $\sqrt{45}$ (b) $\sqrt{176}$ (c) $\sqrt{300}$ (d) $\sqrt{75}$ (e) $\sqrt{60}$ (f) $\sqrt{162}$
(g) $\sqrt{160}$ (h) $\sqrt{192}$
4. (a) $2\sqrt{15}$ (b) $3\sqrt{2}$ (c) 60 (d) $4\sqrt{15}$ (e) $2\sqrt{5}$ (f) $45\sqrt{10}$
(g) $10\sqrt{3}$ (h) $5\sqrt{10}$ (i) 30 (j) $2\sqrt{30}$ (k) $9\sqrt{6}$ (l) $80\sqrt{10}$
5. (a) 9.42 cm^2 (b) 37.68 cm^2 (c) 109.9 cm^2
6. (a) 36 cm^2 (b) 192 cm^2 (c) 340 cm^2
7. (a) 33 cm^2 (b) 300 cm^2 (c) 33 cm^2

Exercise 3-4

1. (a) $10\sqrt{2}$ (b) $7\sqrt{3}$ (c) $6\sqrt{5}$ (d) $5\sqrt{7}$ (e) $4\sqrt{10}$ (f) $4\sqrt{3}$
(g) $11\sqrt{11}$ (h) $2\sqrt{6}$ (i) $9\sqrt{13}$
2. (a) $9\sqrt{3}$ (b) $7\sqrt{2}$ (c) $\sqrt{7}$ (d) $5\sqrt{5}$ (e) 0 (f) $5\sqrt{11}$
3. (a) $18\sqrt{2}$ (b) $15\sqrt{2}$ (c) $3\sqrt{3}$ (d) $5\sqrt{6}$ (e) $5\sqrt{10}$ (f) $9\sqrt{2}$
(g) $5\sqrt{3}$ (h) $4\sqrt{5}$ (i) $13\sqrt{3}$
4. (a) $2\sqrt{2}$ (b) $9\sqrt{5} + 5\sqrt{2}$ (c) $5\sqrt{5} + 6$ (d) $5\sqrt{2}$ (e) $23\sqrt{3}$
(f) $10\sqrt{3}$ (g) $2\sqrt{7}$ (h) $2\sqrt{3} - 15\sqrt{2}$
5. (a) $14\sqrt{2}$ cm (b) $9\sqrt{3}$ cm (c) $4\sqrt{2} + 5\sqrt{5}$ cm
6. (a) $12.56\sqrt{7}$ cm (b) $12.56\sqrt{2}$ cm (c) $18.84\sqrt{3}$ cm

Exercise 3-5

1. (a) 15.7 (b) 3.1 (c) 0.62 (d) 8.6 (e) 76.1 (f) 0.28
(g) 19.3 (h) 1.9
2. (a) 6 (b) 6 (c) 8 (d) 7 (e) 12 (f) 54
(g) 2 (h) 17
3. (a) 1.2 (b) 1.3 (c) $\frac{5}{6}$ (d) $\frac{9}{10}$ (e) 0.5 (f) 0.7
(g) 1.5 (h) $\frac{12}{7}$
4. (a) 30 (b) 50 (c) 0.07 (d) 80 (e) 200 (f) 0.4
(g) 120 (h) 0.04 (i) 6000
5. (a) (iii) (b) (vii) (c) (x) (d) (i) (e) (vi) (f) (ix)
(g) (v) (h) (ii) (i) (iv) (j) (viii)
6. (a) 5.7 (b) 7.7 (c) 4.6 (d) 6.2 (e) 24 (f) 27
(g) 38 (h) 77 (i) 0.65 (j) 0.29 (k) 0.32 (l) 0.085
7. (a) 9.35 (b) 3.56 (c) 7.64 (d) 2.5 (e) 2.4 (f) 37.2
(g) 0.410 (h) 0.196
8. (a) 17.1 (b) 15.6 (c) 1.41 (d) 23.4 (e) 11.3 (f) 38.1
9. (a) 21 cm (b) 27 cm (c) 14 cm

Exercise 3-6

1. (a) 4.795 (b) 8.602 (c) 2116 (d) 7.747 (e) 1521 (f) 7.071
(g) 7.810 (h) 4.123 (i) 6889 (j) 2.828
2. (a) 60 (b) 22.36 (c) 43.59 (d) 97.47 (e) 67.08 (f) 28.28
(g) 51.96 (h) 38.73 (i) 79.37 (j) 86.60
3. (a) 67 (b) 79 (c) 19 (d) 34 (e) 49 (f) 54
(g) 29 (h) 98 (i) 44 (j) 89
4. (a) 3.464 (b) 15.810 (c) 2.95 (d) 6.204 (e) 5.606 (f) 0.606
(g) 3.535 (h) 0.8214
5. (a) 3.640 (b) 23.73 (c) 1.889 (d) 0.7700 (e) 0.8326 (f) 0.05
(g) 30.70 (h) 0.077 27
6. (a) 31.62 (b) 12.648 (c) 18.522 (d) 18.972 (e) 15.492 (f) 21.908

Exercise 3-7

1. (a) 4.9 m (b) 7.6 m (c) 2.9 m (d) 5.5 m (e) 4.9 m
2. (b) (ii) $d_1 = 14.42$ m (iii) $d_2 = 11.31$ m (iv) $d_3 = 15.95$ m (v) $d_4 = 13.20$ m
3. (a) 12 V (b) 120 V (c) 2 amp, $\frac{5}{6}$ amp
4. 1.26 m
5. (a) 4.405 m (b) 4.855 m (b) 5.535 m
6. 33 km
7. (a) 2.0 s (b) 1.0 s

REVIEW EXERCISE

1. **(a)** 0.75, − 2.125, 5.3, − 1.35, 0.049
(b) $\frac{3}{4}$, $-2\frac{39}{100}$, $4\frac{5}{8}$, $-1\frac{19}{50}$, $\frac{1}{40}$
2. **(a)** 3 **(b)** 5 **(c)** $\sqrt{10}$ **(d)** $\sqrt{30}$ **(e)** $6\sqrt{5}$ **(f)** $20\sqrt{7}$
(g) $6\sqrt{10}$ **(h)** $15\sqrt{30}$ **(i)** 9 **(j)** $6\sqrt{2}$ **(k)** $30\sqrt{19}$ **(l)** $24\sqrt{14}$
3. **(a)** $7\sqrt{3}$ **(b)** $4\sqrt{7}$ **(c)** $11\sqrt{2}$ **(d)** $24\sqrt{11}$ **(e)** $14\sqrt{6}$ **(f)** $5\sqrt{15}$
(g) $8\sqrt{17}$ **(h)** $4\sqrt{3}$ **(i)** $\sqrt{5}$ **(j)** 0 **(k)** $\sqrt{7}$ **(l)** 0
4. **(a)** $3\sqrt{2}$ **(b)** $2\sqrt{3}$ **(c)** $2\sqrt{5}$ **(d)** $6\sqrt{2}$ **(e)** $12\sqrt{2}$ **(f)** $4\sqrt{3}$
(g) $9\sqrt{5}$ **(h)** $15\sqrt{7}$ **(i)** $2\sqrt{13}$ **(j)** $25\sqrt{2}$
5. **(a)** $3\sqrt{2}$ **(b)** $2\sqrt{5}$ **(c)** $10\sqrt{3}$ **(d)** $30\sqrt{3}$ **(e)** 168 **(f)** $45\sqrt{10}$
(g) $5\sqrt{10}$ **(h)** $9\sqrt{6}$
6. **(a)** 14 **(b)** $16\sqrt{2}$ **(c)** $5\sqrt{10}$ **(d)** $7\sqrt{10}$ **(e)** $5\sqrt{3}$ **(f)** $23\sqrt{3}$
(g) $\sqrt{2}$ **(h)** $8\sqrt{5} + 5$ **(i)** $5\sqrt{2}$ **(j)** 0 **(k)** $\sqrt{2}$ **(l)** $2\sqrt{2}$
7. **(a)** 8.66 **(b)** 3.87 **(c)** 0.97 **(d)** 68 **(e)** 87 **(f)** 27
(g) 5.4 **(h)** 6.6 **(i)** 0.24
8. **(a)** 19 **(b)** 6.4 **(c)** 0.79 **(d)** 93 **(e)** 3.4 **(f)** 0.29
9. **(a)** 1.44 m **(b)** $d = 1.66$ m
10. 10.6 m

REVIEW AND PREVIEW TO CHAPTER 4

Exercise 1

1. $11x + 13$ **2.** $9a + 7b$ **3.** $19y - 6$
4. $3a + 1$ **5.** $-8a - 12$ **6.** $3x - 7y + 15$
7. $22m - 3mn$ **8.** $7x - 11y$ **9.** $-2a - 5b - 1$
10. $8x^2 - 11x + 6$ **11.** $11xy - 2x - 13y$ **12.** $3a^2 - 3a + 2$
13. $2ab - 3a - 7b$ **14.** $-3a^2 + 6a - 15$ **15.** $-2a - 24x - 8$

Exercise 2

1. $3x + 21$ **2.** $8a - 24$ **3.** $15a^2 + 10a + 10$
4. $2a + 12$ **5.** $3x^2 - 6x - 21$ **6.** $10x - 15y$
7. $12a^2 + 8a - 16$ **8.** $3m - 5$ **9.** $11a - 10$
10. $20x + 38$ **11.** $22b - 8$ **12.** $6x^2 + 11x - 1$
13. $23a^2 + 39a - 13$ **14.** $7b + 18$ **15.** $-7x^2 + 3x - 2$

Exercise 3

1. **(a)** 7 **(b)** 9 **(c)** 10 **(d)** 12 **(e)** 0 **(f)** 5 **(g)** 11 **(h)** 8 **(i)** 7
2. **(a)** 3000 **(b)** 500 **(c)** 48 **(d)** 1200 **(e)** 216

Exercise 4-1

1. (a) 8 (b) 9 (c) 16 (d) 16
2. (a) 100 (b) 54
3. (a) x^8 (b) a^{12} (c) b^3 (d) m^6 (e) a^5b^5 (f) a^7b^4
4. (a) a^4 (b) b (c) n^9 (d) x^3 (e) a^4 (f) x
5. (a) $15a^9$ (b) $-8x^5$ (c) $6a^3b^5$ (d) $10x^4$ (e) $15m^2n$ (f) $12x^5$
   (g) $30y^4$ (h) $-6a^8$ (i) $-14x^8$
6. (a) $4a^2$ (b) $3xy^4$ (c) a (d) $8m$ (e) -5
   (f) $9b^2$ (g) $7q$ (h) -3 (i) $-3x^9y$

Exercise 4-2

1. (a) x^8 (b) a^9 (c) a^6b^3 (d) x^5y^{15} (e) $a^5b^5c^5$ (f) b^{24}
   (g) $8x^9$ (h) $a^{15}b^6$ (i) $9a^{10}$ (j) $27x^3y^6$ (k) $125a^{24}$ (l) $64x^6y^3z^3$
2. (a) $\frac{x^5}{y^5}$ (b) $\frac{a^4}{9}$ (c) $\frac{x^6}{y^3}$ (d) $\frac{a^4}{b^{20}}$ (e) $\frac{9x^2}{y^2}$ (f) $\frac{8x^6}{w^3}$
   (g) $\frac{25a^4}{4b^6}$ (h) $\frac{9a^2}{b^6}$
3. (a) $-8a^9$ (b) $9x^6$ (c) $9x^4$ (d) $6a^7b$ (e) $\frac{a^{15}b^3}{c^{12}}$ (f) $4a^8$
   (g) $-64x^6y^{18}$ (h) $-64a^{15}$ (i) $\frac{-27a^5}{b^3}$
4. (a) $16x^{11}$ (b) $2b^3$ (c) $3x^5y^{13}$ (d) $-4n^6$ (e) $-8xy$ (f) $-4m^4n^2$

Exercise 4-3

1. (a) x^{-2} (b) a^7 (c) y^4 (d) b^{-4} (e) a^8 (f) x
   (g) 1 (h) 1
2. (a) x^5 (b) b^{-3} (c) a^{-3} (d) x^{-5} (e) m^{-5} (f) n^7
   (g) y^{-5} (h) m^{-4}
3. (a) 1 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{10}$ (e) $\frac{1}{9}$ (f) 1
   (g) $\frac{1}{16}$ (h) $\frac{1}{8}$ (i) $\frac{1}{16}$
4. (a) $15x^3$ (b) a^5b^{-3} (c) 6 (d) x^2 (e) $9m^{-2}$ (f) $15a^{-6}$
   (g) m^7 (h) $8a^{15}b^{-9}$ (i) x^5y^2 (j) $35x^4y^{-2}$ (k) $7m^{-2}n^{-2}$ (l) 15
5. (a) a^2 (b) $3x^{-5}$ (c) $4b^{-10}$ (d) $m^{-3}n$ (e) 1 (f) $3a^2b^{-2}$
   (g) b^4 (h) y^{-4} (i) $16x^{-4}$ (j) x^7 (k) $16b^8$ (l) $\frac{a^{-14}}{2}$
6. (a) $1\frac{1}{5}$ (b) $\frac{7}{12}$ (c) $\frac{1}{25}$ (d) $\frac{1}{16}$ (e) 2 (f) 4
   (g) 1 (h) $\frac{9}{20}$ (i) 10 (j) $\frac{1}{100}$ (k) 9 (l) $\frac{1}{100}$

Exercise 4-4

1. (a) 13 (b) 6 (c) 6 (d) 1 (e) 4 (f) 8 (g) 8 (h) 2
2. (a) 6 (b) 15 (c) 0 (d) 24 (e) 23 (f) 16 (g) 18 (h) 0
3. (a) 6 (b) 36 (c) 10 (d) 35 (e) 17 (f) 36
4. (a) 0 (b) -13 (c) 5 (d) 0 (e) 16 (f) 11
5. (a) 6 (b) 21 (c) -12 (d) -13 (e) -15 (f) 1 (g) -1 (h) 17
6. (a) -12 (b) $\frac{5}{6}$ (c) $-\frac{19}{4}$ (d) -6 (e) $\frac{1}{5}$ (f) 0

Exercise 4-5

1. (a) $2x + 6$ (b) $6a + 3$ (c) $4 + 8b$ (d) $15x - 10$
   (e) $-2x + 10$ (f) $-6 + 18a$ (g) $-x - 6$ (h) $-3x + 3y$
2. (a) $3x^2 + x$ (b) $2a + 6a^2$ (c) $-2b^2 - 6b$ (d) $-3m + 6m^2$
   (e) $-2a^2 + 4a$ (f) $ab - abc$ (g) $2x^3 - 4x^2 + 2x$ (h) $-a + 2a^2 + a^3$
3. (a) $5x + 21$ (b) $23a - 9$ (c) $9m - 5$ (d) $-3d - 2$
   (e) $8x^2 - 18x + 11$ (f) $a - 5b$ (g) $10a + 7b - 14c$ (h) $7a^2 - 34a + 12$
4. (a) $x - 29$ (b) $9x - 16y$ (c) $22a - 14$ (d) $-12x^2 + x + 17$
   (e) $11m + 9$ (f) $13a - 12b + 6c$ (g) $9y - 5$ (h) $7x^2 - 4x - 3$
   (i) $11a + 31m - 3$ (j) $-5a - 4b + 14c$
5. (a) $-4x^2 - 17x$ (b) $5a^2 - 4a$ (c) $2x^3 - x^2 - 2x + b$ (d) $b^2 - 28b$
   (e) $a^2 + a + 8$ (f) $-4a^2 + 8ab$ (g) $11x^2 - 19x + 35$ (h) $-4a^2 - 4a - 19$
   (i) $6m^2 - 16mn + 24n^2$ (j) $-14x^2 - 6x + 18$
6. (a) $14x + 22$ (b) $24m - 12$ (c) $23a - 6$ (d) $12x + 12$

Exercise 4-6

1. (a) $x^2 + 4x + 3$ (b) $a^2 + 12a + 35$ (c) $m^2 + 9m + 18$ (d) $t^2 + 14t + 33$
   (e) $d^2 + 13d + 42$ (f) $m^2 + 6m + 8$ (g) $r^2 + 16r + 60$ (h) $s^2 + 13s + 12$
   (i) $a^2 + 6a + 9$ (j) $x^2 + 6x + 9$
2. (a) $m^2 - m - 6$ (b) $x^2 - 11x + 30$ (c) $a^2 + a - 30$ (d) $b^2 + 3b - 18$
   (e) $y^2 - 6y + 9$ (f) $m^2 - 4m - 60$ (g) $d^2 - 9$ (h) $s^2 - 15s + 56$
   (i) $a^2 - 9a + 18$ (j) $-t^2 - t + 30$
3. (a) $2x^2 + 13x + 6$ (b) $6a^2 - 7a + 2$ (c) $2m^2 - 3m - 14$ (d) $9x^2 - 1$
   (e) $4b^2 - 12b + 9$ (f) $2d^2 + 7d - 30$ (g) $5 - 13r - 6r^2$ (h) $4r^2 - 6r - 18$
   (i) $6t^2 - t - 15$ (j) $-15x^2 + 26x - 7$
4. (a) $7x^2 - 19x + 10$ (b) $3m^2 - 12$ (c) $1 - 25b^2$ (d) $9x^2 - y^2$
   (e) $6a^2 + 5ab + b^2$ (f) $a^2 + 2ab + b^2$ (g) $3x^2 - 5xy + 2y^2$ (h) $6b^2 - bc - 2c^2$
   (i) $6m^2 + mn - 2n^2$ (j) $x^2 - y^2$
5. (a) $2x^2 - 3x + 1$ (b) $6a^2 - 5a - 6$ (c) $4m^2 + 12m + 9$
6. (a) $a^3 + 5a^2 + 7a + 3$ (b) $x^3 + 5x^2 + 9x + 6$
   (c) $2m^3 - 5m^2 + m + 2$ (d) $2s^3 - 3s^2 - 12s + 9$
   (e) $2x^3 - 7x^2 - 5x + 4$ (f) $4b^3 + 3b + 18$
   (g) $6a^3 + a^2 - 9a + 10$ (h) $x^4 + x^3 - 6x^2 - 5x - 1$
   (i) $2m^4 - m^3 - 8m^2 + 10m - 3$ (j) $2y^4 + y^3 - 13y^2 + 7y + 3$

Exercise 4-7

1. (a) $a^2 + 2a + 1$ (b) $x^2 + 6x + 9$ (c) $m^2 + 4m + 4$
   (d) $t^2 + 10t + 25$ (e) $n^2 - 12n + 36$ (f) $r^2 - 4r + 4$
   (g) $x^2 - 8x + 16$ (h) $b^2 - 6b + 9$ (i) $r^2 - 14r + 49$
2. (a) $m^2 + 12m + 36$ (b) $y^2 - 2y + 1$ (c) $m^2 + 16m + 64$
   (d) $16 + 8x + x^2$ (e) $25 - 10m + m^2$ (f) $a^2 + 14a + 49$
   (g) $r^2 + 16r + 64$ (h) $t^2 + 4t + 4$ (i) $1 + 2m + m^2$
3. (a) $4x^2 + 4x + 1$ (b) $9m^2 + 12m + 4$ (c) $25a^2 + 10a + 1$
   (d) $9t^2 - 6t + 1$ (e) $9a^2 - 12a + 4$ (f) $16b^2 + 16b + 4$
   (g) $25a^2 + 30a + 9$ (h) $4x^2 - 28x + 49$ (i) $36x^2 - 12x + 1$
4. (a) $x^2 - 12x + 36$ (b) $4y^2 + 12y + 9$ (c) $25t^2 - 30t + 9$
   (d) $16b^2 + 40b + 25$ (e) $36m^2 + 12m + 1$ (f) $1 - 4r + 4r^2$
   (g) $9 - 18t + 9t^2$ (h) $1 + 12a + 36a^2$ (i) $4s^2 - 4s + 1$
5. (a) $2x^2 + 2x + 13$ (b) $2a^2 + 4a + 34$ (c) $13x^2 - 2x + 2$
   (d) $5m^2 + 2m + 58$ (e) $13b^2 + 14b + 26$ (f) $3x^2 + 12x + 14$
   (g) $3a^2 - 8a + 14$ (h) $14x^2 - 10x + 14$

Exercise 4-8

1. (a) $2x + 4$ (b) $6x - 3$ (c) $4a - 20$ (d) $a - 7$ (e) $2 - 6c$
(f) $-3 - 6x$ (g) $-8x + 4$ (h) $-x + 7$ (i) $2a + 4b - 2c$ (j) $-6x - 12y$
2. (a) $5x - 12$ (b) $-5a - 8$ (c) $10m + 9$
(d) $3b + 5$ (e) $25c - 26$ (f) $3x^2 + 9x + 6$
(g) $4x^2 - 14x + 6$ (h) $4x^2 + 24x + 36$ (i) $-2x^2 + 8x + 42$
3. (a) $2a^2 + 6a - 16$ (b) $2x^2 - 5x + 22$ (c) $m^2 + 7m + 5$
(d) $2d^2 - 5d - 42$ (e) $-14b + 7$ (f) $-9m^2 - 4m - 7$
4. (a) $2x^2 + 4x - 15$ (b) $6a^2 - 21a - 30$ (c) $-4t^2 - 13t + 35$
(d) $9b^2 + 9b - 7$ (e) $-x^2 + 42x - 165$ (f) $16m^2 - m + 4$
5. (a) $4x^2 + 4x + 13$ (b) $10m^2 - 13m + 1$ (c) $-10x^2 + 8x$
(d) $8x - 6$ (e) $7m^2 + m + 2$ (f) $-23b + 24$

Exercise 4-9

1. (a) $2ab$ (b) xyt (c) $10b$ (d) $2bc$ (e) nt
(f) $-2dem$ (g) $4c$ (h) -3 (i) $3t$
2. (a) xy (b) $3b$ (c) $2x$ (d) $-4c$ (e) mn
(f) x^4 (g) $-6mn$ (h) $4x^4y$ (i) $-3a^2b$
3. (a) $x + 2$ (b) $ab + 3b$ (c) $2a - b$ (d) $2n + 3t$ (e) $3x - 2$
(f) $2x^2 - 3x + 4$ (g) $1 - 2b + 3a$
4. (a) $-2x + 4$ (b) $-2a + 3b + 4$ (c) $-4 - a + 3a^2$
(d) $1 - 2t + 3t^2$ (e) $3c$ (f) $-2x + 1 - y$

Exercise 4-10

1. (a) $3x^3 + 2x^2 - 2x + 1$ (b) $6a^3 - 3a^2 + 5a + 2$
(c) $5m^4 - 3m^3 + m^2 + 2m + 7$ (d) $-4b^5 + 3b^3 - b^2 + 1$
(e) $-6x^2 - 3x^4 + 2x^2 + 7$
2. (a) $x + 3$ (b) $a - 4$ (c) $m + 7$
(d) $2x - 3$ (e) $2a + 1$ (f) $3m + 1$, R:9
3. (a) $x^2 - x - 1$ (b) $2a^2 - 3a + 1$ (c) $3x^2 - 5x - 1$
(d) $5m^3 + 3m^2 + 5m - 1$ (e) $3b^3 - 2b^2 + 3b - 2$ (f) $x^2 - 2x + 1$, R:5
4. (a) $3m^2 - 2m - 1$ (b) $3x^2 - 2x - 1$ (c) $3t^2 + 3t$
(d) $4a^3 + 2a - 2$ (e) $x^2 + x + 1$ (f) $x^3 - 3$, R:4
5. $2x - 3$
6. $3a^2 - 2a + 1$
7. $2x - 1$

Exercise 4-11 (I)

1. (a) 4 (b) 4 (c) x (d) $4x$ (e) ab (f) $6x$
(g) $4x$ (h) $2xy^2$ (i) $20a$ (j) $10bc$ (k) xb (l) 2

Exercise 4-11 (II)

1. (a) $2(x + 3)$ (b) $3(a - 3)$ (c) $6(2 - b)$
(d) $2(2a - b)$ (e) $3(a + 2b + 3c)$ (f) $5(a^2 - 2b + 3)$
(g) $4(b^2 + 1)$ (h) $3(x^2 + 2x - 1)$ (i) $10(2xy - b)$
(j) $2(t - 2m + 3r)$
2. (a) $2x(2x + 3)$ (b) $a(5b - 6c)$ (c) $3x(2y + t + 3a)$
(d) $2a(a - 2)$ (e) $6a(2a - b)$ (f) $5x(2 - x)$
(g) $2b(4b^2 - 2b + 3)$ (h) $3m(1 - 2m - 3m^2)$ (i) $2b(3b - 1 + 5b^2)$
(j) $3m^2(m^2 - 2m + 4)$
3. (a) $xy(3 + 7t)$ (b) $mn(7 + 6m)$ (c) $ab(12a - 6b + 1)$
(d) $2rt(t + 3t^2 - 2r)$ (e) $5xy(2m - t)$ (f) $pqr(14 - 7p + t)$
(g) $2b(2a + x + y)$ (h) $(x + y)(3 - a)$ (i) $(x + 2)(x + 3)$
(j) $(a - 3)(a - 2)$ (k) $(m - 1)(m - 2)$ (l) $(t + 5)(t + 6)$

Exercise 4-12

1. **(a)** $x - 2$ **(b)** $a + 3$ **(c)** $m - 6$ **(d)** $m + 5$ **(e)** $2x - 1$ **(f)** $4 + n$
2. **(a)** $(x - 7)(x + 7)$ **(b)** $(m - 4)(m + 4)$ **(c)** $(a - 12)(a + 12)$
(d) $(b - 11)(b + 11)$ **(e)** $(c - 10)(c + 10)$ **(f)** $(d - 8)(d + 8)$
3. **(a)** $4(x - 2)(x + 2)$ **(b)** $(1 - y)(1 + y)$ **(c)** $(4m - 1)(4m + 1)$
(d) $(3a - 5)(3a + 5)$ **(e)** $(10 - a)(10 + a)$ **(f)** $9(t - 2)(t + 2)$
(g) $(2m - 7)(2m + 7)$ **(h)** $(1 - 6x)(1 + 6x)$ **(i)** $(7 - 3a)(7 + 3a)$
4. **(a)** $(2a - 3b)(2a + 3b)$ **(b)** $(4x - 5y)(4x + 5y)$ **(c)** $(a - 4b)(a + 4b)$
(d) $16(3m - b)(3m + b)$ **(e)** $(5b - 7x)(5b + 7x)$ **(f)** $(1 - 20t)(1 + 20t)$
(g) $(5a - 4b)(5a + 4b)$ **(h)** $36(2b - t)(2b + t)$ **(i)** $(15x - 1)(15x + 1)$
5. **(a)** 65 **(b)** 27 **(c)** 3 **(d)** 45 **(e)** 36 **(f)** 140 **(g)** 197 **(h)** 296 **(i)** 329

Exercise 4-13

1. **(a)** $(x + 4)$ **(b)** $(x + 5)$ **(c)** $(x - 3)$ **(d)** $(x - 4)$ **(e)** $(a + 5)$
(f) $(m - 3)$ **(g)** $(b + 1)$ **(h)** $(c - 5)$ **(i)** $(t - 4)$ **(j)** $(d - 9)$
2. **(a)** $(x + 3)(x + 2)$ **(b)** $(x + 7)(x + 2)$ **(c)** $(a - 5)(a + 3)$
(d) $(m - 4)(m - 4)$ **(e)** $(b - 3)(b - 7)$ **(f)** $(t + 2)(t + 2)$
(g) $(d - 6)(d + 3)$ **(h)** $(x + 11)(x + 2)$ **(i)** $(n + 10)(n - 2)$
(j) $(a + 1)(a + 1)$
3. **(a)** $(x + 3)(x + 2)$ **(b)** $(a - 4)(a + 3)$ **(c)** $(m - 3)(m - 3)$
(d) $(r - 6)(r + 3)$ **(e)** $(n - 5)(n - 5)$ **(f)** $(b + 5)(b - 3)$
(g) $(t + 4)(t + 1)$ **(h)** $(x + 7)(x + 2)$ **(i)** $(c - 7)(c + 3)$
(j) $(y - 4)(y - 4)$
4. **(a)** $(x + 5)$ **(b)** $(a + 3)$ **(c)** $(b + 1)$ **(d)** $(m - 5)$ **(e)** $(x + 5)$
(f) $(r + 2)$ **(g)** $(t + 4)$ **(h)** $(b + 3)$ **(i)** $(c + 5)$ **(j)** $(x + 1)$
5. **(a)** $(x + 2)(x + 1)$ **(b)** $(a + 5)(a + 3)$ **(c)** $(t + 4)(t + 2)$
(d) $(x + 5)(x + 5)$ **(e)** $(m + 6)(m + 1)$ **(f)** $(b - 4)(b + 2)$
(g) $(r + 6)(r - 3)$ **(h)** $(d - 4)(d - 3)$ **(i)** $(n - 5)(n - 1)$
(j) $(a - 5)(a + 3)$ **(k)** $(r - 2)(r - 2)$ **(l)** $(h - 9)(h + 2)$
6. **(a)** $(x - 5)(x - 4)$ **(b)** $(a + 12)(a + 2)$ **(c)** $(m - 11)(m + 2)$
(d) $(t + 9)(t + 4)$ **(e)** $(d + 10)(d + 10)$ **(f)** $(n + 7)(n - 3)$
(g) $(x + 3)(x - 1)$ **(h)** $(a - 11)(a + 8)$ **(i)** $(m + 5)(m - 4)$
(j) $(t - 5)(t + 4)$ **(l)** $(a + 10)(a + 3)$
(k) $(h - 5)(h - 6)$ **(b)** $(a - 7)(a - 7)$ **(c)** $(r + 12)(r + 12)$
7. **(a)** $(x - 10)(x + 7)$ **(e)** $(b + 4)(b + 4)$ **(f)** $(t - 6)(t + 4)$
(d) $(m - 8)(m - 3)$ **(h)** $(m - 11)(m + 3)$ **(i)** $(x + 10)(x - 4)$
(g) $(d + 7)(d - 5)$ **(k)** $(r + 8)(r - 7)$ **(l)** $(t + 3)(t - 2)$
(j) $(h - 10)(h - 5)$ **(b)** $3(m + 4)(m + 3)$ **(c)** $5(a - 4)(a + 2)$
8. **(a)** $2(x + 3)(x + 2)$ **(e)** $10(x + 5)(x - 3)$ **(f)** $3(b + 5)(b + 5)$
(d) $2(t - 3)(t - 3)$ **(h)** $5(x - 4)(x + 3)$ **(i)** $4(r - 2)(r - 2)$
(g) $7(x - 3)(x + 1)$
(j) $2(m + 3)(m + 2)$

REVIEW EXERCISE

1. **(a)** 125 **(b)** 81 **(c)** 32 **(d)** 1 **(e)** $\frac{1}{2}$ **(f)** $\frac{1}{3}$
(g) 1 **(h)** $\frac{1}{10}$ **(i)** $\frac{1}{4}$ **(j)** $\frac{8}{27}$ **(k)** 9 **(l)** 5
(m) 1 **(n)** $\frac{1}{2}$
2. **(a)** a^4 **(b)** x^6 **(c)** x^2y^3 **(d)** $w^3y^3z^2$ **(e)** 5^3 **(f)** $3^2 \times 2^3$
3. **(a)** x^8 **(b)** a^4 **(c)** y^7 **(d)** b^5 **(e)** b^5 **(f)** x^{20}
(g) w^{-5} **(h)** x^7y^{10} **(i)** x^{-6} **(j)** x^0 **(k)** a^{-4} **(l)** x^{-3}
4. **(a)** x^9 **(b)** a **(c)** b^7 **(d)** w^{-2} **(e)** b^6 **(f)** x^{-5}
(g) y^{-5} **(h)** k^{-5} **(i)** z^{-3} **(j)** a^{-17} **(k)** y^{-6} **(l)** x^5

5. (a) 12 (b) 13 (c) 3 (d) -18 (e) 36 (f) 5
6. (a) $3x + 12$ (b) $4a - 2$ (c) $-3m - 9$ (d) $-2a^2 - 3a$ (e) $y - xy$
(f) $-x + 3$ (g) $2x^2 - 2x + 2$ (h) $-4m^2 + 8m$ (i) $-x^2 + x + 7$
7. (a) $2abc$ (b) $2y$ (c) m^3 (d) c (e) $-5m$ (f) 0 (g) -1 (h) $-3ab$ (i) $3bc$
8. (a) $15a^7$ (b) $28y^3$ (c) a^7b^4 (d) $-12x^8$ (e) $35a^8$ (f) $-15a^2$
(g) $7a^8b$ (h) $-15a^3b^5$ (i) $45a^2$ (j) $-20x^2y^3$ (k) $15a^{-9}$ (l) $-18a^5b$
9. (a) $3b^2$ (b) $5x^6$ (c) $9b^5$ (d) $3y^2$ (e) $4x^7$ (f) $-2a$
(g) $9b^2$ (h) $6x^5$ (i) $5a^2$ (j) $17y^4$ (k) $-8w^5$ (l) $a^{-5}b^3$
10. (a) $5x^4$ (b) $15a^4$ (c) x^6y^{10} (d) $3x^2$ (e) $4x^3$ (f) $72x^7$
(g) $25a^2$ (h) $5mn^{-1}$
11. (a) $\frac{1}{9}$ (b) 4 (c) $\frac{4}{3}$ (d) $1\frac{1}{2}$ (e) $\frac{5}{6}$ (f) 4
(g) 3 (h) $\frac{3}{8}$
12. (a) 110.8 (b) 32.2 (c) 19.19 (d) 535.4 (e) 5.56 (f) 2.84
(g) 20.84 (h) 0.496 (i) 3.479 (j) 0.9714
13. (a) $x^2 + 5x + 6$ (b) $m^2 + 6m + 9$ (c) $b^2 - 6b + 8$ (d) $t^2 - 9$
(e) $r^2 - 4r + 4$ (f) $1 + 2m + m^2$ (g) $b^2 - b - 12$ (h) $d^2 - 9d + 20$
(i) $x^2 - 2x + 1$ (j) $a^2 - 25$
14. (a) $2(x + 3)$ (b) $2x(x - 2y)$ (c) $5m(n - 2)$ (d) $(x - 4)(x + 4)$
(e) $(m - 11)(m + 11)$ (f) $(d - 8)(d + 8)$ (g) $(a - 3)(a + 3)$ (h) $(1 - b)(1 + b)$
(i) $(c - 7)(c + 7)$
15. (a) 24 (b) 4 (c) 0 (d) 60 (e) 1 (f) -9 (g) 4 (h) -7
16. (a) $7x - 5$ (b) $-8a^2 + 8a + 3$ (c) $-a - b + 3c$ (d) $5m^2 - 3m + 6$
(e) $3b^2 - 5b - 7$ (f) $-7x^2 + 11x$ (g) $17 - 10c$
17. (a) $2a^2 + 9a - 18$ (b) $6x^2 + 15x - 9$ (c) $2m^2 + 28m + 98$
(d) $-2 - 4x + 30x^2$ (e) $9b^2 - 18b + 37$ (f) $x^2 + 7x - 19$
(g) $y - 13y^2$ (h) $-a^2 - 29a - 26$ (i) $9x^2 - 13x - 7$
(j) $6d^2 - 60d + 86$ (k) $-4m^2 - 4m - 39$
18. (a) $(x - 7)$ (b) $(a - 5)$ (c) $(x + 3)$
(d) $b^2 - 3b + 2$ (e) $2c^2 - 2c + 1$ (f) $2x^2 - 3x + 4$
(g) $4a^2 + 2a + 1$ (h) $x^3 + 2x^2 - x - 3$ (i) $3 - 2b + 2b^2$
19. (a) $4(x - 3)$ (b) $3a(a + 3)$ (c) $10a(2 - a)$
(d) $2b^2(b^2 - 3b + 1)$ (e) $3xy(1 - 2y + 4x)$ (f) $3abc(4a - 2b + 1)$
(g) $(x + 2)(3 - x)$ (h) $(x - y)(m - n)$ (i) $m(x + y)(3n - 2)$
20. (a) $(m - 11)(m + 11)$ (b) $(x + 3)(x + 6)$ (c) $(b - 6)(b - 6)$
(d) $(2r - 7)(2r + 7)$ (e) $(c - 9)(c + 3)$ (f) $(a + 6)(a - 2)$
(g) $(1 - 7ab)(1 + 7ab)$ (h) $(1 - 6d)(1 - 2d)$ (i) $(b + 14)(b - 4)$
(j) $(y + 16)(y + 5)$ (k) $(6a - 11b)(6a + 11b)$ (l) $(4x^2 - y)(4x^2 + y)$
(m) $(x - 10)(x + 9)$ (n) $(a + 23)(a - 4)$ (o) $(a + 20)(a + 20)$
21. (a) $2x^3 - x^2 - 25x - 12$ (b) $x^4 + 2x^3 - 13x^2 - 14x + 49$
(c) $x^3 - x^2 + 12$
22. (a) $(x + y)^2 = x^2 + 2xy + y^2$ (b) $(x + 2y)^2 = x^2 + 4xy + y^2$
(c) $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$ (d) $(2x)^2 = 4x^2$
(e) $(5x)^2 = 25x^2$ (f) $4(4x^2) = 16x^2$

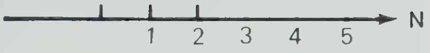
REVIEW AND PREVIEW TO CHAPTER 5

Exercise 1

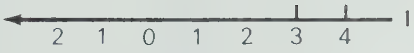
1. (a) 6 (b) 9 (c) 0 (d) -6 (e) -9 (f) -15 (g) $3a$
2. (a) 5 (b) 11 (c) 17 (d) -11 (e) -9 (f) -3 (g) $2a - 3$
3. (a) 1 (b) 17 (c) 7 (d) -1 (e) 1 (f) 199 (g) 199 (h) $2x^2 - 1$

Exercise 2

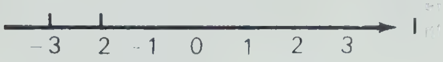
1. (a)



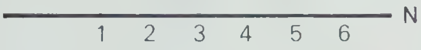
(b)



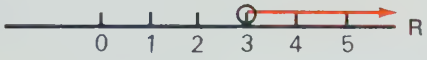
(c)



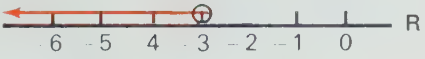
(d)



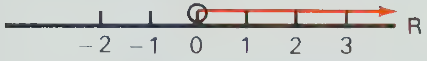
(e)



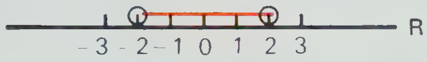
(f)



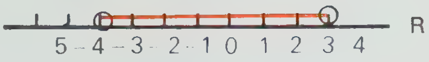
(g)



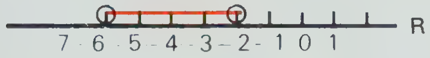
2. (a)



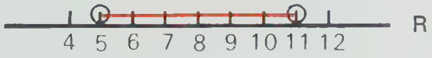
(b)



(c)



(d)



Exercise 3

1. $\{x \mid x > 6, x \in N\}$
2. $\{x \mid x < 7, x \in N\}$
3. $\{x \mid x < 4, x \in I\}$
4. $\{x \mid -2 \leq x \leq 4, x \in I\}$
5. $\{x \mid x \geq 3, x \in R\}$
6. $\{x \mid x < 2, x \in R\}$
7. $\{x \mid -1 \leq x \leq 4, x \in R\}$
8. $\{x \mid -3 < x < 1, x \in R\}$
9. $\{x \mid -2 \leq x \leq 5, x \in R\}$
10. $\{x \mid 1 < x \leq 7, x \in R\}$

Exercise 5-1

1. **(a)**

Triangles	Sides
1	3
2	6
3	9
•	•
•	•
•	•
100	300

(b) {1,2,3, ... , 100}
(c) {3,6,9, ... , 300}
(d) 30 triangles have a total of 90 sides
(e) (17,51),(9,27)
(f) ordered pairs

2. **(a)**

ℓ	km
3	21
5	35
6	42
20	140
30	210

(b) {(3,21), (5,35), (6,42), (20,140), (30,210)}
(c) {3,5,6,20,30}
(d) {21,35,42,140,210}
(e) 10 ℓ are required for 70 km
(f) (2,14), (45,315)

3. **(a)**

s	m
2	660
3	990
5	1650
30	9900
40	13 200

(b) {(2,660), (3,990), (5,1650), (30,9900), (40,13 200)}
(c) {2,3,5,30,40}
(d) {660,990,1650,9900,13 200}
(e) $(3\frac{1}{2},1155)$
(f) (5,1650)
(g) In $5\frac{1}{2}$ s sound travels 1815 m

4. **(a)**

Chirps per minute	Temperature °C
56	12.2
80	15.6
92	17.2
96	17.8
100	18.3
40	10

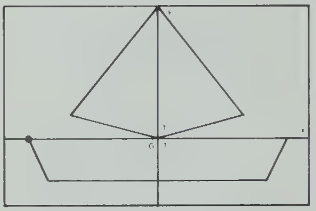
Temperature is to three figure accuracy

(b) {(40,10), (56,12.2), (80,15.6), (92,17.2), (96,17.8), (100,18.3)}
(c) {40,56,80,92,96,100}
(d) {10,12.2,15.6,17.2,17.8,18.3}

Exercise 5-2

1. A(4,2), B(-3,1), C(-4,-4), D(2,-3), E(3,0), F(0,3), G(0,-2), H(-2,-2)
2. (i)C, (ii)D, (iii)H, (iv)J, (v)G, (vi)A, (vii)E, (viii)F
3. **(a)** Parallelogram, **(b)** Isosceles triangle, **(c)** Square, **(d)** Rectangle, **(e)** Trapezoid, **(f)** Pentagon
4. {(2,4), (3,6), (4,8), (-3, -6)} They lie in a straight line
5. {(3,1), (6,2), (9,3), (-12,-4)} They lie in a straight line
6. {(5,7), (6,9), (8,10), (-3,-1)} They lie in a straight line
7. {(3,2), (4,3), (5,4), (-2,-3)} They lie in a straight line
8. {(2,2), (4,4), (6,6), (-2,-2)} They lie in a straight line
9. {(0,4), (2,6), (-3,3), (-7,-4)} They lie in a straight line
10. The second element is two more than the first element
11. {(4,2), (1,5), (-4,10), (6,0)} They lie in a straight line
12. {(6,4), (8,6), (-3,-5), (0,-2)} They lie in a straight line
13. {(7,3), (11,5), (1,0), (-7,-4)} They lie in a straight line

14.

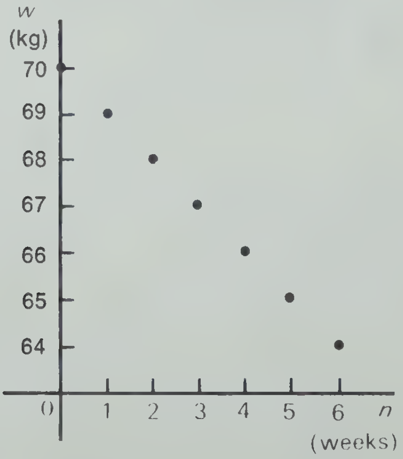


Exercise 5-3

1. **(a)** Domain: {3,4,5,6}, range: {2,7,12,17}
(b) Domain: {6,8,10,15,17}, range: {2,9,12,14,20}
(c) Domain: {2}, range: {3,4,5,6}
(d) Domain: {3,4,5,6}, range: {1}
(e) Domain: {*a,b,c,d*}, range: {*a,b,c,d*}

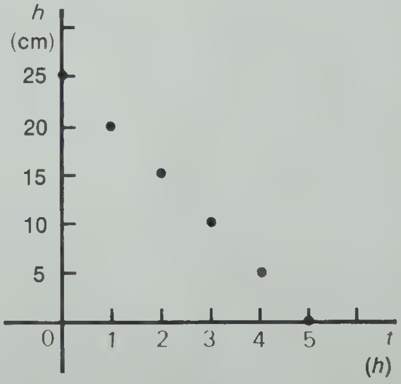
2. **(a)** (0,1), (1,2), (2,3), (3,4)
(b) (0,−1), (1,1), (2,3), (3,5)
(c) (−2,−4), (−1,−1), (0,2), (1,5)
(d) (1,3), (2,6), (3,9), (4,12)
(e) (−1,−8), (0,−7), (1,−6), (2,−5)
(f) (0,6), (1,5), (2,4), (3,3)

3. **(a)**



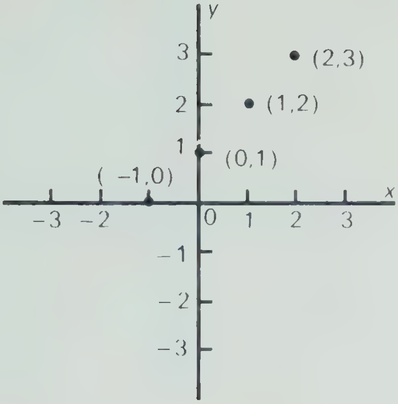
(b) Domain: {0,1,2,3,4,5,6}
range: {64, 65, 66, 67, 68, 69, 70}

4. **(a)**

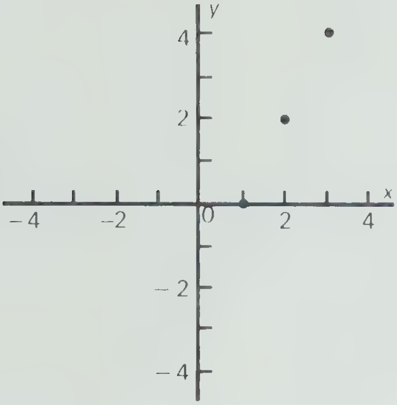


(b) Domain: {0,1,2,3,4,5}
range: {0, 5, 10, 15, 20, 25}

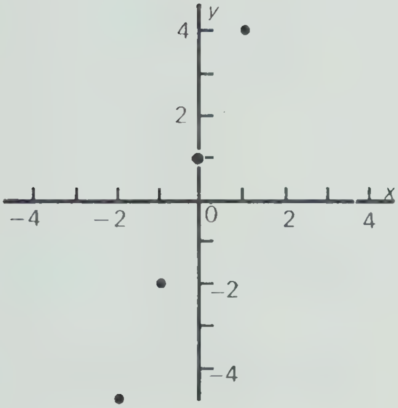
5. (a)


y
x
3
2
1
−1
−2
−3
−3
−2
0
1
2
3
(2,3)
(1,2)
(0,1)
(−1,0)

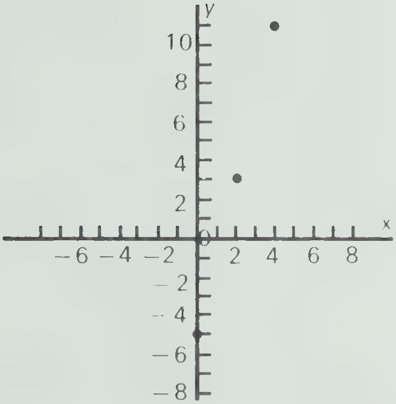
(b)


y
x
4
2
−2
−4
−4
−2
0
2
4

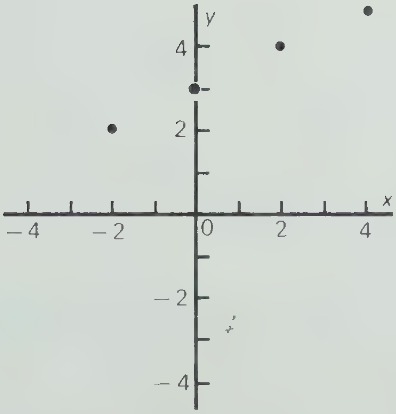
(c)


y
x
4
2
−2
−4
−4
−2
0
2
4

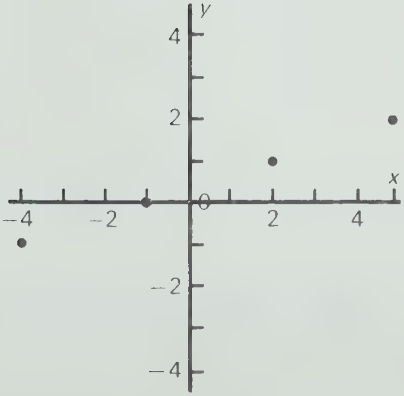
(d)


y
x
10
8
6
4
2
−2
−4
−6
−8
−6
−4
−2
0
2
4
6
8

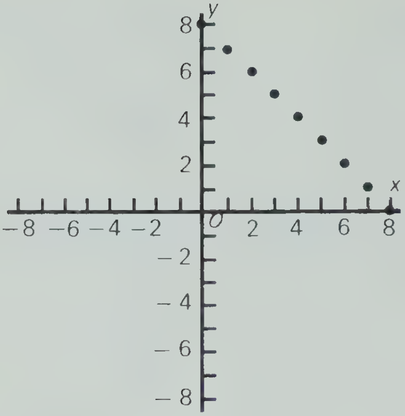
(e)


y
x
4
2
−2
−4
−4
−2
0
2
4

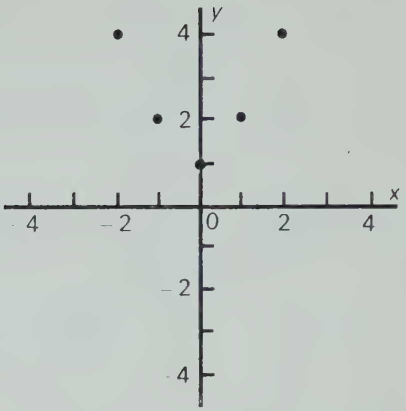
(f)


y
x
4
2
−2
−4
−4
−2
0
2
4

(g)



(h)

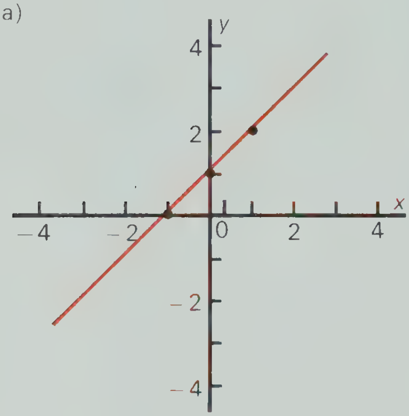


6. **(a)** $y = x + 1$ **(b)** $y = x - 4$ **(c)** $y = 2x$ **(d)** $y = -\frac{1}{2}x$

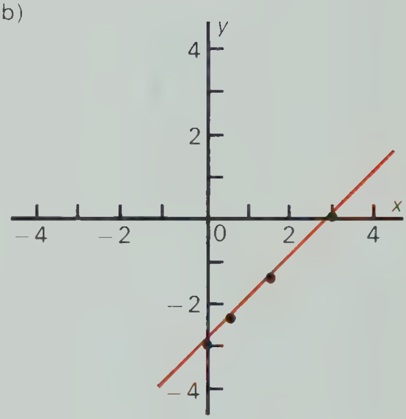
Exercise 5-4

1. Functions are (a), (b), (f), (i), (j), (l)

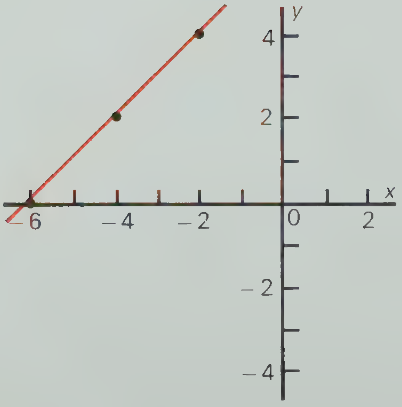
2. (a)



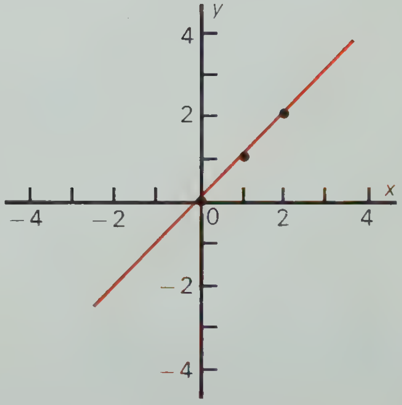
(b)



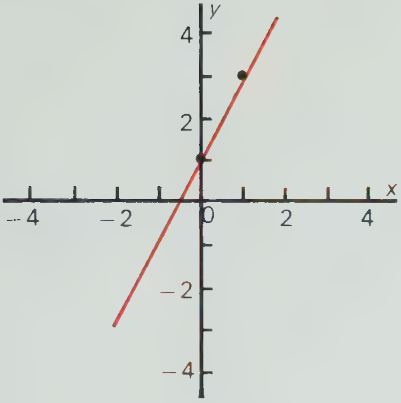
(c)



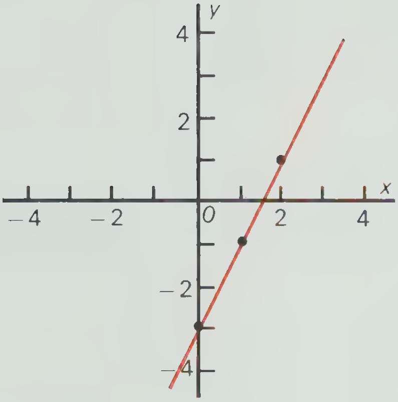
(d)



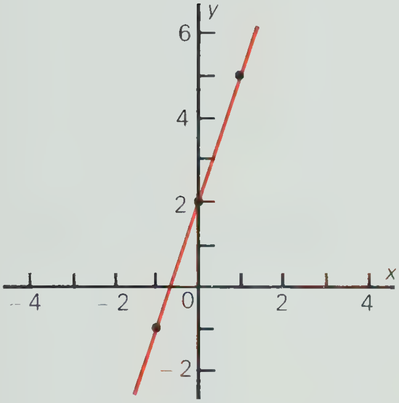
3. (a)


y
4
2
x
– 4
– 2
0
2
4
– 2
– 4

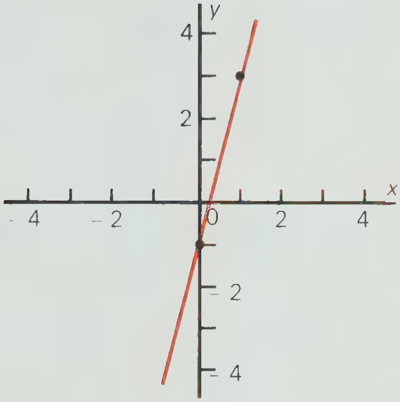
(b)


y
4
2
x
– 4
– 2
0
2
4
– 2
– 4

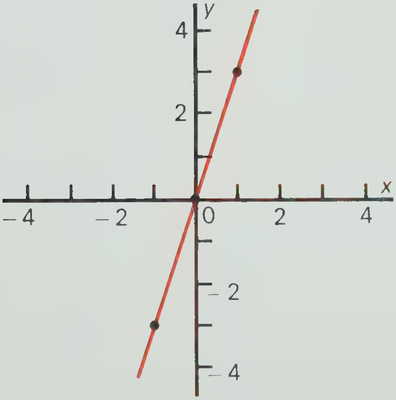
(c)


y
6
4
2
x
– 4
– 2
0
2
4
– 2

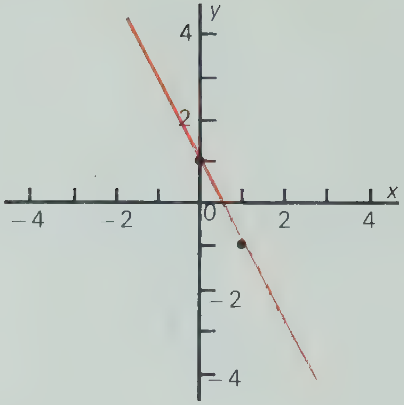
(d)


y
4
2
x
– 4
– 2
0
2
4
– 2
– 4

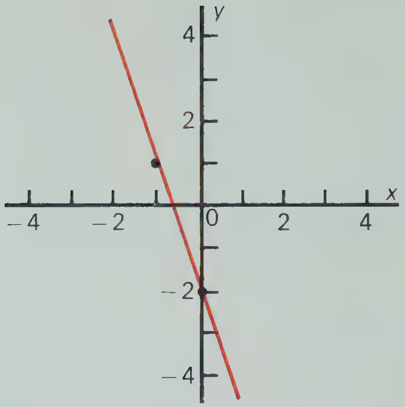
(e)


y
4
2
x
– 4
– 2
0
2
4
– 2
– 4

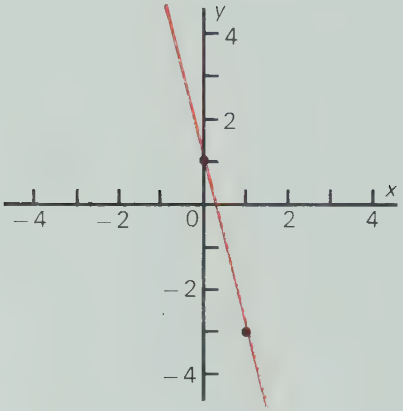
4. (a)


y
4
2
x
−4
−2
0
2
4
−2
−4

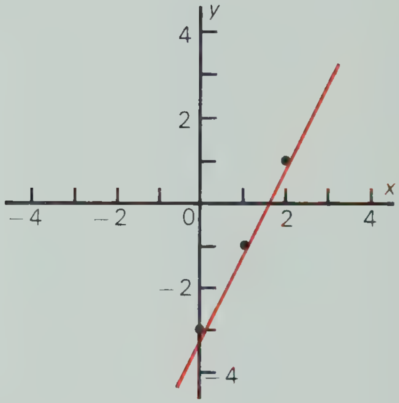
(b)


y
4
2
x
−4
−2
0
2
4
−2
−4

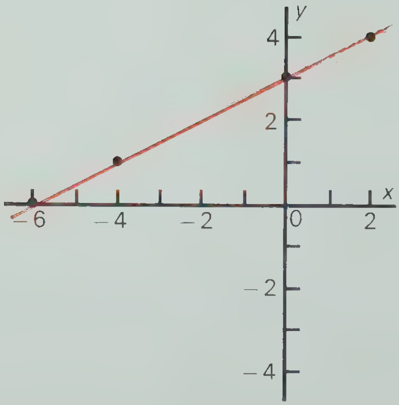
(c)


y
4
2
x
−4
−2
0
2
4
−2
−4

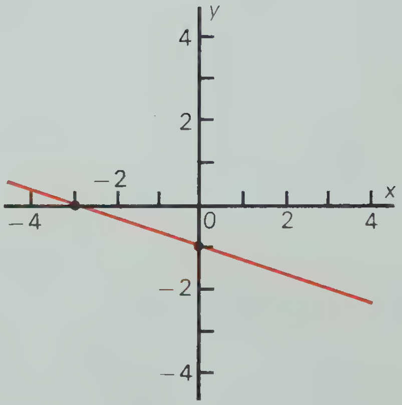
(d)


y
4
2
x
−4
−2
0
2
4
−2
−4

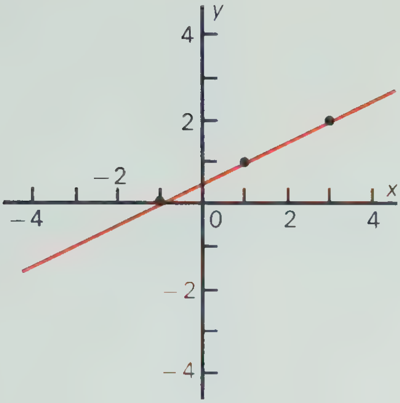
5. (a)


y
4
2
x
−6
−4
−2
0
2
−2
−4

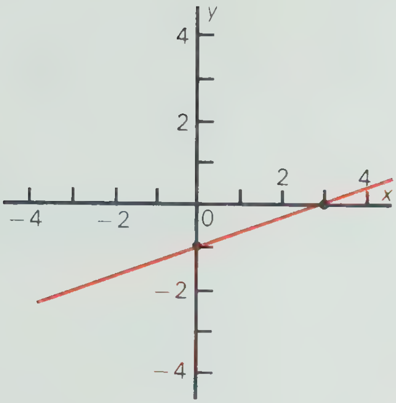
(b)


y
4
2
−2
x
−4
0
2
4
−2
−4

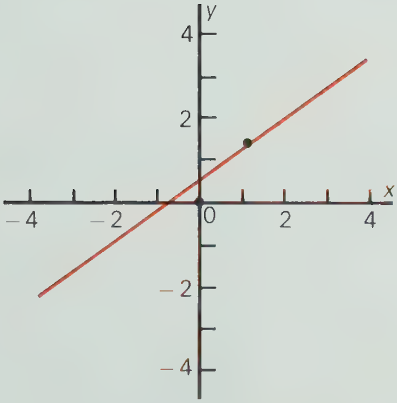
(c)



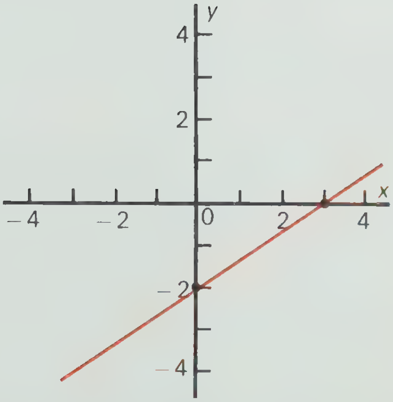
(d)



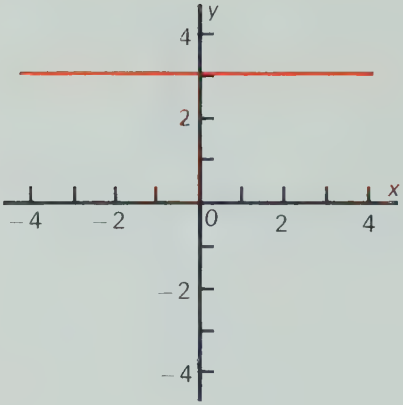
(e)



(f)

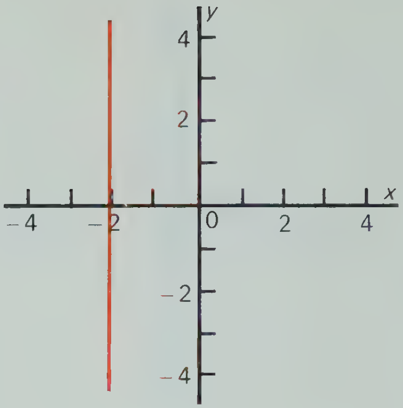


6. (a)



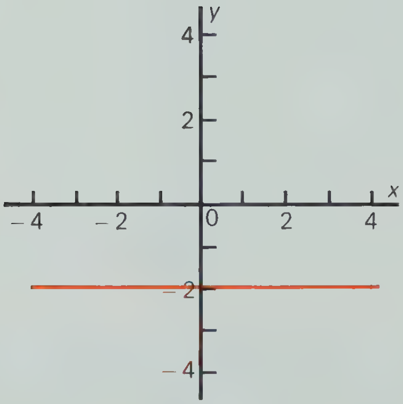
Domain is R
Range is {3}

(b)



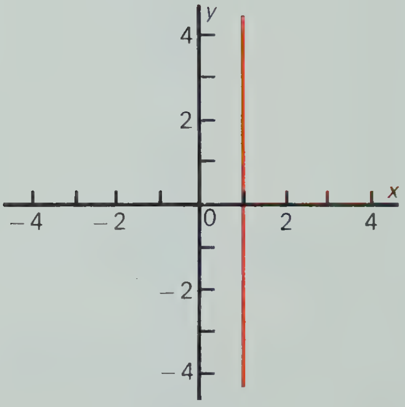
Domain is {−2}
Range is R

(c)



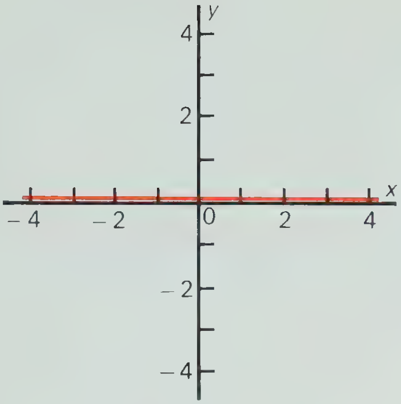
Domain is R
Range is {−2}

(d)



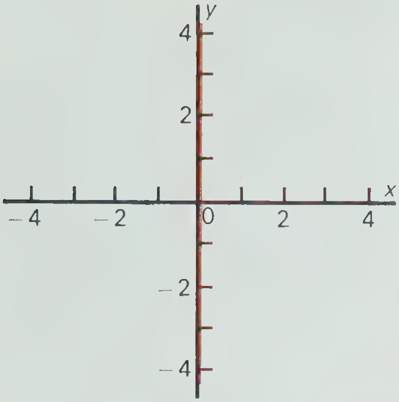
Domain is {1}
Range is R

(e)



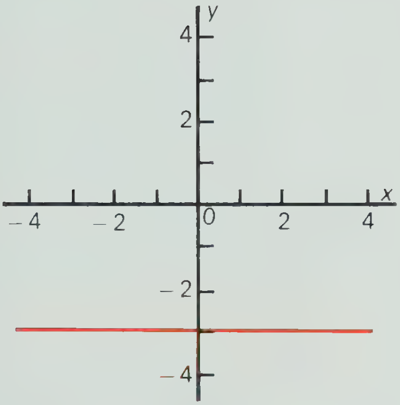
Domain is R
Range is { 0 }

(f)



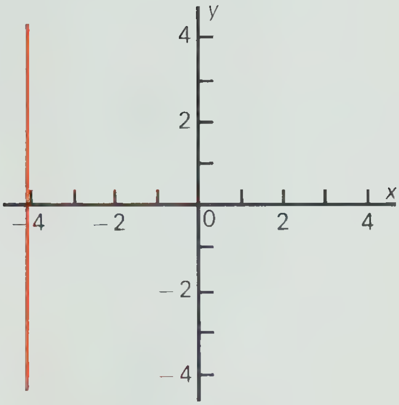
Domain is { 0 }
Range is R

(g)



Domain is R
Range is {–3}

(h)



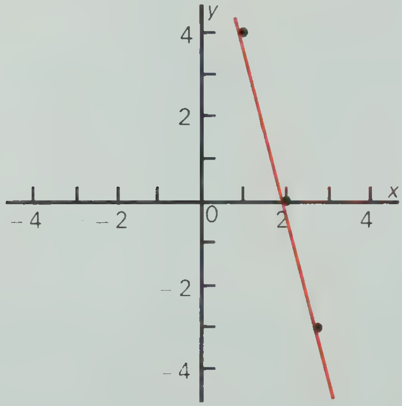
Domain is {–4}
Range is R

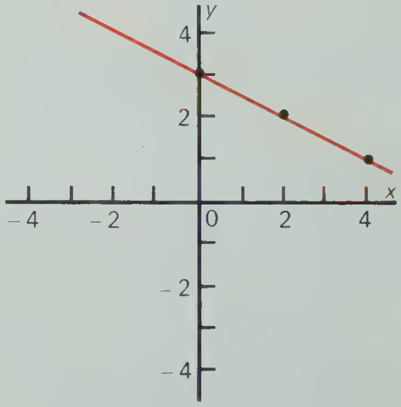
Exercise 5-5

1. **(a)** 18, 32, 38, 42, 58
(b) 1, 9, 12, 30, 36
(c) 2.5, 5, 2, 4, 7.25
2. **(a)** 4.5, 3.5, 2.5, 12
(b) 13.8, 9.8, 4, 4
(c) 9, 3, 3, 9, 9, 3
3. **(a)** (1,5), (3,11), (5,17), (5.5, 18.5)
(b) (4,1), (7,2.5), (10,4), (14,6)
(c) (1,3), (3.5, 14.3), (4,18)
(d) (1,8), (2.5, 2.8), (4,– 7)
4. **(a)** (30,16), (30,27)
(b) (63,36), (63,78); (84,49), (84,118)
(c) (24,12), (24,17)

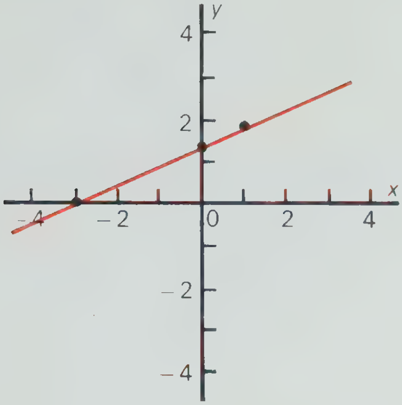
Exercise 5-6

1. (a) $y = 2x + 3,\ m = 2,\ b = 3$
   (b) $y = -3x - 2,\ m = -3,\ b = -2$
   (c) $y = 4x + 6,\ m = 4,\ b = 6$
   (d) $y = -\frac{1}{2}x,\ m = -\frac{1}{2},\ b = 0$
   (e) $y = -3x + 8,\ m = -3,\ b = 8$
   (f) $y = 2x - 7,\ m = 2,\ b = -7$
2. (a) $y = -3x + 7,\ m = -3,\ b = 7$
   (b) $y = 4x + 6,\ m = 4,\ b = 6$
   (c) $y = -3x + 9,\ m = -3,\ b = 9$
   (d) $y = 4x + 6,\ m = 4,\ b = 6$
   (e) $y = 5x - 3,\ m = 5,\ b = -3$
   (f) $y = 3x + 2,\ m = 3,\ b = 2$
3. (a) $y = -3x - 2,\ m = -3,\ b = -2$
   (b) $y = 4x + 7,\ m = 4,\ b = 7$
   (c) $y = -2x - 6,\ m = -2,\ b = -6$
   (d) $y = -8x + 3,\ m = -8,\ b = 3$
   (e) $y = 3x - 12,\ m = 3,\ b = -12$
   (f) $y = -5x + 7,\ m = -5,\ b = 7$
4. (a) $y = 2x + 3,\ m = 2,\ b = 3$
   (b) $y = -5x + 4,\ m = -5,\ b = 4$
   (c) $y = -\frac{3}{2}x + 2,\ m = -\frac{3}{2},\ b = 2$
   (d) $y = -\frac{7}{2}x + 6,\ m = -\frac{7}{2},\ b = 6$
   (e) $y = 2x + 3,\ m = 2,\ b = 3$
   (f) $y = -3x + 4,\ m = -3,\ b = 4$
5. (a) $y = \frac{3}{2}x - 4,\ m = \frac{3}{2},\ b = -4$
   (b) $y = -\frac{4}{5}x + \frac{7}{5},\ m = -\frac{4}{5},\ b = \frac{7}{5}$
   (b) $y = -\frac{8}{3}x + 2,\ m = -\frac{8}{3},\ b = 2$
   (d) $y = \frac{3}{5}x + \frac{6}{5},\ m = \frac{3}{5},\ b = \frac{6}{5}$
   (e) $y = -\frac{4}{5}x - \frac{6}{5},\ m = -\frac{4}{5},\ b = -\frac{6}{5}$
   (f) $y = -\frac{3}{2}x + \frac{17}{2},\ m = -\frac{3}{2},\ b = \frac{17}{2}$
6. (a) $y = -2x + 4,\ m = -2,\ b = 4$
   (b) $y = \frac{3}{2}x - 3,\ m = \frac{3}{2},\ b = -3$
   (c) $y = \frac{7}{3}x - 2,\ m = \frac{7}{3},\ b = -2$
   (d) $y = -\frac{3}{4}x + \frac{3}{2},\ m = -\frac{3}{4},\ b = \frac{3}{2}$
   (e) $y = \frac{8}{3}x + \frac{2}{3},\ m = \frac{8}{3},\ b = \frac{2}{3}$
   (f) $y = \frac{3}{4}x - 3,\ m = \frac{3}{4},\ b = -3$
7. (a) $y = -3x + 6$
   (b) $y = -\frac{1}{2}x + 3$

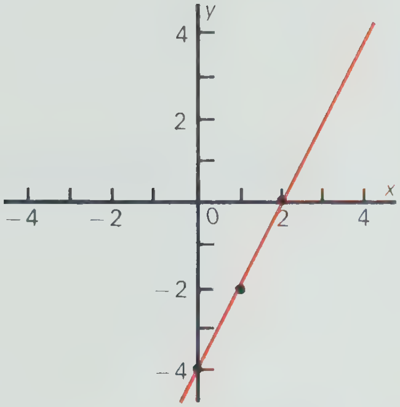




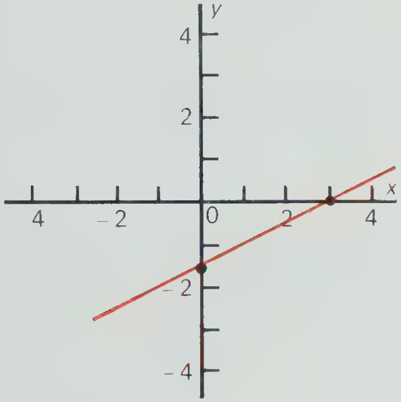
(c) $y = \frac{1}{2}x + 3$



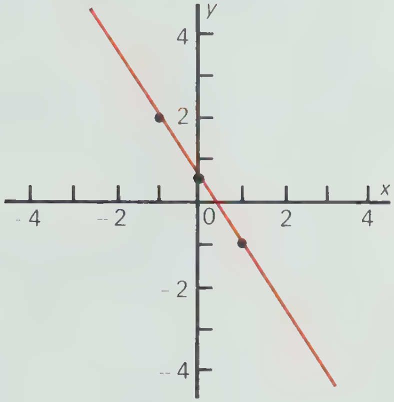
(d) $y = 2x - 4$



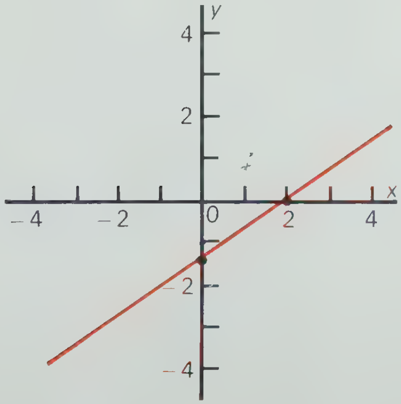
(e) $y = \frac{1}{2}x - 3$



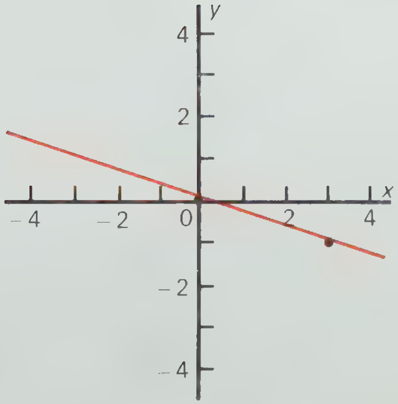
(f) $y = -\frac{3}{2}x + \frac{1}{2}$



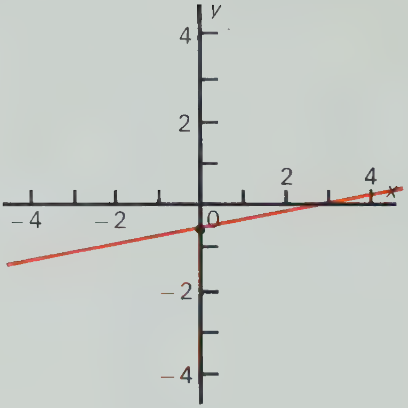
(g) $y = \frac{x}{3} - \frac{2}{3}$



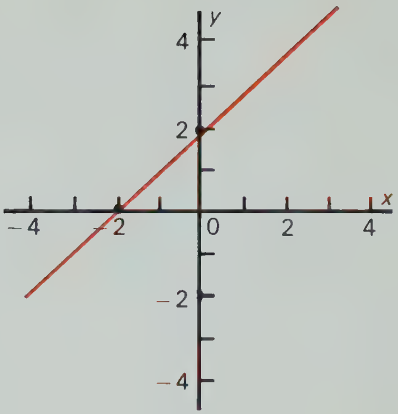
(h) $y = -\frac{1}{3}x$



(i) $y = \frac{x}{4} - \frac{3}{4}$



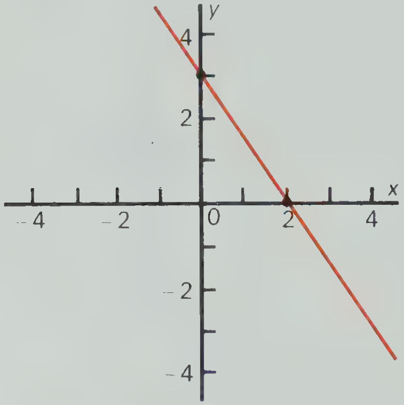
(j) $y = x + 2$



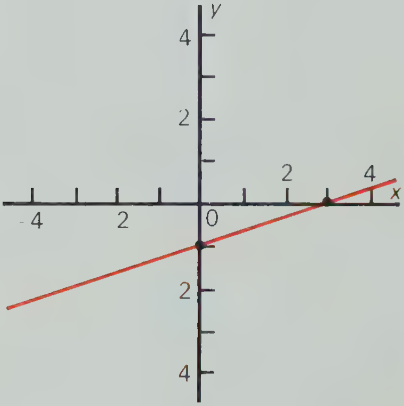
Exercise 5-7

1. **(a)** $x = 4, y = 3$ **(b)** $x = 3, y = 2$ **(c)** $x = 2, y = 5$ **(d)** $x = 5, y = 3$
   (e) $x = 4, y = 2$ **(f)** $x = 2, y = 6$ **(g)** $x = 3, y = -6$ **(h)** $x = 2, y = -3$
   (i) $x = 3, y = -4$ **(j)** $x = 9, y = -3$

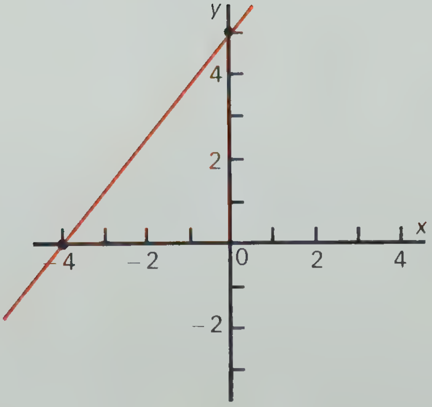
2. (a)



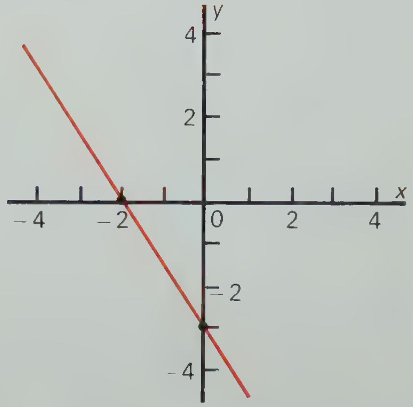
(b)



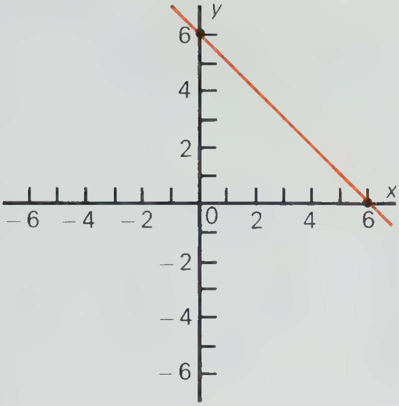
(c)



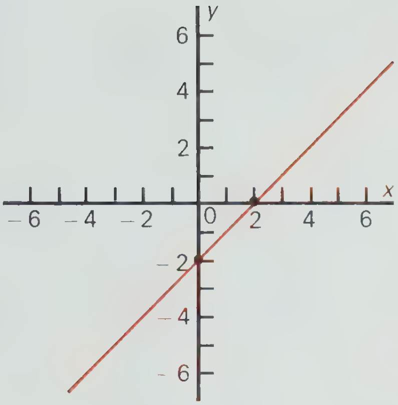
(d)



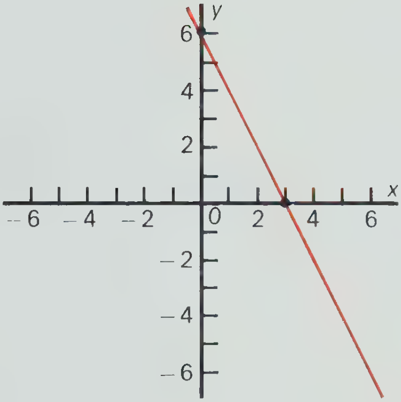
3. (a)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

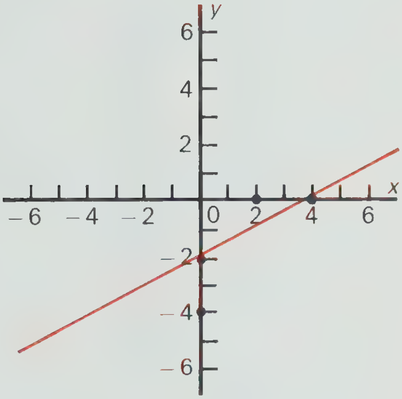
(b)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

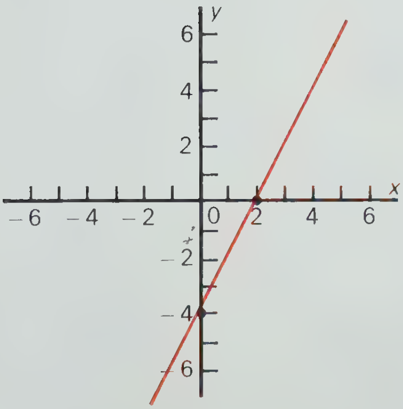
(c)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

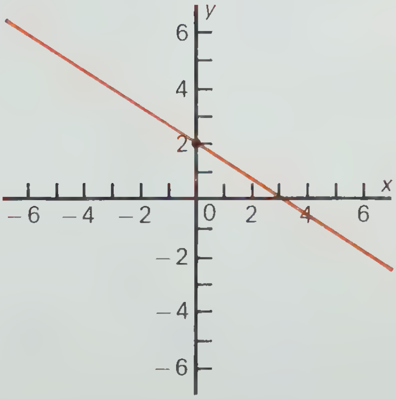
(d)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

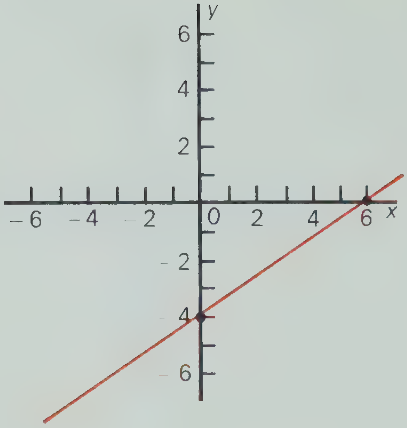
(e)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

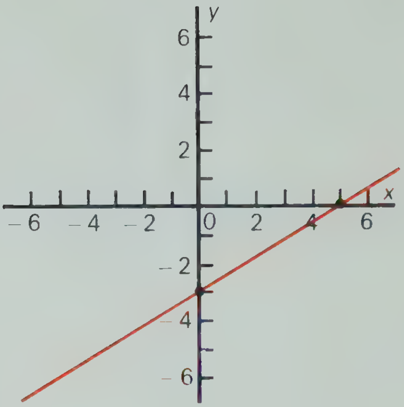
(f)


y
x
6
4
2
−2
−4
−6
−6 −4 −2 0 2 4 6

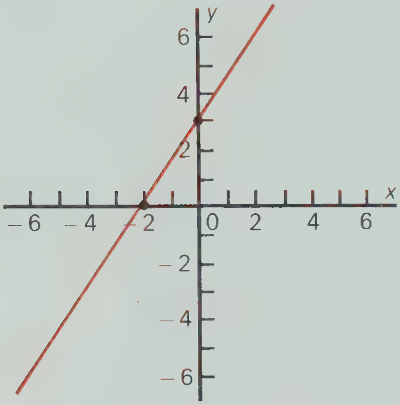
(g)


y
x
6
4
2
0
-2
-4
-6

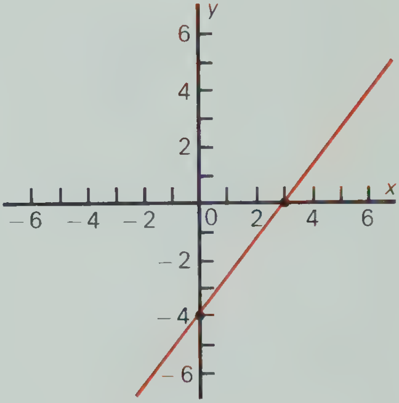
(h)


y
x
6
4
2
0
-2
-4
-6

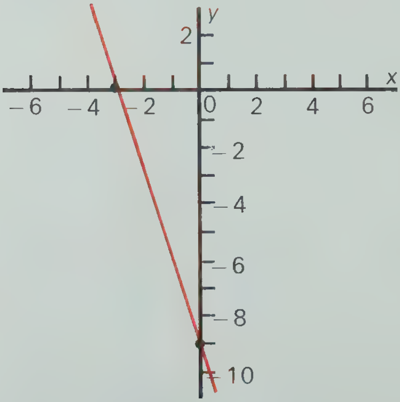
(i)


y
x
6
4
2
0
-2
-4
-6

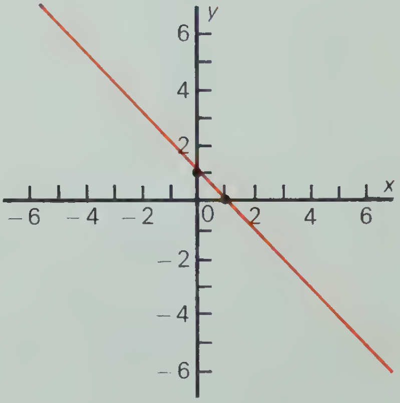
(j)


y
x
6
4
2
0
-2
-4
-6

(k)


y
x
2
-6
-4
-2
0
4
6
-4
-6
-8
-10

(l)

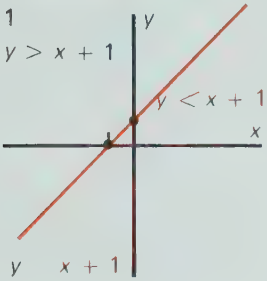

y
x
6
4
2
0
-2
-4
-6

Exercise 5-8

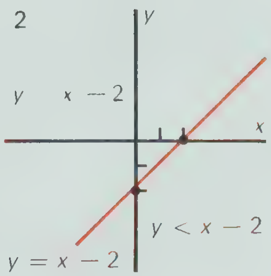
1. (a), (d), (e) **2.** (b), (c), (d)
3. (a), (c), (e), (f) **4.** (b), (c), (e)
5. **(a)** NO **(b)** YES **(c)** NO **(d)** YES **(e)** YES **(f)** YES **(g)** NO **(h)** NO
6. **(a)** YES **(b)** YES **(c)** NO **(d)** NO **(e)** NO **(f)** YES **(g)** NO **(h)** NO

Exercise 5-9

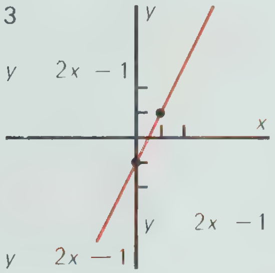
1



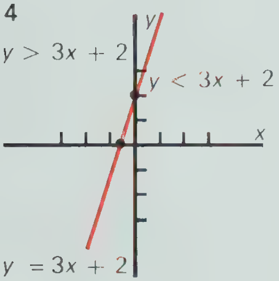
2



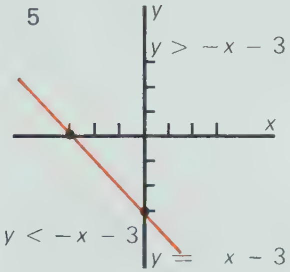
3



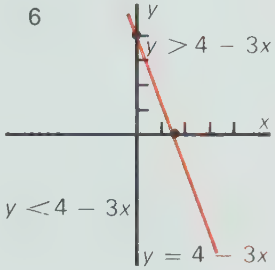
4



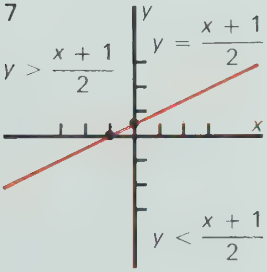
5



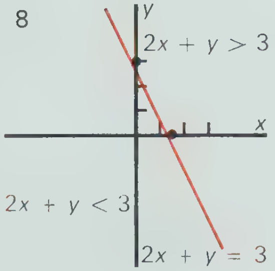
6



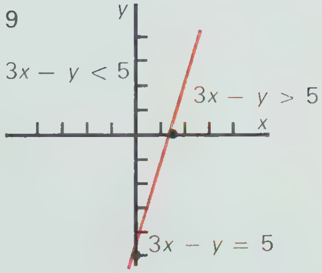
7



8

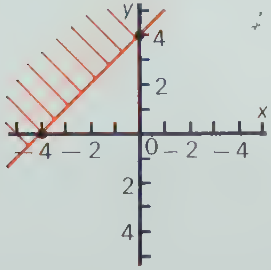


9

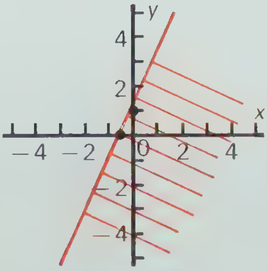


Exercise 5-10

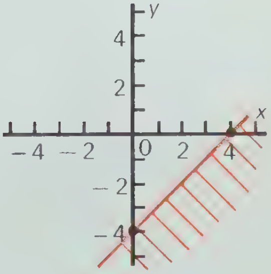
1.(a)



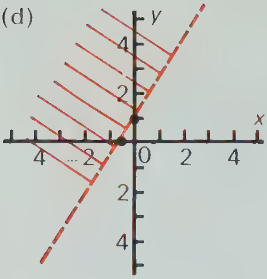
(b)



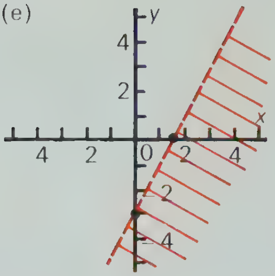
(c)



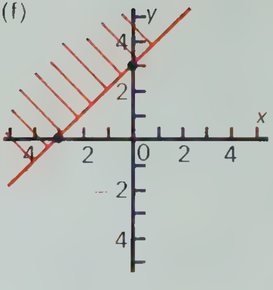
(d)



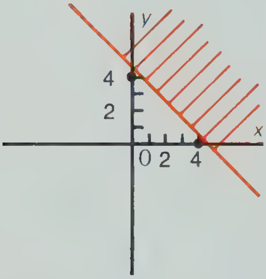
(e)



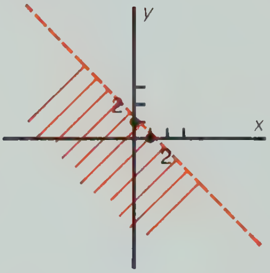
(f)



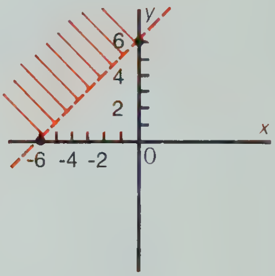
2. (a)



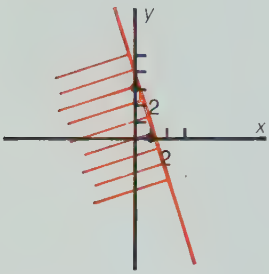
(b)



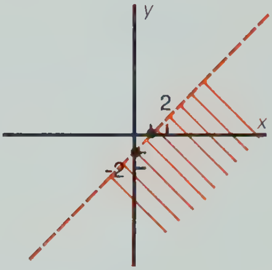
(c)



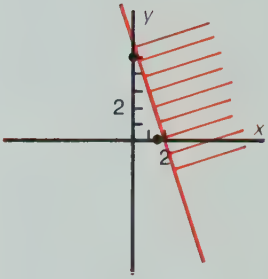
(d)



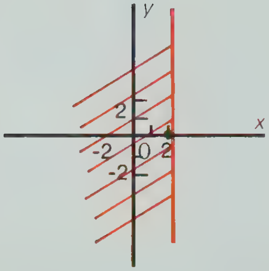
(e)



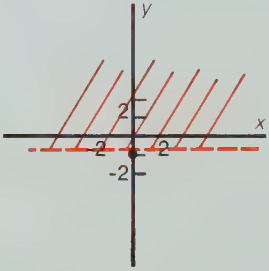
(f)



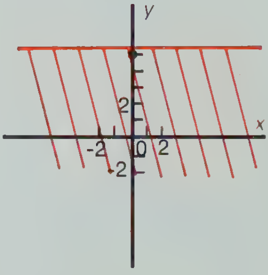
3. (a)



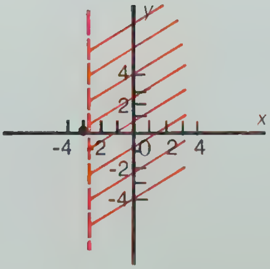
(b)



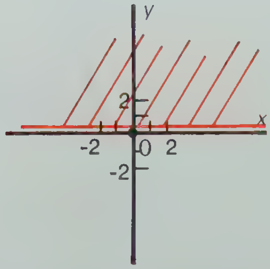
(c)



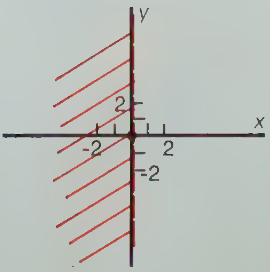
(d)



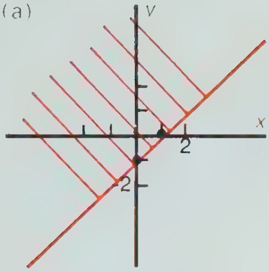
(e)



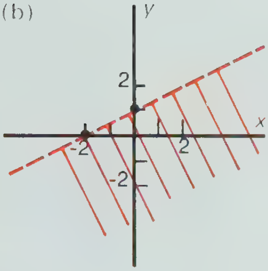
(f)



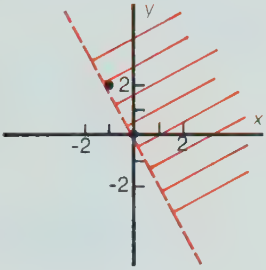
4. (a)



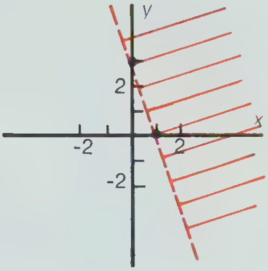
(b)



(c)

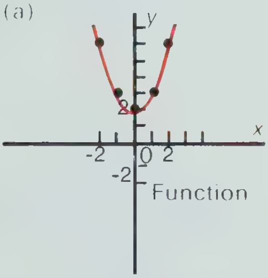


(d)

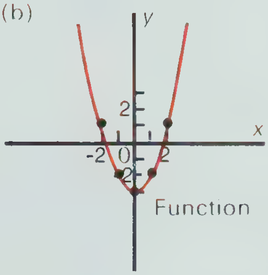


Exercise 5-11

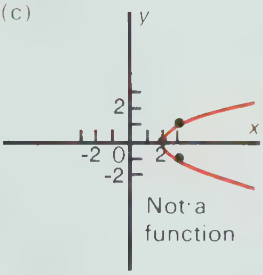
5. (a)



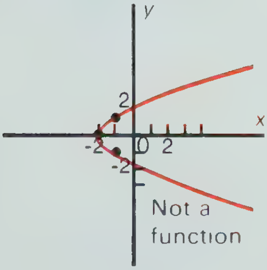
(b)



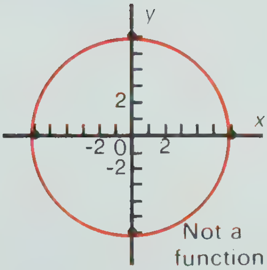
(c)



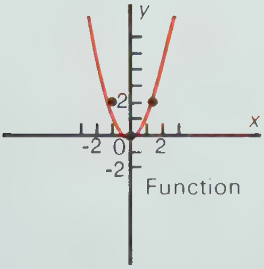
(d)



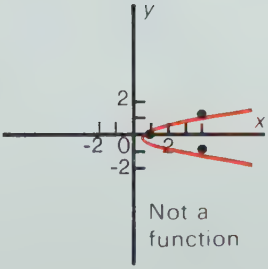
(e)



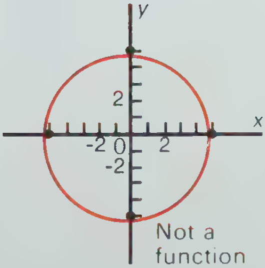
(f)



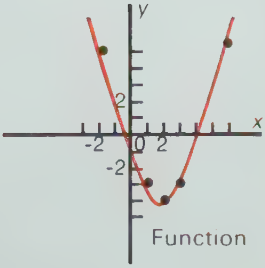
(g)



(h)



(i)



REVIEW EXERCISE

1. $D = \{1,2,3,4\}$ for all parts.

(a) $R = \{2,4,6,8\}$, $y = 2x$ **(d)** $R = \{3,5,7,9\}$, $y = 2x + 1$

(b) $R = \{1,4,9,16\}$, $y = x^2$ **(e)** $R = \{1,4,7,10\}$, $y = 3x + 2$

(c) $R = \{3,4,5,6\}$, $y = x + 2$ **(f)** $R = \{2,5,10,17\}$, $y = x^2 + 1$

2. **(a)** $x = 7$, $y = 2$ **(b)** $x = 10$, $y = -2$

(c) $x = 0$, $y = 0$ **(d)** $x = 3$, $y = -7$

(e) $x = -5$, $y = 4$ **(f)** $x = 3$, $y = -1$

(g) $x = 2$, $y = 4$ **(h)** $x = 3$, $y = -9$

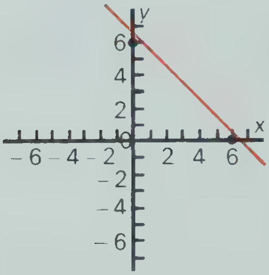
3. **(a)** YES **(b)** NO **(c)** NO **(d)** YES **(e)** YES

(f) NO **(g)** YES **(h)** YES **(i)** YES **(j)** NO

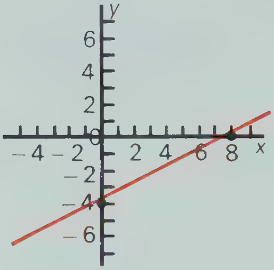
4. **(a)** Not a function **(b)** Function **(c)** Function

(d) Not a function **(e)** Not a function **(f)** Function

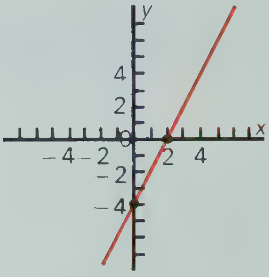
5. (a)



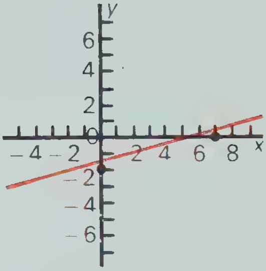
(b)



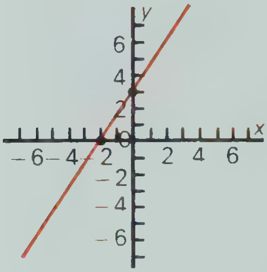
(c)



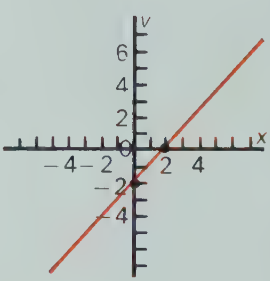
(d)



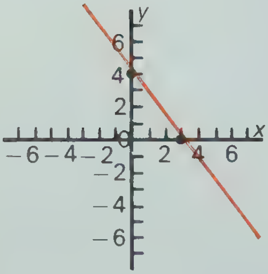
(e)



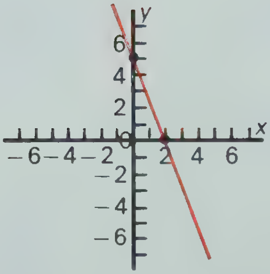
(f)



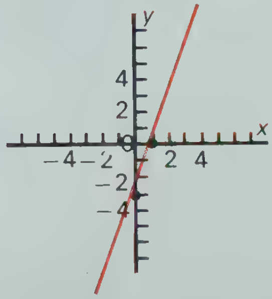
(g)



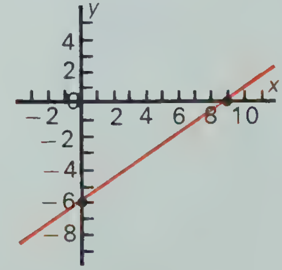
(h)



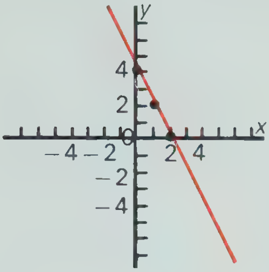
(i)



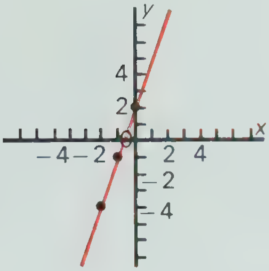
(j)



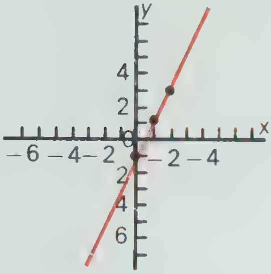
6. **(a)** $y = -2x + 4$



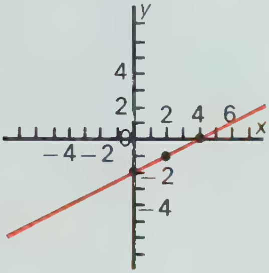
(b) $y = 3x + 2$



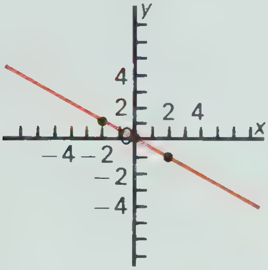
(c) $y = 2x - 1$



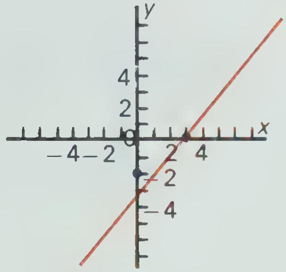
(d) $y = \frac{1}{2}x - 2$



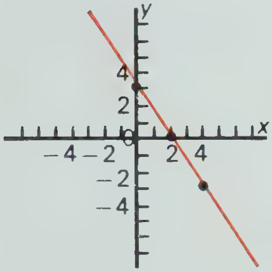
(e) $y = -\frac{1}{2}x$



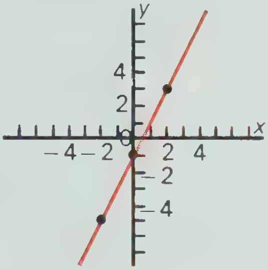
(f) $y = x - 3$



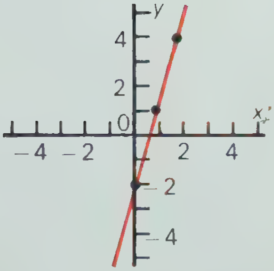
(g) $y = -\frac{3}{2}x + 3$



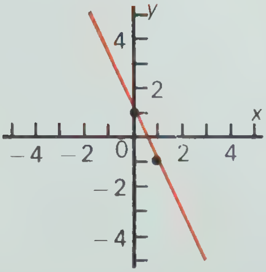
(h) $y = 2x - \frac{3}{2}$



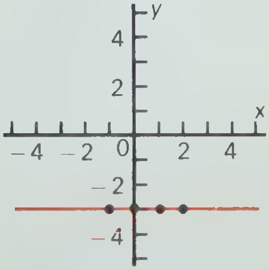
7. (a)



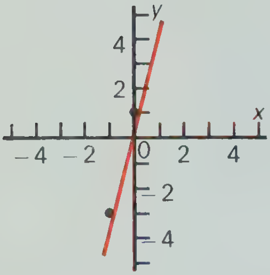
(b)



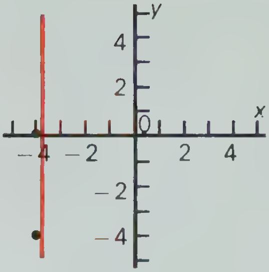
(c)



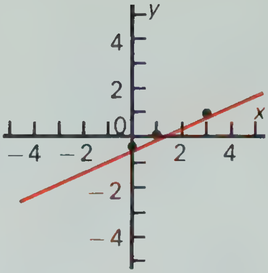
(d)



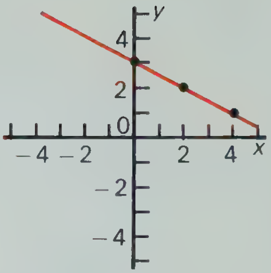
(e)



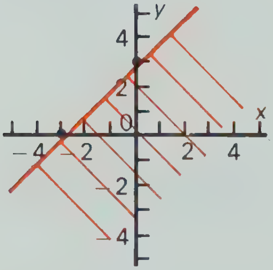
(f)



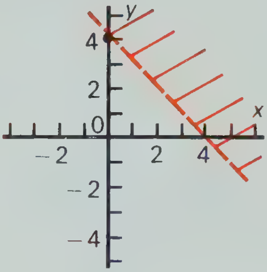
(g)



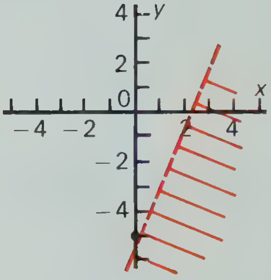
8.(a)



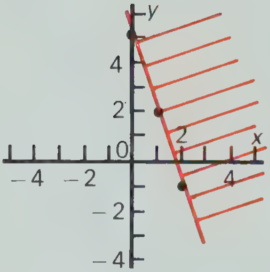
(b)



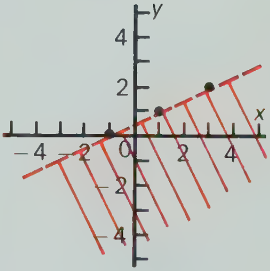
(c)



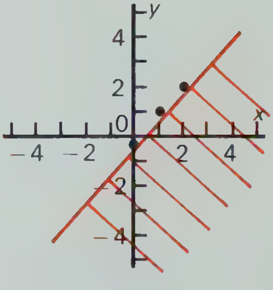
(d)



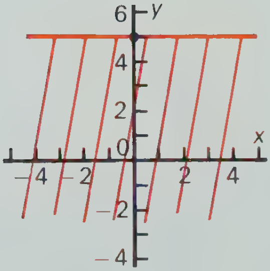
(e)



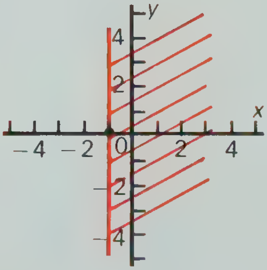
(f)



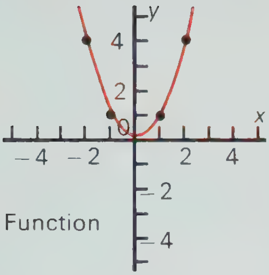
(g)



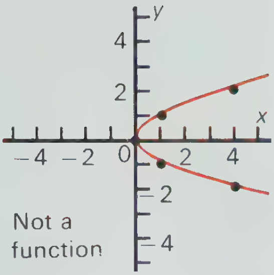
(h)



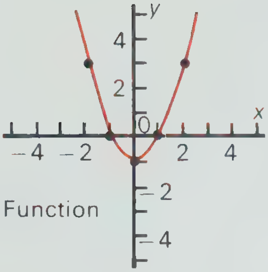
9. (a)



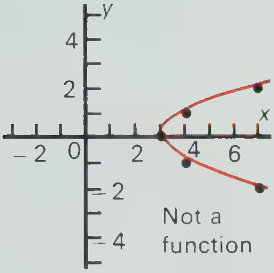
(b)



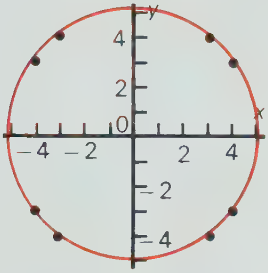
(c)



(d)

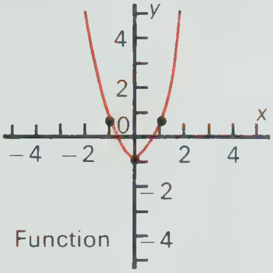


(e)

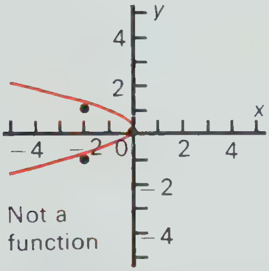


Not a function

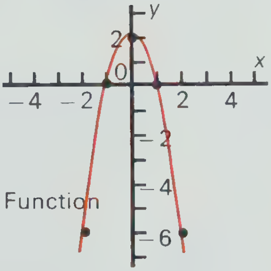
(f)



(g)



(h)



10. (6,$734), (10,$1450), (14,$2420), (16,$3030)

11. (2,9.8), (5,9.1), (7,6.4), (9,7.0), (10,− 1)

REVIEW AND PREVIEW TO CHAPTER 6

Exercise 1

1. **(a)** 7 **(b)** $7a - 21$ **(c)** $19t - 31$
   (d) $8m + 2$ **(e)** $-x + 10$
2. **(a)** $2x^2 - 4x - 36$ **(b)** $m^2 + 13m + 54$ **(c)** $26b^2 + 20b$
   (d) $-4y^2 + 13y - 16$ **(e)** $4a - 7$

Exercise 2

1. **(a)** 9 **(b)** 3 **(c)** 4 **(d)** 14 **(e)** 7
   (f) 19 **(g)** −9 **(h)** −13 **(i)** −6 **(j)** −3
2. **(a)** 5 **(b)** 5 **(c)** 4 **(d)** −4 **(e)** 7
   (f) 5 **(g)** −5 **(h)** 0 **(i)** −3 **(j)** −1

Exercise 3

1. **(b)** {5,6,7,. . .} **(c)** {9,10,11,. . .} **(d)** {. . .,3,4,5}
   (e) {. . .,4,5,6} **(f)** {5,6,7,. . .} **(g)** {. . .,19,20,21}
   (h) {2,3,4,. . .} **(i)** {. . .,− 6,− 5,− 4} **(j)** {. . .,− 7,− 6,− 5}
   (k) {− 4,− 3,− 2,. . .}

2. (a) $\{5,6,7,\ldots\}$ (b) $\{5,6,7,\ldots\}$ (c) $\{2,3,4,\ldots\}$
(d) $\{3,4,5,\ldots\}$ (e) $\{\ldots,-4,-3,-2\}$ (f) $\{-1,0,1,\ldots\}$
(g) $\{7,8,9,\ldots\}$ (h) $\{\ldots,33,34,35\}$ (i) $\{\ldots,-9,-8,-7\}$
(j) $\{0,1,2,\ldots\}$

CHAPTER 6

Exercise 6-1

1. (a) 4 (b) 3 (c) 9 (d) 17 (e) 8 (f) -7 (g) -7 (h) 2
2. (a) 3 (b) 9 (c) -8 (d) -3 (e) 6 (f) 0 (g) 7 (h) -3
3. (a) 2 (b) -11 (c) 2 (d) -5 (e) $-\frac{1}{2}$
4. (a) 8 (b) -9 (c) -4 (d) 1 (e) -1
5. (a) 22 (b) -5 (c) -2 (d) 1 (e) 2
6. (a) $\frac{11}{2}$ (b) $\frac{1}{10}$ (c) $-\frac{5}{2}$ (d) $-\frac{5}{14}$ (e) 2

Exercise 6-2

1. (a) 12 (b) -15 (c) 10 (d) 12 (e) -21 (f) 12
2. (a) $\frac{10}{3}$ (b) $\frac{9}{4}$ (c) -3 (d) -6 (e) 7
3. (a) 8 (b) -1 (c) -8 (d) 5 (e) $\frac{3}{4}$
4. (a) 5 (b) 7 (c) $-\frac{5}{11}$ (d) $\frac{14}{15}$ (e) 7

Exercise 6-3

1. (a) 3 (b) 4 (c) 1 (d) 2 (e) 8 (f) -5
2. (a) 3 (b) -1 (c) 1 (d) -2 (e) -3 (f) 2
3. (a) -2 (b) -2 (c) 0 (d) -1 (e) 1 (f) -1
4. (a) 1 (b) -3 (c) -1 (d) -1 (e) -1 (f) 3

Exercise 6-4

1. (a) $(1,4)$ (b) $(1,3)$ (c) $(2,1)$ (d) $(-2,-1)$ (e) $(-2,6)$ (f) $(1,2)$
2. (a) $(2,1)$ (b) $(3,3)$ (c) $(6,0)$ (d) $(4,5)$ (e) $(-3,-2)$ (f) $(-2,2)$
3. (a) $(\frac{1}{2},4)$ (b) $(\frac{1}{2},\frac{1}{2})$ (c) $(-\frac{1}{2},2)$ (d) $(2\frac{1}{2},-3)$ (e) $(\frac{1}{2},\frac{1}{3})$ (f) $(\frac{1}{3},\frac{1}{5})$
4. (a) No intersection (b) No intersection (c) No intersection
(d) Infinite number of points of intersection

Exercise 6-5

1. (a) $y = 7 - 2x$ (b) $y = 3x + 2$ (c) $y = 4x + 5$
(d) $y = 2 - 4x$ (e) $y = 4x - 2$ (f) $y = 3x - 2$
(g) $y = -2x + \frac{5}{2}$ (h) $y = -\frac{5}{3}x + 4$ (i) $y = \frac{1}{2}x + \frac{7}{4}$
(j) $y = \frac{x}{5} + \frac{1}{5}$ (k) $y = 3x - 2$ (l) $y = \frac{3x}{2} - 3$
2. (a) $(2,5)$ (b) $(3,8)$ (c) $(4,11)$ (d) $(-1,5)$
(e) $(5,-17)$ (f) $(-2,-5)$ (g) $(\frac{3}{4},\frac{9}{2})$ (h) $(\frac{1}{2},\frac{7}{2})$
3. (a) $(2,4)$ (b) $(12,4)$ (c) $(7,3)$ (d) $(-1,-1)$
(e) $(1,-\frac{1}{2})$ (f) $(16,11)$
4. (a) $(1,1)$ (b) $(2,3)$ (c) $(2,-1)$ (d) $(1,-2)$
(e) $(-\frac{1}{2},1)$ (f) $(-\frac{1}{2},-\frac{1}{3})$ (g) $(-\frac{2}{3},\frac{1}{2})$ (h) $(\frac{2}{5},-\frac{3}{4})$

Exercise 6-6

1. **(a)** 3, 1 **(b)** 6, 4 **(c)** 2, 2 **(d)** 3, 3
(e) 2, 0 **(f)** 1, 1 **(g)** 1, 1 **(h)** 1, 3
2. **(a)** 4, 2 **(b)** 3, 2 **(c)** 3, 3 **(d)** 4, 2
(e) 7, 2 **(f)** 2, 3 **(g)** 2, 2 **(h)** 2, 1
3. **(a)** 3, 2 **(b)** 1, 2 **(c)** 2, 3 **(d)** 1, 2
(e) 2, 2 **(f)** 2, 1 **(g)** 5, 1 **(h)** 4, 2
4. **(a)** −1, −2 **(b)** −2, 1 **(c)** −2, 5 **(d)** −1, 5
(e) −2, −3 **(f)** −1, −2 **(g)** 6, 2 **(h)** −3, −3
5. **(a)** 2, 3 **(b)** 2, 5 **(c)** 3, −3 **(d)** −4, −2
(e) 3, 2 **(f)** 4, 3

Exercise 6-7

1. **(a)** $3x$ **(b)** $\frac{1}{2}x$ **(c)** $x + 7$ **(d)** $y + 12$
(e) $b + 12$ **(f)** $2m$ **(g)** $m - 4$ **(h)** $x - 8$
(i) $4x + 6$ **(j)** $2x - 7$ **(k)** $b + 7$ **(l)** $3x - 6$
2. **(a)** $x + y = 12$ **(b)** $x + y = 26$ **(c)** $x - y = 6$ **(d)** $x + y = 28$
(e) $x + y = 156$ **(f)** $x - y = 4$ **(g)** $2x + 6y = 88$ **(h)** $5x - 3y = 18$
(i) $x - y = 56$ **(j)** $x + y = 77$
3. 78, 53 **4.** 27 **5.** 48 **6.** 10, 14 **7.** 15 **8.** 9, 3
9. 14 **10.** 12 **11.** 65 **12.** 80 **13.** 20 **14.** 320
15. 81 **16.** 2, 3 **17.** 38, 52

Review Exercise

1. **(a)** 7 **(b)** 7 **(c)** 10 **(d)** −4 **(e)** 0 **(f)** −1 **(g)** −4 **(h)** 1
(i) −9 **(j)** 16 **(k)** 27 **(l)** −20 **(m)** −7 **(n)** 0 **(o)** 2
2. **(a)** $y = -3x + 9$ **(b)** $y = 4x + 2$ **(c)** $y = x + 11$
(d) $y = -3x - 2$ **(e)** $y = -2x - 4$ **(f)** $y = x - 3$
(g) $y = -2x + 3$ **(h)** $y = -\frac{2}{3}x + \frac{7}{3}$ **(i)** $y = \frac{5}{2}x - \frac{11}{2}$
3. **(a)** $x = 7, y = 1$ **(b)** $x = 4, y = 1$ **(c)** $x = 4, y = 8$
(d) $x = 4, y = 0$ **(e)** $x = 1, y = 3$ **(f)** $x = 3, y = -3$
4. **(a)** 5 **(b)** 4 **(c)** −3 **(d)** 4 **(e)** 15 **(f)** 2 **(g)** 2 **(h)** 4 **(i)** 2
5. **(a)** 19 **(b)** $\frac{9}{4}$ **(c)** 36 **(d)** 4 **(e)** ∅ **(f)** 4 **(g)** 26 **(h)** −5
6. **(a)** −2 **(b)** 1 **(c)** 1 **(d)** $-\frac{4}{5}$ **(e)** −1 **(f)** 3 **(g)** −1
7. **(a)** $x = 8, y = 23$ **(b)** $x = 2, y = 0$ **(c)** $x = -2, y = 0$
(d) $a = 3, b = -2$ **(e)** $x = 1, y = 0$ **(f)** $m = 3, n = 2$
(g) $r = -2, t = -1$ **(h)** $b = 4, d = 3$
8. 6, 9 **9.** 20, 12 **10.** 7, 2
11. 19 units **12.** $a = 6$ units, $b = 10$ units
13. $a = 2, b = 3$
14. **(a)** $x = 14°, y = 4°$ **(b)** $x = 110°, y = 70°$

REVIEW AND PREVIEW TO CHAPTER 7

Exercise 1

1.

km	m	cm	mm
1	1000	100 000	1 000 000
0.001	1	100	1000
0.000 01	0.01	1	10
0.000 001	0.001	0.1	1
0.1	100	10 000	100 000
0.001	1	100	1000
0.01	10	1000	10 000

2.

km	m	cm	mm
2.5	2500	250 000	2 500 000
0.093	93	9300	93 000
0.576	576	57 600	576 000
0.0032	3.2	320	3200
0.007 86	7.86	786	7860
0.000 352	0.352	35.2	352
0.000 1073	0.1073	10.73	107.3
0.000 007 9	0.0079	0.79	7.9

3. 13 km
4. 8.5 cm^2
5. 3.125 m^2
6. 7.065 cm^2
7. 85.3 km/h
8. 50.4 m^3
9. 14 130 cm^3

Exercise 2

1.

ℓ	cl	ml
1	100	1000
0.01	1	10
0.001	0.1	1
1	100	1000
0.01	1	10
0.001	0.1	1

2.

ℓ	cl	ml
560	56 000	560 000
0.0375	3.75	37.5
0.0016	0.16	1.6
1.365	136.5	1365
0.79	79	790
0.000 85	0.085	0.85

3.

kg	g
0.1	100
10	10 000
0.01	10
1.5	1500
5.75	5750
0.0375	37.5
0.000 875	0.875

4. 76.7 kg **5.** 0.12¢ **6.** **(a)** 0.2895 kg **(b)** 0.000 007 1 m^3 **7.** 38.4 ℓ

CHAPTER 7

Exercise 7-1

1. **(a)** 2:1 **(b)** 6:1 **(c)** 1:7 **(d)** 4:3 **(e)** 5:2 **(f)** 3:2
(g) 3:1 **(h)** 5:2

2. **(a)** 4:2:1 **(b)** 2:1:1 **(c)** 1:2:4 **(d)** 6:2:3 **(e)** 3:1:2 **(f)** 2:5:8
(g) 2:2:5 **(h)** 4:2:1 **(i)** $\frac{2}{3}$ **(j)** $\frac{5}{6}$ **(k)** $\frac{5}{4}$ **(l)** $\frac{3}{5}$
(m) $\frac{2}{1}$ **(n)** $\frac{7}{2}$ **(o)** $\frac{1}{6}$

3. **(a)** 1 **(b)** 3 **(c)** $\frac{1}{2}$ **(d)** 100 **(e)** 12 **(f)** $\frac{1}{4}$

4. **(a)** 3:50 **(b)** 15:2 **(c)** 500:23 **(d)** 25:1 **(e)** 8:5 **(f)** 9:4

5.

Ratio	Simplification	Simplest form
$\frac{1}{2}:3$	$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$	1:6
$5:\frac{2}{3}$	$5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2}$	15:2
$1\frac{1}{4}:\frac{5}{6}$	$\frac{5}{4} \div \frac{5}{6} = \frac{5}{4} \times \frac{6}{5} = \frac{3}{2}$	3:2
$2:7\frac{1}{2}$	$2 \div \frac{15}{2} = 2 \times \frac{2}{15} = \frac{4}{15}$	4:15
$1\frac{1}{3}:2\frac{7}{15}$	$\frac{4}{3} \div \frac{37}{15} = \frac{4}{3} \times \frac{15}{37} = \frac{20}{37}$	20:37
$4\frac{2}{5}:3$	$\frac{22}{5} \div 3 = \frac{22}{5} \times \frac{1}{3} = \frac{22}{15}$	22:15

6.

Ratio	Simplification	Simplest form
$\frac{1}{5}:\frac{1}{3}$	$15 \times \frac{1}{5}:15 \times \frac{1}{3}$	3:5
0.5:0.3	$10 \times 0.5:10 \times 0.3$	5:3
2.1:1:0.3	$10 \times 2.1:10 \times 1:10 \times 0.3$	21:10:3
$\frac{3}{5}:\frac{1}{2}:5$	$10 \times \frac{3}{5}:10 \times \frac{1}{2}:10 \times 5$	6:5:50
2.4:0.8:2	$10 \times 2.4:10 \times 0.8:10 \times 2$ $= 24 \div 4:8 \div 4:20 \div 4$	6:2:5
$1\frac{1}{2}:3:\frac{1}{4}$	$4 \times \frac{3}{2}:4 \times 3:4 \times \frac{1}{4}$	6:12:1

7. \$3977.75 and \$4546 **8.** Pete: 384 votes; Kareen: 480 votes; Bill: 288 votes.

9. 240 m^3 of cement, 320 m^3 of sand, and 800 m^3 of crushed gravel.

10. (a), (d) and (f) are equivalent ratios. **11.** 1:4 **12.** **(a)** 27 cm^3 **(b)** 216 cm^3 **(c)** 1:8

Exercise 7-2

1. **(a)** 9 **(b)** 42 **(c)** $\frac{3}{4}$ **(d)** 14 **(e)** 26
(f) 3 **(g)** 2.25 **(h)** 25 **(i)** ±2

2. **(a)** $x = 1, y = 24$ **(b)** $a = 7.5, b = 3.75$ **(c)** $x = 1\frac{2}{3}, y = 3\frac{1}{3}$ **(d)** $a = 7.8, b = 13$
(e) $x = 4.5, y = 4$ **(f)** $a = \frac{10}{3}, y = \frac{18}{5}$

3. $2\frac{1}{4}$

4. 12

5. (a) ± 4 (b) $\pm 2\sqrt{15}$ (c) $\pm 2\sqrt{5}$ (d) $\pm 3\sqrt{2}$

6. 12.5 cm

7. 31.4 cm

8. 37.5 kg

9. 85 teeth

10. Gear *A*, 36 teeth and Gear *C*, 18 teeth.

Exercise 7-4

1.

Statement	Notation	Equation	Proportion
"*a*" varies directly as "*b*"	$a \propto b$	$a = kb$	$\frac{a_1}{b_1} = \frac{a_2}{b_2}$
"*c*" varies as "*d*"	$c \propto d$	$c = kd$	$\frac{c_1}{d_1} = \frac{c_2}{d_2}$
"*x*" varies as "*y*"	$x \propto y$	$x = ky$	$\frac{x_1}{y_1} = \frac{x_2}{y_2}$
V is proportional to *T*	$V \propto T$	$V = kT$	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$
W varies directly as d^2	$W \propto d^2$	$W = kd^2$	$\frac{W_1}{{d_1}^2} = \frac{W_2}{{d_2}^2}$
F varies as *M*	$F \propto M$	$F = kM$	$\frac{F_1}{M_1} = \frac{F_2}{M_2}$
A is proportional to r^2	$A \propto r^2$	$A = kr^2$	$\frac{A_1}{{r_1}^2} = \frac{A_2}{{r_2}^2}$

2. (a) (b)

(ii) $v = 15t$ (ii) $c = 3.1d$

3.

	Statement	Equation	Given values	Variation constant *k*	Equation
(a)	*x* varies directly as *y*	$x = ky$	$x = 3$ $y = 5$	$k = \frac{3}{5}$	$x = \frac{3}{5}y$ or $5x = 3y$
(b)	$V \propto T$	$V = kT$	$V = 550$ $T = 300$	$550 = 300 \cdot k$ $\therefore\ k = \frac{11}{6}$	$V = \frac{11}{6}T$ or $6V = 11T$
(c)	*d* varies as *t*	$d = kt$	$d = 10.5$ $t = 2.5$	$10.5 = 2.5 \times k$ $\therefore\ k = \frac{21}{5}$	$d = \frac{21}{5}t$ or $5d = 21t$
(d)	*a* varies directly as b^3	$a = kb^3$	$a = 24$ $b = 2$	$24 = 2^3 \cdot k$ $\therefore\ k = 3$	$a = 3b^3$

4. **(a)** $\frac{2}{3}$ **(b)** 36 5. 616 cm^2

6. 19.6 kg
7. $26.25

8. **(a)** $5\frac{1}{3}$ kg **(b)** 40 cm

9. 198 cm^3
10. **(a)** 44.1 m **(b)** 34.3 m

Exercise 7-5

1.

a_1	b_1	a_2	b_2
6	5	3	10
4	15	12	5
$7\frac{1}{2}$	2	3	5
5	3	2	$7\frac{1}{2}$
$3\frac{1}{2}$	$\frac{6}{7}$	$1\frac{1}{2}$	2
2.6	5.1	4.4	3
4.6	0.6	9.2	0.3

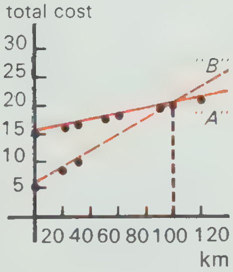
2. 3 d 3. 25 m^3 4. 400 r/min
5. $1\frac{3}{4}$ 6. 640 vibrations per second
7. 6 cm 8. 160 r/min 9. 1.2 m

Exercise 7-6

1. **(a)** $26.60 **(b)** $T = 5 + 0.09n$ where T represents the total charge in dollars, and n represents the number of kilometres driven.
2. **(a)** Plan *A*, $5 (27.50–22.50)
   (b)

kilometres	20	30	50	60	90	120
Cost "A" ($)	16.00	16.50	17.50	18.00	19.50	21.00
kilometres	20	30	50	60	90	120
Cost "B" ($)	8.00	9.50	12.50	14.00	18.50	23.00

Plan A becomes the better plan on trips over 100 km.



3. **(a)** $400, $650, $1150 **(b)** 100, 140, 180

4. **(a)**

t	0	1	2	3	4	5
v	0	2.5	5	7.5	10	12.5

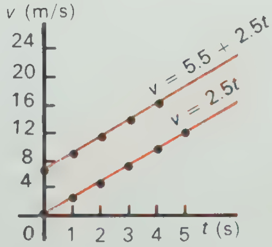
(c) direct

(d)

t	0	1	2	3	4	5
v	5.5	8	10.5	13	15.5	18

(e) partial

(b)



Review Exercise

1. (a) 1:2 (b) 1:3 (c) 4:1 (d) 9:4 (e) 5:3 (f) 9:4
(g) 1:2:4 (h) 8:9:3 (i) 5:12:1 (j) 2:7:8 (k) 3:4:1

2. 7:28 and 4:16, 14:2 and 56:8, 9:6 and 27:18, 8:10 and 32:40.

3. The 3, 2, 6, and 4 are called the first, second, third and fourth proportionals respectively.

4. The mean proportional between 3 and 12.

5. (a) 4:3 (b) 3:4 (c) 3:7 (d) 4:7

6. (a) Direct (b) Partial (c) Inverse

7. (a) 9:100 (b) 375:1 (c) 5:6 (d) 3:1
(e) 3:4 (f) 1:5 (g) 29:19 (h) 26:1

8. (a) 9 (b) 18 (c) 1 (d) 35
(e) 18 (f) 3 (g) 4.5 (h) 90

9. (a) $a = 1$, $b = 6$ (b) $x = 1.5$, $y = 7.5$ (c) $y = 4.5$, $z = 18$ (d) $a = 12$, $b = 1.5$

10. (a) 24 (b) 65 (c) 12 (d) 5
(e) 0.06 (f) $x = 7.5$, $y = 12.5$ (g) $a = 36$, $b = 2.4$ (h) $w = 3.2$, $r = 4$
(i) $x = 1.8$, $y = 2$

11. 32 cm 12. 21 weeks 13. $1\frac{1}{4}$ ℓ

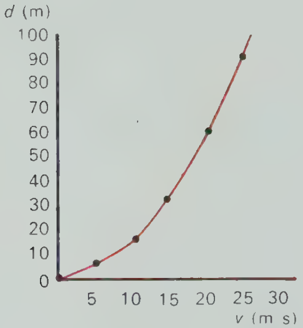
14. $a = 8.1$ cm, $b = 4.1$ cm 15. 45 V 16. (a) 160 kg (b) 3.75 m

17. (a) $k = 0.15$

(b)

v (m/s)	0	5	10	15	20	25
d (m)	0	3.75	15	33.75	60	93.75

(c)



(d)

v (m/s)	7.5	14	11	15	19
d (m)	8.4	30	18.	35	55

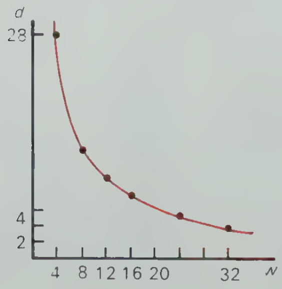
18. 14.3 m/s²

19. (a) $k = 112$

(b)

w	4	8	12	16	24	32
s	28	14	9.3	7	4.7	3.5

(c)



(d)

d	5	14.5	24.9	16
w	22.4	7.8	4.5	7

REVIEW AND PREVIEW TO CHAPTER 8

1. (a) 0.07 (b) 0.05 (c) 0.45 (d) 0.065 (e) 0.125
(f) 1.0 (g) 0.0775 (h) 0.75 (i) 0.085 (j) 0.0325
(k) 0.091 25 (l) 0.053 (m) 0.0075 (n) 0.021 25 (o) 0.1275
(p) 0.005 (q) 0.015 (r) 0.0475 (s) 0.003 75 (t) 0.1025
2. (a) 25% (b) 75% (c) 62.5% (d) 6% (e) 7.5%
(f) 14% (g) 5.5% (h) 55% (i) 4.75% (j) 6.125%
(k) 45.5% (l) 0.375% (m) 1.25% (n) 3.75% (o) 10.5%
(p) 8.75%
3. (a) $28.20 (b) $45 (c) $15 (d) $54 (e) $33.75
(f) $107.50 (g) $8.94 (h) 70¢ (i) $34 (j) $1.31
(k) $10.65 (l) $770 (m) $266.25 (n) $669.50 (o) $1127.50
4. (a) $41.67 (b) $35.71 (c) $306.25 (d) $338.18 (e) $2291.67
(f) $236.67 (g) $400 (h) $150 (i) $1500 (j) $520
(k) $5250 (l) $250.23
5. (a) $74.60, $5.22 (b) $1118.95, $78.33
(c) $12 620.70, $883.45 (d) $1327.54, $92.93
(e) $224.66, $15.73 (f) $1914.63, $134.02
6. (a) $279.85 (b) $291.05 (c) $3118.35 (d) $46.23 (e) $22.90
(f) $872.50 (g) $88.78 (h) $448.61 (i) $97.87

CHAPTER 8

Exercise 8-1

1. Balance on hand.
2. Enter in different columns. Receipts are added to the balance. Payments are subtracted from the balance.
3. Month end balance $583.31.
6. 20¢

Exercise 8-3

1. (a) $25 (b) $385.69 (c) 10% (d) 12%
(e) $196.16 (f) $100 (g) 7 months (h) 73 d
2. $864, $2564
3. $4.10 4. $137 5. $2556

Exercise 8-4

1. (a) 11% (b) 16% (c) 10% (d) 7.5% (e) 9.0%
2. 19% 3. 14.4% 4. 10% 5. 20.9%

Exercise 8-5

2. Total amount repaid was $223.54.
3. (a) 1.6% (b) 19.2%
4. Payments $11.20, $11.04, $10.88, $10.72, $10.56, $10.40.
5. Payments $122.50, $121.75, $102.25, $101.50, $100.75.
6. Principal at the end of each month $1434.55, $1367.79, $1301.49, $1234.51, $1167.38, $1099.57.
7. Unpaid Balance $2416.15, $2331.75, $2246.80, $2161.31, $2095.27, $1988.67.

Exercise 8-6

1. (a) $228.65 (b) $268.18 (c) $366.84
2. (a) $125.56 (b) $137.46 (c) $181.30
3. (a) $304.62 (b) $104.49 (c) $224.20 (d) $192.58 (e) $321.69
4. (a) $230.16 (b) $176.96 (c) $327.78
5. $343.52
6. $534.17
7. (a) $406.71 (b) $14 641.56 (c) $3391.56
8. (a) $647.94 (b) 23 325.84 (c) $5403.34

REVIEW EXERCISE

1. Final balance $768.54. 2. Amount of deposit $77.51.
4. (a) $2.47 (b) $25 (c) $912.50 (d) $632.66 (e) 1.5% (f) 3.0%
(g) three days (h) sixteen days
5. 10% 6. 17.1%

7. (a) **Balance column**	(b) **Monthly payment column**
$2433.75	$103.75
$2367.00	$103.11
$2299.75	$102.47
$2232.00	$101.83
$2163.74	$101.20

(c) **Balance Part (a)**
$2163.74
Balance Part (b)
$2075.00

8. Monthly payment $162.43, total interest paid $830.30.

REVIEW AND PREVIEW TO CHAPTER 9

1. (a) (i) $915.43
(ii) $263.93
(iii) $1107.30
(iv) $182.28
(b) $2468.94
2. (a) (i) $131.76 (ii) $698.73
(iii) $549.73 (iv) $1049.40 (Cr.)
(b) (i) $140.98 (ii) $747.64
(iii) $588.21 (iv) $1122.86 (Cr.)
3. (a) (i) $842.68 (ii) $1209.90
(iii) $4146.82 (iv) $2804.05
(b) $9003.45, $278.67

CHAPTER 9

Exercise 9-1

1. (a) 2 × 3 (b) 3 × 3 (c) 3 × 2 (d) 1 × 4 (e) 3 × 1 (f) 2 × 4
(g) 1 × 1 (h) 1 × 2 (i) 2 × 2
2. (a) 3 × 4 (b) 3,9,1 (c) 7,1,8,9 (d) 4,5,7 (e) 2 (f) 9
(g) 3 (h) 6
3. (a) $25.50 (b) $20.40 (c) $8.50 (d) $34.50 (e) $49.50 (f) $53.90

4. (a) 5664 km (b) 4917 km (c) 201 km

(d) $\begin{pmatrix} 1331 & 3606 & 2333 & 2192 & 3266 & 2108 & 2397 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 3427 \\ 1821 \\ 549 \\ 397 \\ 1482 \\ 0 \\ 4505 \\ 2108 \end{pmatrix}$

(f) $\begin{pmatrix} 7302 & 2546 & 0 & 402 & 1866 & 1098 & 9460 & 4666 \end{pmatrix}$

(g) $\begin{pmatrix} 2108 \\ 12\,006 \\ 9460 \\ 9152 \\ 11\,328 \\ 9010 \\ 0 \\ 4794 \end{pmatrix}$

5. (a) 21 (b) 2 (c) 4 (d) $\begin{pmatrix} 15 & 12 & 3 & 7 \end{pmatrix}$, 37

(e) $\begin{pmatrix} 19 \\ 21 \\ 16 \\ 5 \\ 12 \end{pmatrix}$, 73

(f) Big M: 48, Astro: 44, Cosmic: 55, Sim's: 30, Pete's: 37

(g) 214

Exercise 9-2

1. (a) $\begin{pmatrix} 9 & 9 \\ 11 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 6 & 9 & 18 \\ 10 & 21 & 23 \end{pmatrix}$ (d) $\begin{pmatrix} -6 & -1 & 4 \\ 18 & 9 & 4 \\ 6 & -3 & 33 \end{pmatrix}$

(f) $\begin{pmatrix} -3 & 62 \\ -16 & -116 \\ 109 & -164 \end{pmatrix}$ (g) $\begin{pmatrix} 1148 & 932 \\ 1258 & 1724 \end{pmatrix}$ (h) $\begin{pmatrix} 1032 & 807 \\ 1115 & 1259 \\ 1584 & 954 \end{pmatrix}$

2. (a) $\begin{pmatrix} 31 & 7 \\ 4 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 13 \\ 16 \\ 12 \end{pmatrix}$

(c) $\begin{pmatrix} 56 & 13 \\ 19 & -14 \\ -4 & 8 \end{pmatrix}$ (d) $\begin{pmatrix} 22 & 10 & 2 \\ -7 & 12 & 22 \\ -16 & 11 & 20 \end{pmatrix}$

(e) $\begin{pmatrix} 1074 & 428 & 807 \end{pmatrix}$ (f) $\begin{pmatrix} 14\,137 & 5145 \\ 3375 & 5704 \end{pmatrix}$

3. (a) Nov $= \begin{pmatrix} 7 & 4 & 1 \\ 6 & 4 & 2 \\ 5 & 6 & 1 \\ 4 & 8 & 0 \end{pmatrix}$, Dec $= \begin{pmatrix} 3 & 9 & 2 \\ 5 & 8 & 1 \\ 8 & 6 & 0 \\ 10 & 3 & 1 \end{pmatrix}$, Jan $= \begin{pmatrix} 5 & 5 & 0 \\ 7 & 2 & 1 \\ 3 & 6 & 1 \\ 4 & 6 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 15 & 18 & 3 \\ 18 & 14 & 4 \\ 16 & 18 & 2 \\ 18 & 17 & 1 \end{pmatrix}$

(c) Blues

(d) Aces

Exercise 9-3

1. (a) $\begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 7 & 11 \\ 8 & 2 & 11 \end{pmatrix}$ (c) $\begin{pmatrix} -8 & 10 \\ -7 & -2 \end{pmatrix}$

   (e) $\begin{pmatrix} -3 & 3 \\ -13 & 12 \\ 3 & 6 \end{pmatrix}$ (f) $\begin{pmatrix} 6 & 5 & -3 \\ -8 & -2 & -5 \end{pmatrix}$ (g) $\begin{pmatrix} -4 & 11 & -3 \\ 4 & -2 & -2 \\ -4 & 4 & -11 \end{pmatrix}$

2. (a) $\begin{pmatrix} 6 & 2 \\ 7 & -8 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 \\ 9 & 13 \\ 1 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} 29 \\ -6 \\ -6 \end{pmatrix}$

   (d) $\begin{pmatrix} 4 & 15 \\ -8 & -27 \end{pmatrix}$ (e) $\begin{pmatrix} 19 & 53 & 15 \\ -25 & -3 & 15 \\ -3 & -12 & 21 \end{pmatrix}$ (f) $\begin{pmatrix} 88 & 81 \\ 4 & -19 \end{pmatrix}$

Exercise 9-4

1. (a) $\begin{pmatrix} 2 & 10 \\ 12 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 12 \\ 4 \\ -24 \end{pmatrix}$ (c) $\begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$

   (d) $\begin{pmatrix} -6 & 0 \\ 2 & -8 \end{pmatrix}$ (e) $\begin{pmatrix} -3 & 0 & 18 \end{pmatrix}$ (f) $\begin{pmatrix} 20 & 30 & -60 \\ 40 & -10 & -70 \end{pmatrix}$

2. (a) $\begin{pmatrix} 12 & 21 \\ 8 & 24 \end{pmatrix}$ (b) $\begin{pmatrix} 10 & 8 \\ 11 & 18 \\ 28 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & 2 \\ 12 & -17 \end{pmatrix}$

   (d) $\begin{pmatrix} 6 & 18 & -12 \\ 12 & 3 & 5 \\ 4 & 6 & 21 \end{pmatrix}$ (e) $\begin{pmatrix} 3 \\ 20 \\ -28 \end{pmatrix}$ (f) $\begin{pmatrix} 20 & 10 \\ -17 & 23 \\ -21 & 2 \end{pmatrix}$

3. (a) $\begin{pmatrix} 1 & 4 \\ -1 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 7 & -5 \\ 5 & 8 \end{pmatrix}$ (c) $\begin{pmatrix} 3 & -2 \\ 7 & 4 \end{pmatrix}$ (d) $\begin{pmatrix} -6 & 9 \\ -6 & -4 \end{pmatrix}$

   (e) $\begin{pmatrix} 5 & -1 \\ 10 & 8 \end{pmatrix}$ (f) $\begin{pmatrix} -1 & 15 \\ -1 & 8 \end{pmatrix}$ (g) $\begin{pmatrix} -5 & 18 \\ 1 & 4 \end{pmatrix}$ (h) $\begin{pmatrix} -8 & 8 \\ -9 & -8 \end{pmatrix}$

4. (a) $\begin{pmatrix} 0 & 513 & 272 & 453 & 746 & 587 \\ 513 & 0 & 640 & 586 & 557 & 62 \\ 272 & 640 & 0 & 733 & 1024 & 704 \\ 453 & 586 & 733 & 0 & 298 & 595 \\ 746 & 557 & 1024 & 298 & 0 & 579 \\ 587 & 62 & 704 & 595 & 579 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & 1016 & 544 & 906 & 1492 & 1174 \\ 1016 & 0 & 1280 & 1172 & 1114 & 124 \\ 544 & 1280 & 0 & 1466 & 2048 & 1408 \\ 906 & 1172 & 1466 & 0 & 596 & 1190 \\ 1492 & 1114 & 2048 & 596 & 0 & 1158 \\ 1174 & 124 & 1408 & 1190 & 1158 & 0 \end{pmatrix}$

Exercise 9-5

1. (a) $\begin{pmatrix} 16 & 14 \\ 22 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 33 \\ 27 \end{pmatrix}$ (c) $\begin{pmatrix} 34 \\ 18 \\ 31 \end{pmatrix}$ (e) $\begin{pmatrix} 5 & 7 \\ 3 & 4 \end{pmatrix}$

   (f) $\begin{pmatrix} 60 & 17 & 22 \\ 101 & 32 & 34 \\ 92 & 22 & 31 \end{pmatrix}$

2. (a) $\begin{pmatrix} 72 & 6 \\ 81 & 2 \\ 94 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ 15 \end{pmatrix}$ (c) Thurs: \$306, Fri: \$273, Sat: \$327.

   (d) \$906

3. (a) $\begin{pmatrix} 29 & 31 & 6 \\ 29 & 29 & 8 \\ 32 & 28 & 6 \\ 28 & 30 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ (c) Aces: 64, Blades: 66, Blues: 70, Flyers: 64.

4. (a) $\begin{pmatrix} 2100 & 8134 & 2824 \\ 3090 & 9213 & 1101 \\ 2520 & 8475 & 1316 \end{pmatrix}$ (b) $\begin{pmatrix} 9 \\ 7 \\ 4 \end{pmatrix}$ (c) Fri: \$87 134, Sat: \$96 705, Sun: \$87 269.
(d) \$271 108
(e) \$69 390

REVIEW EXERCISE

1. (a) $\begin{pmatrix} 7 & 9 \\ 16 & 19 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 3 & 12 \\ -10 & 7 & 2 \\ 1 & 2 & 13 \end{pmatrix}$ (d) $\begin{pmatrix} 14 & 10 & 2 \\ 13 & 5 & -25 \end{pmatrix}$
(e) $\begin{pmatrix} -2 \\ -2 \\ 12 \\ 6 \end{pmatrix}$

2. (a) $\begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 9 & 11 \\ 1 & 4 & 16 \\ 7 & 5 & 17 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & -4 \\ -1 & -3 \end{pmatrix}$
(d) $\begin{pmatrix} 4 & 17 \\ 4 & 5 \\ 4 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 12 & 6 \\ 9 & 21 \end{pmatrix}$ (f) $\begin{pmatrix} -16 & -2 & -10 \\ -8 & 4 & -6 \\ -16 & 2 & 0 \end{pmatrix}$
(g) $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ (h) $\begin{pmatrix} -3 & 4 \\ 11 & 32 \end{pmatrix}$ (i) $\begin{pmatrix} -4 & -4 \\ -10 & 10 \\ 20 & 37 \end{pmatrix}$

3. (a) $\begin{pmatrix} 18 & 15 \\ 14 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 22 & 8 \\ 47 & 37 \end{pmatrix}$ (c) $\begin{pmatrix} 5 & 6 \\ 8 & 7 \end{pmatrix}$
(d) $\begin{pmatrix} 4 & 15 & 6 \\ 4 & 14 & 7 \\ 3 & 9 & 3 \end{pmatrix}$ (e) (45) (f) $\begin{pmatrix} 16 & 22 \\ 6 & 3 \end{pmatrix}$

4. (a) $\begin{pmatrix} 35 & 12 & 24 \\ 26 & 11 & 21 \\ 31 & 13 & 22 \end{pmatrix}$ (b) $\begin{pmatrix} 9 \\ 21 \\ 35 \end{pmatrix}$ (c) South: \$1407; Westside: \$1200; Eastern: \$1322.
(d) \$2345 (e) \$3929

REVIEW AND PREVIEW TO CHAPTER 10

1. (a) 80° (b) 130° (c) 150° (d) 105° (e) 60°
2. (a) 50° (b) 10° (c) 65° (d) 45° (e) 15°
3. (a) 75° (b) 130° (c) 90° (d) 30° (e) 30° (f) 25°
4. (a) $a = 60°, b = 120°, c = 110°$ (b) $a = 60°, b = 90°, c = 40°$
(c) $a = 138°, b = 42°, c = 138°$
5. (a) 27.0 (b) 14.1 (c) 20.3 (d) 289 (e) 6.25 (f) 8.1
(g) 18.6 (h) 22.9 (i) 29 (j) 5.3

CHAPTER 10

Exercise 10-1

1. (a) 180° (b) 240° (c) 45° (d) 2052° (e) 1260°
(f) 1665° (g) 2261° (h) 9.72°
2. (a) 96°, 8° (b) 264°, 22° (c) 180°, 15° (d) 1080°, 90°
3. (a) 57° (b) 27° (c) 65° (d) 37° (e) 76°

4. **(a)** $\frac{1}{12}$ r/s **(b)** 5 r/min

5. **(a)** 45° **(b)** 135° **(c)** 135° **(d)** 0° **(e)** $22\frac{1}{2}°$
(f) $112\frac{1}{2}°$ **(g)** 90° **(h)** $22\frac{1}{2}°$

Exercise 10-2

1.

Polygon	No. of sides	No. of triangles	Angle sum of the polygon	No. of angles in the polygon	Angle of the regular polygon
triangle	3	1	180°	3	60°
quadrilateral	4	2	360°	4	90°
pentagon	5	3	540°	5	108°
hexagon	6	4	720°	6	120°
septagon	7	5	900°	7	$128\frac{4}{7}°$
octagon	8	6	1080°	8	135°
decagon	10	8	1440°	10	144°
dodecagon	20	18	3240°	20	162°
n-gon	n	$n - 2$	$(n - 2)180°$	n	$\frac{(n-2)180°}{n}$

Exercise 10-3

1. **(a)** $\triangle GHJ$ **(b)** $\triangle KLM$
2. **(a)** $BC = DF$ or $\angle BAC = \angle DEF$ **(b)** $\angle QPS = \angle RPS$ or $QS = SR$
(c) $MP = OP$ or $\angle MNP = \angle ONP$, or $\angle M$ or $\angle D$ **(d)** $AE = EB$ or $\angle A = \angle B$ or $\angle C = \angle D$
(e) $AD = BC$ or $\angle B = \angle D$ or $\angle BAC = \angle ACD$ **(f)** $XW = XY$ or $\angle WZX = \angle XYZ$
3. SAS 4. SSS 5. **(a)** none **(b)** $\triangle ABD \cong \triangle ACD$
(c) $\triangle AEB \cong \triangle BEC \cong \triangle DEC \cong \triangle DEA$ $\boxed{\text{ASA}}$ $\left\{\begin{array}{l}\triangle BCD \cong \triangle ABD \quad \boxed{\text{SSS}} \\ \triangle ABC \cong \triangle ADC\end{array}\right.$
6. 65°, 50°, 65°, 65°, SAS.

Exercise 10-4

1. **(a)** **(i)** (d,e), (h,c) **(ii)** (b,e), (a,h), (d,g), (c,f) **(iii)** (h,d), (c,e)
(b) **(i)** (w,y), (x,z)
(c) **(iii)** (k,l), (m,j), (k,j), (l,m)
(d) **(iii)** (p,q), (q,r)
2. **(a)** $x = 30°$, $y = 60°$, $z = 90°$ **(b)** $a = 130°$, $b = 50°$, $c = 130°$
(c) $m = 135°$, $n = 110°$ **(d)** $x = 120°$, $y = 120°$
(e) 35° **(f)** $d = 110°$, $e = 70°$, $f = 110°$
3. **(a)** $a = b$ **(b)** $w = z$, $x = y$ **(c)** $a = b = 90°$ **(d)** $x = y$
4. **(a)** 105° **(b)** 140° **(c)** 90°
5. **(a)** $a = 72°$, $b = c = 108°$ **(b)** $a = 85°$, $c = 85°$, $b = 30°$ **(c)** $a = 30°$, $b = 10°$, $c = 50°$
6. **(a)** 55° **(b)** 54° **(c)** 53° **(d)** 52° **(e)** 51° **(f)** 50° **(g)** $180° - x°$
7. **(a)** Corresponding angles are equal.
8. **(a)** 108° **(b)** 109° **(c)** 110° **(d)** 112° **(e)** 114°
10. **(a)** 60° **(b)** 20° **(c)** 100° **(d)** 120°
11. **(a)** $\angle MAB = \angle ABC$, $\angle NAC = \angle ACB$

Exercise 10-5

1. **(a)** yes **(b)** yes **(c)** yes **(d)** no **(e)** no **(f)** yes **(g)** yes **(h)** yes

2.

a	*b*	*c*
4	3	5
4	4	$\sqrt{32}$
12	5	13
6	8	10

a	*b*	*c*
7	24	25
15	8	17
1	$\sqrt{3}$	2
1	1	$\sqrt{2}$

3. **(a)** 4, 5
(b) 7, 8
(c) 9, 10
(d) 12, 13
(e) 24, 25
(f) 13, 14

4. **(a)** 45, 6.71 **(b)** 52, 7.21 **(c)** 34, 5.83 **(d)** 14, 3.74
5. 21.2 cm **6.** 45 m **7.** 8.95 m **8.** 4 m
9. **(a)** 5.66 m **(b)** 0.47 m **10.** 583 km
11. **(a)** $b \doteq 1.73$, 8.66 **(b)** $k \doteq 1.41$, 7.07
12. 13 cm **13.** **(b)** 13 in. **14.** 65.6 cm
15. $AJ \doteq 6.08$ cm, $DK \doteq 5.39$ cm, $EL \doteq 5.00$ cm, $FM \doteq 5.00$ cm, $GN \doteq 5.39$ cm, $HB \doteq 6.08$ cm

Exercise 10-6

1. **(a)** 5 m **(b)** 10 m **(c)** 16 m **(d)** 628 m
2. **(a)** 2.25 m^2 **(b)** 314 m^2 **(c)** 22.4 mm^2 **(d)** 45.0 m^2
3. **(a)** 27 cm^3 **(b)** 3768 m^3 **(c)** 42 m^3 **(d)** 524 m^3
4. 1.8 m^2 **5.** 0.774 m^2 **6.** 15.6 m
7. 4.5 m, \$1.13 **8.** 9.67 m **9.** 36 m
10. **(a)** 60 m^2 **(b)** 60 m^2 **(c)** 24 m^2
11. **(a)** 17 cm^2 **(b)** 51.4 cm^2 **(c)** 48.86 m^2
12. **(a)** 75 360 cm^3 **(b)** 64 000 cm^3 **(c)** 139 360 cm^3
13. **(a)** 11 squares **(b)** 160 m^2 **14.** **(a)** 12 426 m^2 **(b)** \$311 000
15. **(a)** 37.68 m^2 **(b)** 7.74 m^2 **(c)** 22.86 cm^2
16. **(a)** 75.36 cm^3 **(b)** 9 kg **17.** 523.3 m^3

Review Exercise

1. **(a)** 115° **(b)** 55° **(c)** 60° **2.** **(a)** SAS **(b)** SAS
3. **(a)** $a = 50°, b = 130°, c = 50°$ **(b)** $w = 30°, x = 100°, y = 50°$
4. **(a)** 22.4 **(b)** 9.87 **(c)** 20.7 **5.** 28 m
6. 37.7 m **7.** 13 cm
8. **(a)** 10.6 m **(b)** 16.5 cm **(c)** 19.56 cm
9. **(a)** 14.5 cm^2 **(b)** 44 m^2 **(c)** 5.17 cm^2
10. **(a)** 66.6 cm^3 **(b)** 33.4 cm^3 **(c)** 0.377 m^3
11. **(a)** 14 cm^2 **(b)** 339 cm^2 **(c)** 10 m

REVIEW AND PREVIEW TO CHAPTER 11

1. **(a)** 0.385 **(b)** 0.292 **(c)** 0.280 **(d)** 6.22 **(e)** 0.688
(f) 9.45 **(g)** 31.3 **(h)** 14.4 **(i)** 34.0 **(j)** 2.44
2. **(a)** 3.74 **(b)** 7.63 **(c)** 5.21 **(d)** 187 **(e)** 34.9
(f) 2.19 **(g)** 14.3 **(h)** 20.5 **(i)** 234 **(j)** 2.25
3. **(a)** 50 units **(b)** 4.04 units **(c)** 1.41 units **(d)** 13 units
4. **(a)** 63° **(b)** 30°, 60° **(c)** 45° **(d)** 26°

CHAPTER 11

Exercise 11-1

1. **(a)** $\frac{8}{12} = \frac{20}{30}$ **(b)** $\frac{a}{b} = \frac{5}{7}$ **(c)** $\frac{a}{b} = \frac{5}{10}$ **(d)** $\frac{2}{6} = \frac{3}{9}$
2. **(a)** 25 units **(b)** 9 units **(c)** 16 cm **(d)** 3.33 mm **(e)** 1.2 m **(f)** 5.25 m
3. $f = 7.5$ units, $i = 7$ units **4.** $a = 10$ units, $b = 13.3$ units **5.** 9.0 m
6. **(a)** 26.7 cm **(b)** 8.54 cm, 28.5 cm **7.** 112 m

Exercise 11-2

1.

	Hypotenuse	Opposite	Adjacent
(a)	*c*	*a*	*b*
(b)	*d*	*e*	*f*
(c)	*g*	*h*	*i*
(d)	*l*	*k*	*j*
(e)	*p*	*r*	*q*
(f)	*t*	*u*	*s*

2. (a) $\frac{3}{5}$ (b) $\frac{5}{4}$ (c) $\frac{4}{7}$ (d) $\frac{2}{3}$ (e) $\frac{7}{4}$ (f) $\frac{3}{8}$

3. (a) 70.0 m (b) 13.7 m (c) 25.2 cm (d) 168 m (e) 59.6 cm (f) 37.1 m
4. (a) 128 m (b) 14.0 m (c) 83.9 m (d) 3 m (e) 46.6 m (f) 250 m
5. (a) 0.750, 37° (b) 1.333, 53° (c) 2.400, 67°
(d) 1.00, 45° (e) 1.7, 60° (f) 1.875, 62°
6. 22° 7. 18.5 m 8. 209.6 m

Exercise 11-3

1. (a) $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}$ (b) $\frac{8}{10}, \frac{6}{10}, \frac{8}{6}$ (c) $\frac{1.0}{2.0}, \frac{1.7}{2.0}, \frac{1.0}{1.7}$
(d) $\frac{x}{10}, \frac{y}{10}, \frac{x}{y}$ (e) $\frac{y}{r}, \frac{x}{r}, \frac{y}{x}$ (f) $\frac{b}{c}, \frac{a}{c}, \frac{b}{a}$

2 (a) $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$ (b) $\frac{30}{34}, \frac{16}{34}, \frac{30}{16}$ (c) $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$
(d) $\frac{5}{7}, \frac{5}{7}, 1$ (e) $\frac{12}{13}, \frac{5}{13}, \frac{12}{5}$ (f) $\frac{y}{r}, \frac{x}{r}, \frac{y}{x}$

3. (a) 57.4 m (b) 17.7 cm (c) 26.8 m, 22.5 m
(d) 164 m (e) 110 m, 42.3 m

4. (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 5. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$

6. (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$

Exercise 11-4

3. (a) 0.000 to 1.000 (b) 1.000 to 0.000 (c) 0.000 increases without limit
4. (a) 13.2 m (b) 102 m (c) 136 m
5. (a) 24.6 cm (b) 44.1 cm (c) 64.3 m
6. (a) 14.0 cm (b) 9.62 cm (c) 1.46 cm
7. (a) 1.16 cm (b) 0.942 cm (c) 4.24 m
8. (a) $a = 3.26$ cm, $c = 2.64$ cm (b) $f = 60$ m, $\angle E = 53°$, $\angle F = 37°$
(c) $r = 197$ m, $p = 226$ m, $\angle p = 49°$
9. $a = 7.62$ m, $\angle B = 67°$, $\angle c = 23°$ 10. $\angle F = 55°$, $d = 11.5$ m, $e = 16.4$ m

Exercise 11-6

1. $b = 2.13$ cm, $a = 2.45$ cm 2. 6.18 cm 3. 18° 4. 6°, 12°
5. (a) 3° (b) 9.03 cm 6. 2.83 cm 7. 3.848 cm, 4.925 cm
8. 8.23 cm 9. 63° 10. 2.57 cm

Exercise 11-7

1. (a) 1.390 (b) 0.0175 (c) 1.004 (d) 0.3249 (e) 0.5774 (f) 1.414
(g) 1.051 (h) 0 (i) 0.8192 (j) 1 (k) 5.759 (l) 7.115
2. (a) 67° (b) 49° (c) 54° (d) 1° (e) 22° (f) 28°
3. (a) 25.6 m (b) 2.0 cm (c) 8.1 mm (d) 1.3 m (e) 212.1 mm (f) 1.6 m
4. 151.0 m
5. 9.3 m

Review Exercise

1. (a) 24 (b) 36

2.

	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
(a)	0.500	0.865	0.578	2.000	1.156	1.730
(b)	0.630	0.778	0.810	1.588	1.285	1.235
(c)	0.773	0.636	1.214	1.294	1.571	0.824

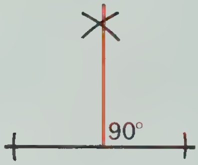
3. (a) 32.2 units (b) 35.5 units

4. 742.5 m 5. 29.7 m 6. 8.20 cm 7. 22° 8. 76°

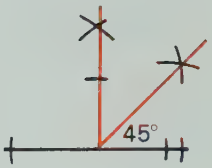
REVIEW AND PREVIEW TO CHAPTER 12

Exercise

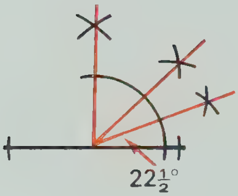
1. (a)

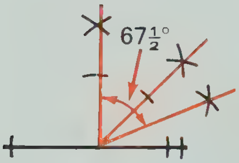


(b)

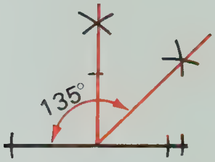


(c)

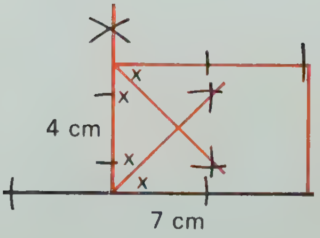




(d)

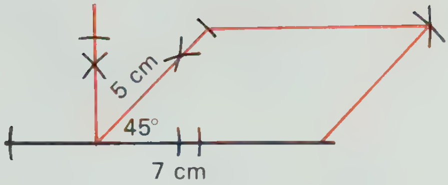


2. (a)

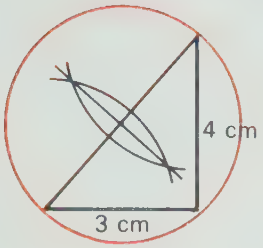


(b) no

3. **(a)**



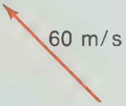
4.



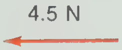
CHAPTER 12

Exercise 12-1

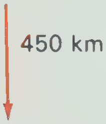
1. **(a)** 6 cm ↓ **(b)** 4.5 cm ↘ **(c)** 1.5 cm ↑ **(d)** 2.25 cm ↑ **(e)** 3.75 cm ←
2. **(a)** 5 cm **(b)** 7.5 cm **(c)** 3.75 cm **(d)** 6.25 cm **(e)** 2 cm
3. **(a)** 3 cm **(b)** 2 cm **(c)** 3.75 cm **(d)** 1.25 cm **(e)** 5.5 cm
4. **(a)** 4 cm **(b)** 5 cm **(c)** 2.5 cm **(d)** 6 cm **(e)** 4 cm
5. **(a)** 10 m/s = 1 cm



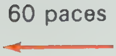
(b) 10 N = 1 cm



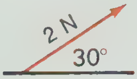
(c) 100 km = 1 cm



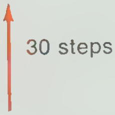
(d) 10 paces = 1 cm



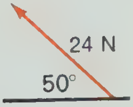
(e) 2 N = 1 cm



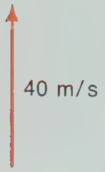
(f) 10 steps = 1 cm



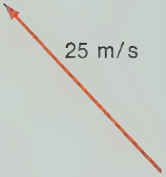
(g) 10 N = 1 cm



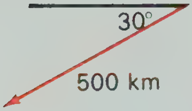
6. **(a)**



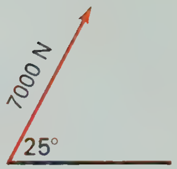
(b)



(c)



(d)



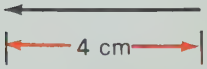
Exercise 12-2

1. **(a)** 15 N right **(b)** 9 N right **(c)** 17 N right **(d)** 10 N left **(e)** 3 N, left

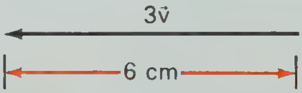
2. **(a)** 6 N, right **(b)** 4 N, right **(c)** 7 N, right **(d)** 28 N, left **(e)** 0 N

3. **(a)** 3 N, right **(b)** 8 N, north

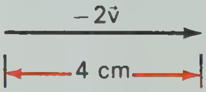
4. **(a)**



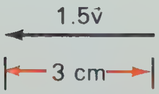
(b)



(c)



(d)

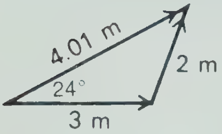


(e)

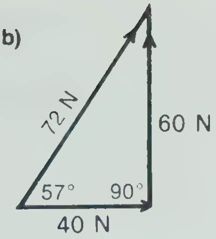
5. **(a)** the vector from B to A
   (b) $\overrightarrow{O}$
   (c) $\overrightarrow{AB}$, $\overrightarrow{AD}$, $\overrightarrow{BC}$, $\overrightarrow{AE}$, $\overrightarrow{EC}$, $\overrightarrow{DE}$, $\overrightarrow{EB}$, $\overrightarrow{DC}$
   (d) (i) $\overrightarrow{DC}$, (ii) $\overrightarrow{AD}$, (iii) $\overrightarrow{EC}$, (iv) $\overrightarrow{DE}$
   (e) $\overrightarrow{AE}$, $\overrightarrow{EC}$

Exercise 12- 3

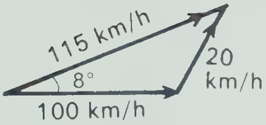
1. **(a)**



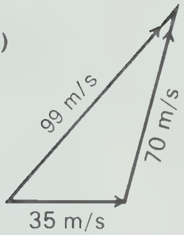
(b)



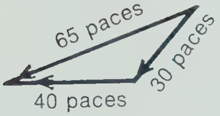
(c)



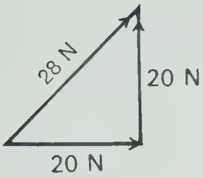
(d)



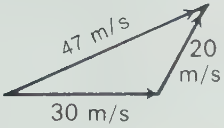
(e)



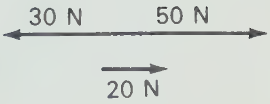
(f)



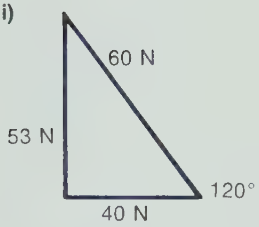
(g)



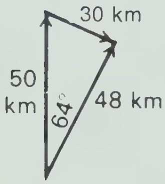
(h)



(i)



(j)



2. **(a)** 215 N **(b)** 210 m

3. 14 kN **4.** 10.8 km/h, 22°

5. 327 km, N 24° W **6.** 15.5 N

7. 14.3 N, 43°

8. 31°, 108°, 41°

9. 15.8 km/h, 18°

Exercise 12-4

1. 109 kn, N 72° W **2.** 11°, 20.4 kn **3.** N 19° E, 141 kn
4. **(a)** 12° **(b)** 19.6 kn **5.** **(a)** N 16° W **(b)** 180 kn **6.** S 43° E, 165 kn

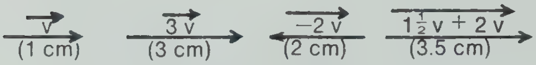
7. S 40° W, 119 kn **8.** S 13° E, 143 kn **9.** N 37° E, 170 kn

10. E 15° S, 106 N

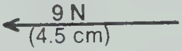
Review Exercise

1. A scalar has only magnitude, a vector has magnitude and direction.

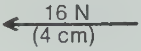
2.



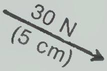
3. **(a)** 2 N = 1 cm



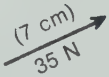
(b) 4 N = 1 cm



(c) 6 N = 1 cm

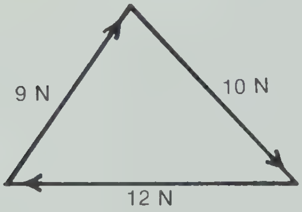


(d) 5 N = 1 cm

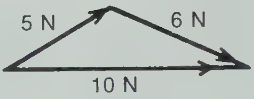


4. **(a)** 24 N **(b)** 16.5 N **(c)** 36 N **(d)** 8.5 N **(e)** 14 N
5. **(a)** 7 N **(b)** 22 N **(c)** 14 N **(d)** 3 N **(e)** 18 N
6. 23 N, 26° **7.** **(a)** 1300 **(b)** 11°

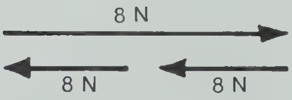
8. **(a)**



(b)



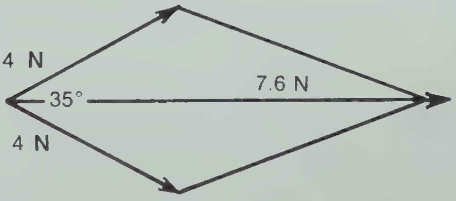
(c)



(d) not possible

9. S 62° E, 174 kn

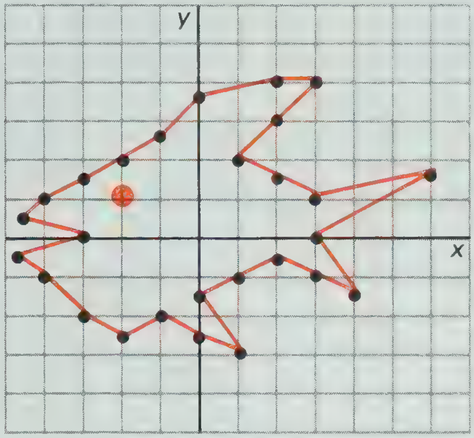
10.



REVIEW AND PREVIEW TO CHAPTER 13

1. $A(2,3)$, $B(-1,4)$, $C(2,0)$, $D(5,4)$, $E(-6,0)$, $F(5,-2)$, $G(0,-2)$, $H(-2,-1)$, $J(-1,-1)$, $K(-1,-4)$, $L(4,-4)$, $M(0,5)$

2.



3. (a) C
   (b) Q, R
   (c) P
   (d) S, T, V
   (e) P, R
   (f) A, C
4. (a) $1,-1$ (b) $5,10$ (c) $5,-5$ (d) $4,3$ (e) $10,-8$
   (f) -2, no y-intercept (g) $0,0$ (h) no x-intercept, 8
5. (a) $v = \frac{d}{t}$ (b) $r = \frac{c}{2\pi}$ (c) $b = P\text{-}a\text{-}c$
   (d) $P = \frac{I}{rt}$ (e) $b = y - mx$ (f) $u = v - at$
   (g) $h = \frac{V}{\pi r^2}$ (h) $b = \frac{2A}{h}$ (i) $l = \frac{P - 2w}{2}$
   (j) $t = \frac{A - P}{Pr}$

Exercise 13-1

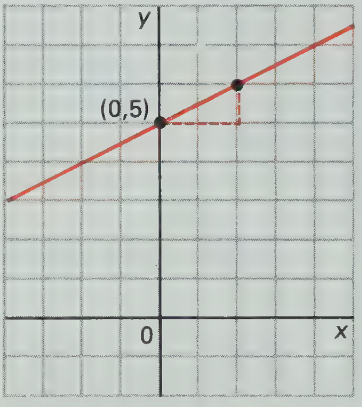
1. (a),(b)

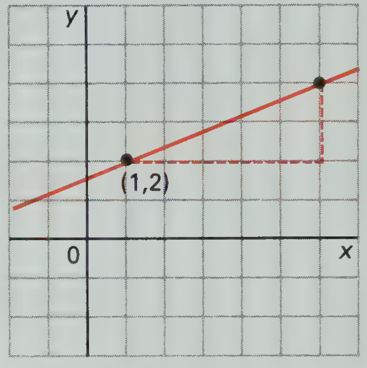
	rise	run	slope
AB	3	5	$\frac{3}{5}$
CD	0	8	0
EF	-2	3	$-\frac{2}{3}$
GH	-2	5	$-\frac{2}{5}$
IJ	-5	0	undefined
KL	5	1	5
MN	1	-6	$-\frac{1}{6}$

2. (a) 2 (b) 2 (c) $\frac{3}{4}$ (d) 0 (e) 5 (f) $\frac{7}{6}$
   (g) $\frac{1}{3}$ (h) $\frac{8}{3}$ (i) $-\frac{1}{2}$ (j) $-\frac{3}{4}$ (k) $\frac{2}{3}$ (l) $-\frac{5}{2}$
   (m) 0.7 (n) -1 (o) $-\frac{2}{5}$ (p) -1
3. (b) $M_{PQ} = \frac{2}{3}$ (c) $M_{QR} = \frac{2}{3}$ (d) collinear
4. (a) collinear (b) collinear (c) collinear (d) not collinear
5. (a) 45 m (b) 2.7 km
6. The 450 m hill.
7. 90 m

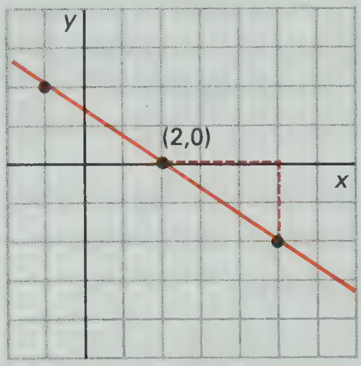
Exercise 13-2

1. (a) $y = -3x + 5, m = -3, b = 5$ (b) $y = -2x + 3, m = -2, b = 3$
   (c) $y = 2x - 6, m = 2, b = -6$ (d) $y = 5x + 2, m = 5, b = 2$
   (e) $y = -x - 5, m = -1, b = -5$ (f) $y = -\frac{3}{4}x + \frac{1}{2}, m = -\frac{3}{4}, b = \frac{1}{2}$
   (g) $y = \frac{2}{5}x + \frac{3}{5}, m = \frac{2}{5}, b = \frac{3}{5}$ (h) $y = -\frac{1}{3}x + \frac{5}{6}, m = -\frac{1}{3}, b = \frac{5}{6}$
   (i) $y = \frac{1}{4}x - \frac{1}{3}, m = \frac{1}{4}, b = -\frac{1}{3}$ (j) $y = \frac{1}{3}x - \frac{4}{3}, m = \frac{1}{3}, b = -\frac{4}{3}$
   (k) $y = -x + \frac{2}{5}, m = -1, b = \frac{2}{5}$ (l) $y = x - \frac{1}{3}, m = 1, b = -\frac{1}{3}$
2. $y = 0x + 5, m = 0, b = 5$
3. (a)

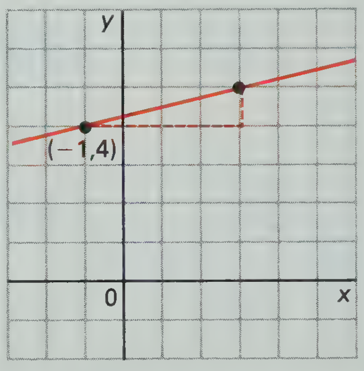




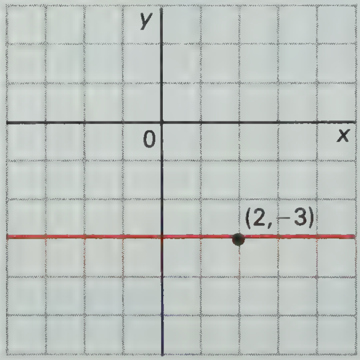
(c)



(d)



(e)



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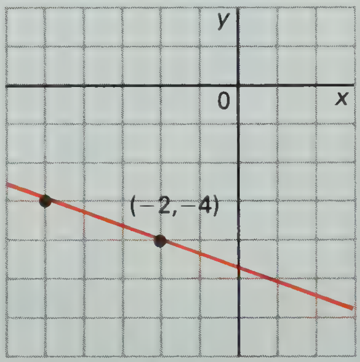
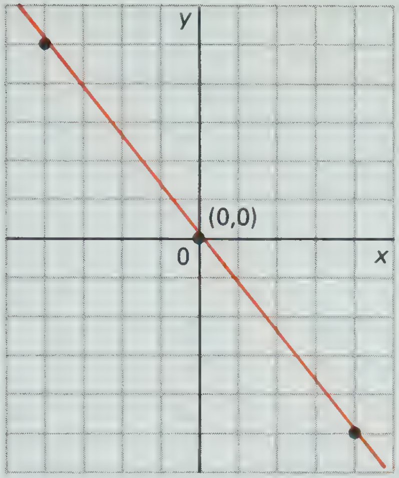
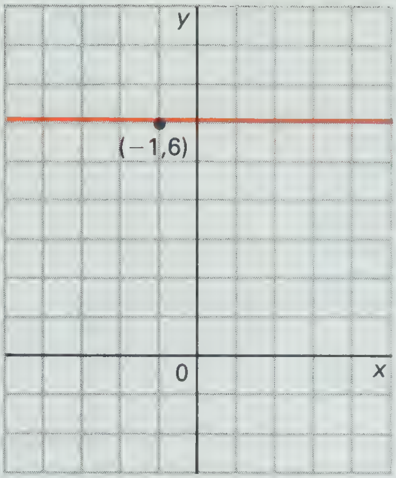


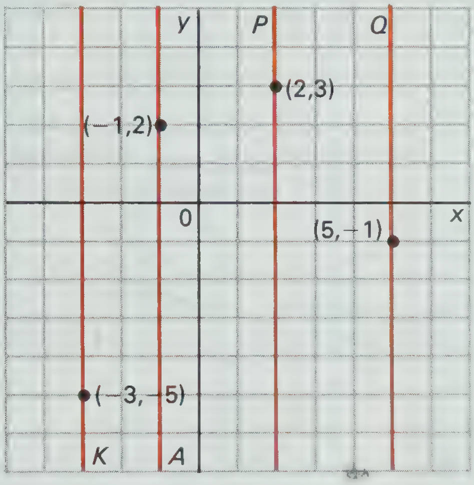
(g)



(h)



4.



5. (a) $y = 3x + 5$
(b) $y = \frac{1}{2}x - 2$
(c) $y = \frac{3}{4}x + 6$
(d) $y = -3x + 4$
(e) $y = \frac{2}{3}x - 1$
(f) $y = -5x$
(g) $y = 0.6x + 1.8$
(h) $y = -2.5x + 400$

Exercise 13-3

1. (a) 5 (b) $\frac{2}{3}$ (c) -1 (d) 0 (e) 2 (f) -5
   (g) -1 (h) -7
2. (a) 3 (b) 2 (c) 7 (d) 10 (e) 7 (f) 0
3. The segments in (a), (d), and (f) are parallel.
4. (a) $M_{PQ} = 0, M_{MN} = 0$
   (b) parallel
   (c) 0
5. (a) undefined
   (b) parallel
   (c) undefined
6. The lines in (a), (c) and (e) are parallel.
8. (a) $y = 6$ (b) $y = 3$ (c) $x = 4$ (d) $x = 9$ (e) $y = -1$ (f) $x = 15$

Exercise 13-4

1. (a) $-\frac{1}{3}$ (b) $-\frac{5}{2}$ (c) 4 (d) $-\frac{6}{5}$ (e) 1 (f) $-\frac{2}{3}$
   (g) $\frac{1}{5}$ (h) -1 (i) undefined (j) 0
2. If one of the lines is parallel to either the x-axis or the y-axis.
3. (a) $-\frac{1}{3}$ (b) $\frac{1}{5}$ (c) -2 (d) $\frac{2}{3}$ (e) 1 (f) $\frac{1}{3}$
   (g) $-\frac{1}{5}$ (h) 2 (i) $-\frac{1}{2}$ (j) $\frac{1}{4}$ (k) -4 (l) $-\frac{8}{3}$
4. (a) $\frac{1}{2}$ (b) -1 (c) $\frac{5}{2}$ (d) $\frac{7}{3}$ (e) $-\frac{4}{3}$ (f) $\frac{2}{5}$
   (g) $-\frac{3}{8}$ (h) -1
5. (a) $< PRQ$
   (b) $< CAB$
   (c) $< MNO$
6. (a) perpendicular (b) parallel
   (c) parallel (d) perpendicular

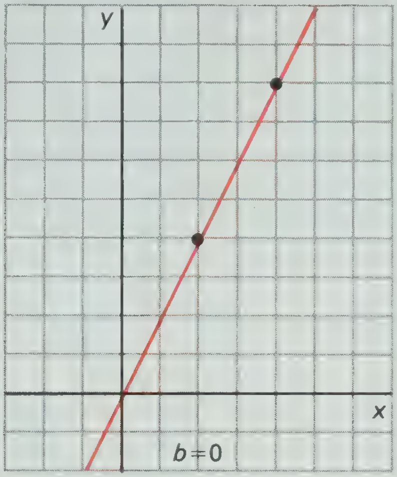
Exercise 13-5

1. $AB = 5, CD = 5, EF = 3, GH = 5, JK = 4$
2. $AB = \sqrt{29}$ $CD = \sqrt{13}$
   $EF = 4\sqrt{2}$ $GH = \sqrt{37}$
   $JK = 3\sqrt{2}$
3. (a) 5 (b) $2\sqrt{13}$ (c) 2 (d) $\sqrt{5}$ (e) $\sqrt{41}$ (f) $2\sqrt{5}$
   (g) $\sqrt{65}$ (h) $2\sqrt{26}$ (i) 4 (j) 10 (k) $\sqrt{82}$ (l) $\sqrt{109}$
4. $AB = CD = 3\sqrt{2}$
   $AD = BC = 2\sqrt{10}$
5. $PM = QM = \sqrt{29}$
6. $AM = MB = \sqrt{13}$

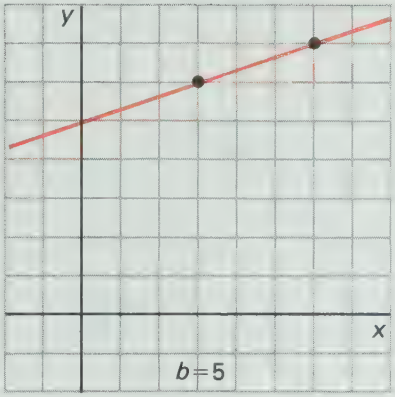
Exercise 13-6

1. (a) 0.5 (b) 2 (c) -1 (d) 11 (e) -2 (f) 0.05

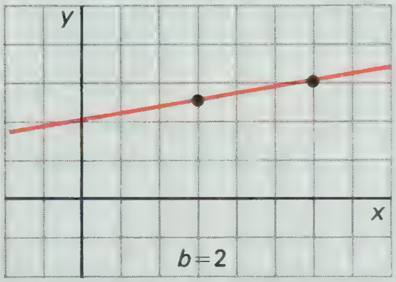
2. (a)



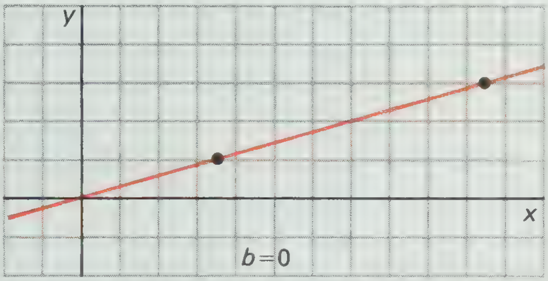
(b)



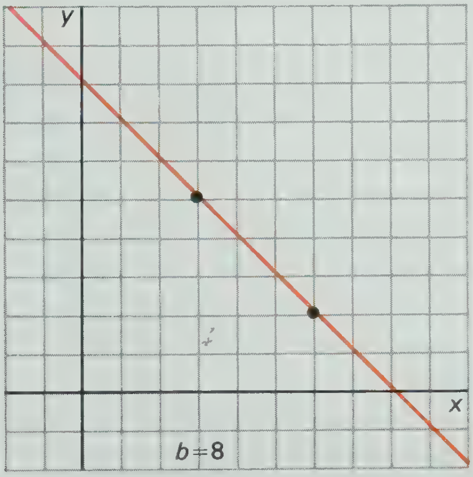
(c)



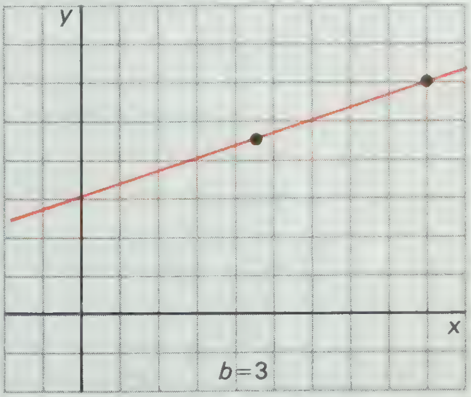
(d)



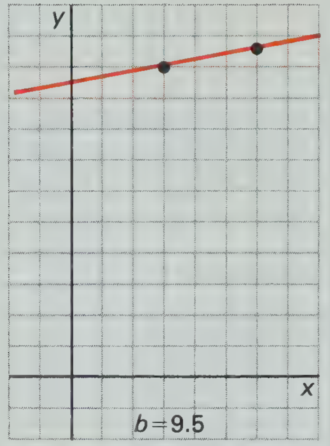
(e)



(f)



(g)



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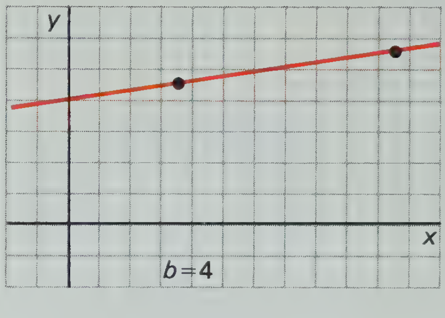
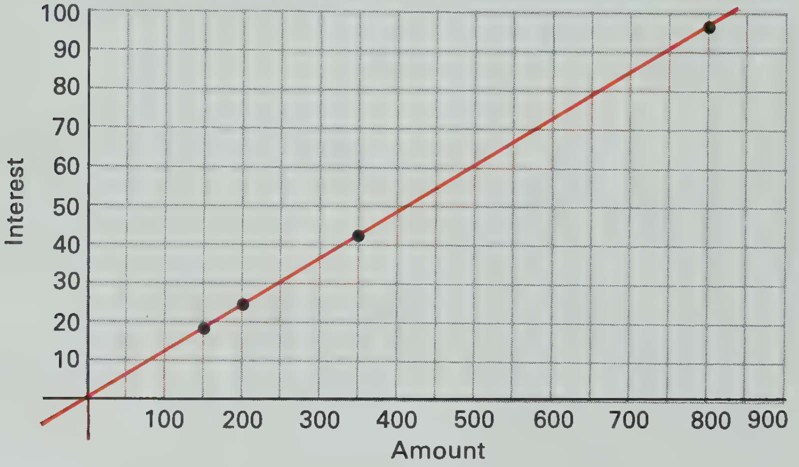
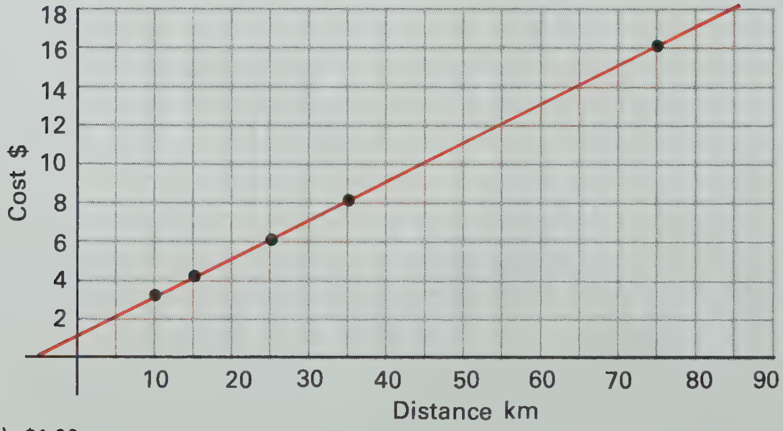


3. **(a)**



3. **(b)** $m = \frac{3}{25}$, 12% **(c)** \$20.00, \$47.50 **(d)** \$100.00, \$825.00

4. **(a)**

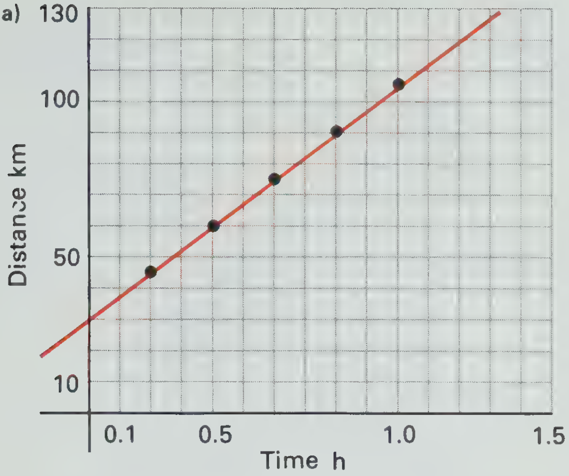


4. **(b)** \$1.00

(c) $m = 0.2$, which represents a cost of \$0.20/km.

(d) $c = 0.2d + 1$ **(e)** \$13.00

5. **(a)**

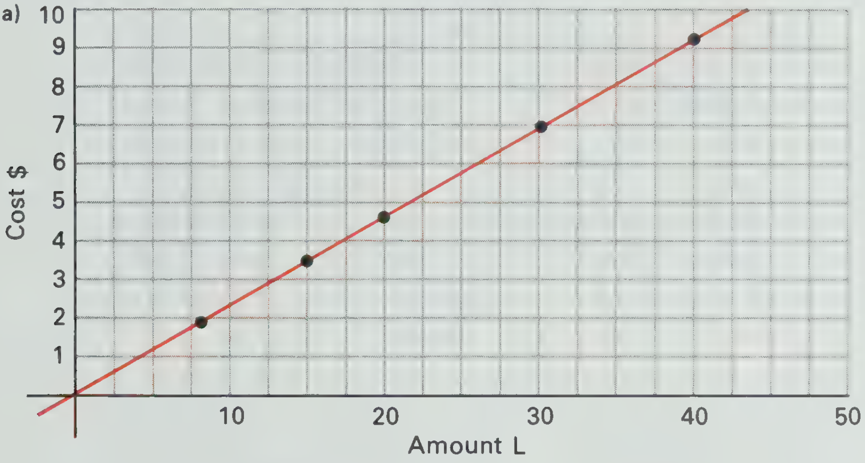


5. **(b)** 15 km

(c) $m = 75$, 75 km/h

(d) $d = 75t + b$

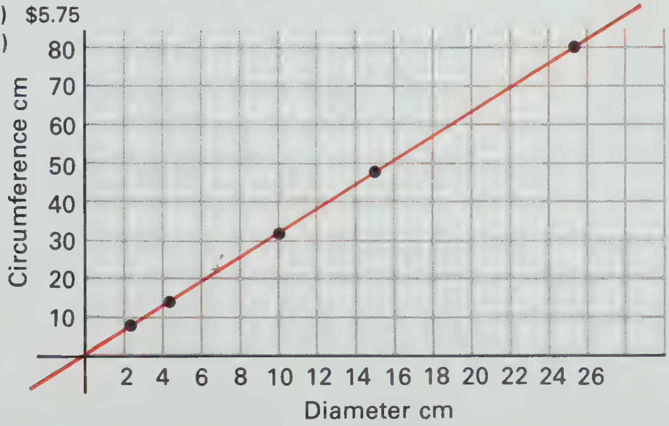
6. **(a)**



6. **(b)** $m = 0.23$. The cost is $0.23 per litre.

(c) $5.75

7. **(a)**

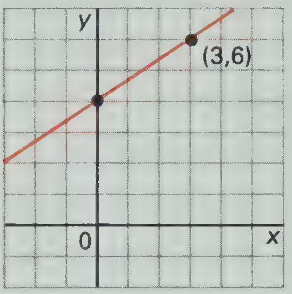


7. **(b)** $m = 3.1, \pi$

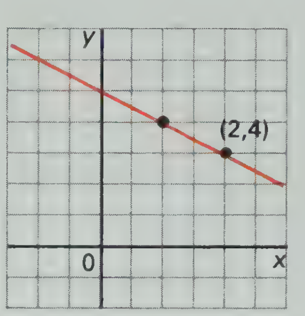
(c) $c = 3.1d$

Review Exercise

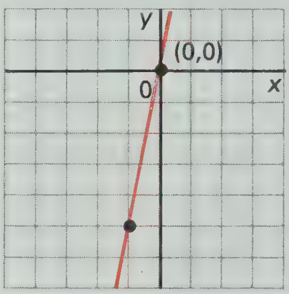
1. (a) 2 (b) 1 (c) -3 (d) $\frac{1}{10}$ (e) $-\frac{2}{5}$ (f) undefined
   (g) undefined (h) 0 (i) 0.5 (j) -1.5 (k) 10 (l) $\frac{2}{3}$
2. (a) collinear (b) not collinear
   (c) collinear (d) collinear
3. (a) 14 m (b) 7.5 m
4. (a)



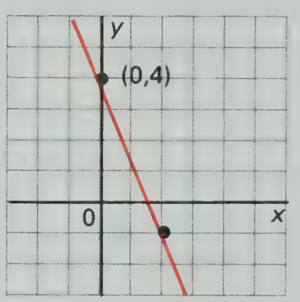
(b)



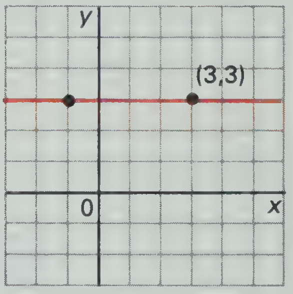
(c)



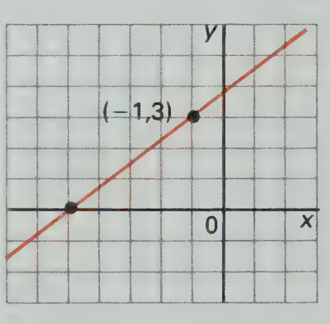
(d)



(e)



(f)



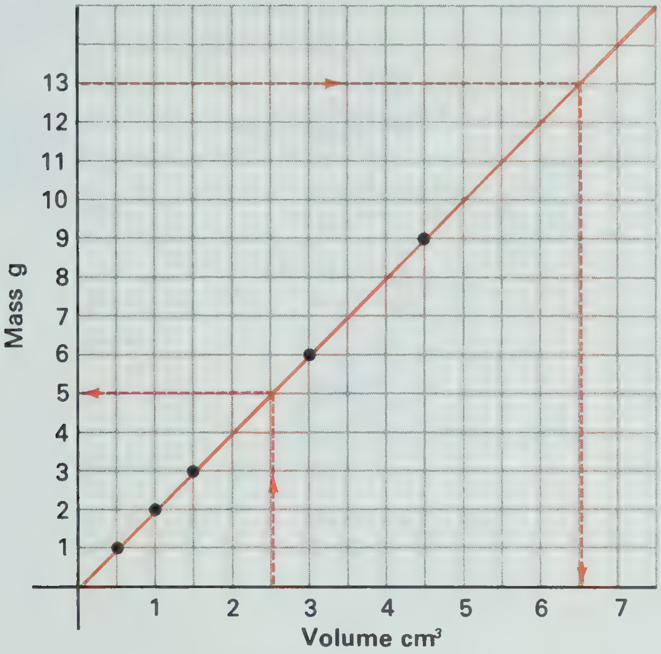
5. (a) $y = -2x + 7, m = -2, b = 7$ (b) $y = 5x - 2, m = 5, b = -2$
   (c) $y = -x + 10, m = -1, b = 10$ (d) $y = -\frac{3}{2}x + 3, m = -\frac{3}{2}, b = 3$
   (e) $y = 4x + 3, m = 4, b = 3$ (f) $y = 2x - 5, m = 2, b = -5$
   (g) $y = 5x + 2, m = 5, b = 2$ (h) $y = 3x, m = 3, b = 0$
6. (a) $y = 7$ (b) $y = 6$ (c) $y = 13$ (d) $x = 7$ (e) $x = -3$ (f) $y = 4$
   (g) $y = 5$ (h) $x = 1$

7. **(a)** parallel **(b)** perpendicular
(c) perpendicular **(d)** parallel
(e) parallel

8. **(a)** 5 **(b)** 13 **(c)** 10 **(d)** 10 **(e)** $\sqrt{2}$ **(f)** $\sqrt{58}$
(g) $3\sqrt{5}$ **(h)** $\sqrt{116}$ **(i)** $\sqrt{2}$ **(j)** $2\sqrt{5}$ **(k)** $\sqrt{65}$ **(l)** $3\sqrt{2}$

9. 30 units

10. **(a)**



(b) $m = 2$
(c) 5 g
(d) 6.5 cm^3

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ISBN 0-07-077842-6